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Tatra Mountains Mathematical Publications
DOI: 10.1515/tmmp-2016-0021
Tatra Mt. Math. Publ. **66** (2016), 73–80

A NOTE ON L-FUZZY BAGS AND THEIR EXPECTED VALUES

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ABSTRACT. A definition of L-fuzzy bags is introduced and studied. In this approach, according to the concept given by M. Delgado et al. (2009), each bag has two parts: function and summary information. Then, the definition of L-fuzzy bag expected value is introduced. In the case $L = [0, 1]$, several integral-based fuzzy bag expected values are prepared. By some examples, the new concepts are illustrated.

1. Introduction

The initial notion of bags, an alternative name for the multisets, was introduced by Yager [9] as an algebraic set-like structure where an element can appear more than once. So far, several works have been done using this new concept. Moreover, bags have been employed in practice, for instance: in flexible querying, representation of relational information, decision problem analysis, criminal career analysis, and even in such a field as biology.

However, due to some existing drawbacks in the first definition of bags [9], the necessity of a revision of this notion revealed. The proposed definition by Delgado et al. [2] has corrected these drawbacks. By some examples, they showed that the given definition of bags by Yager has some deficiencies and it was not well suited for representing and reasoning with real-world information. Then, they proposed new definitions of bags and fuzzy bags.

As it is shown in [5], the lattice of all fuzzy bags defined by Delgado et al. [2] is a complete Boolean algebra which is not compatible with the nature of fuzziness. Improving this incompatibility, in the present paper, we quote a revised definition of fuzzy bags as a special case of L-fuzzy bags based on the proposed definition of bags in [2]. Also we introduce the concept of the L-fuzzy bag expected value.

2010 Mathematics Subject Classification: 03B52, 94D05.
Keywords: bags, expected value, L-fuzzy bags.
Research of the third author was supported by the grant VEGA 1/0420/15.

2. Preliminaries

In this section, some basic concepts which are needed in the sequel are given. For more details, see [2].

Definition 1 ([2])**.** Let P and O be two universes (sets) called "*properties*" and "*objects*", respectively. A (crisp) bag $\mathcal{B}^f$ is a pair (f, B^f), where $f\colon P \to \mathcal{P}(O)$ is a function and B^f is the following subset of $P \times \mathcal{N}_0$

$$B^f = \Big\{\Big(p, card\big(f(p)\big)\Big)\Big| \, p \in P\Big\}.$$

Here, $\mathcal{N}_0 = \{0, 1, \dots\} = \{0\} \cup \mathcal{N}$, where $\mathcal{N}$ is the set of natural numbers, $\mathcal{P}(O)$ is the power set of O, $card(X)$ is the cardinality of set X.

We will use the convention here that $card(\emptyset) = 0$.

In this characterization, a bag $\mathcal{B}^f$ consists of two parts. The first one is the function f that can be seen as an information source about the relation between objects and properties. The second part B^f is a summary of the information in f obtained by means of the count operation $card(.)$. This summary corresponds to the classical view of bags in the sense of [9].

Notation 1. We set $\mathbf{B}(P, O)$ as the set of all bags $\mathcal{B}^f = (f, B^f)$ defined in Definition 1.

Definition 2. Define $\mathcal{B}^0 = (0, B^0)$ and $\mathcal{B}^1 = (1, B^1)$, where $0(p) = \emptyset$, $1(p) = O$ for all $p \in P$, $B^0 = \big\{(p, 0), p \in P\big\}$ and $B^1 = \big\{(p, card(O)), p \in P\big\}$. Clearly, $\mathcal{B}^0, \mathcal{B}^1 \in \mathbf{B}(P, O)$.

Example 1 ([2]). Let $O = \{\text{John, Ana, Bill, Tom, Sue, Stan, Ben}\}$ and $P = \{17, 21, 27, 35\}$ be the set of objects and the set of properties, respectively. Let $f\colon P \to \mathcal{P}(O)$ be the function in Table 1 with $f(p) \subseteq O$ for all $p \in P$.

Table 1. Function: age-people.

p	17	21	27	35
$f(p)$	$\{\text{Bill, Sue}\}$	$\{\text{John, Tom, Stan}\}$	$\emptyset$	$\{\text{Ben}\}$

So, we can define bag $\mathcal{B}^f = (f, B^f)$, where $B^f = \big\{(17, 2), (21, 3), (27, 0), (35, 1)\big\}$.

Definition 3 ([2])**.** a) A bag $\mathcal{B}^f$ is a sub bag of $\mathcal{B}^g$, denoted by $\mathcal{B}^f \sqsubseteq \mathcal{B}^g$ if $f(p) \subseteq g(p)$ for all $p \in P$.

b) Two bags $\mathcal{B}^f$ and $\mathcal{B}^g$ are equal, denoted by $\mathcal{B}^f = \mathcal{B}^g$ if $\mathcal{B}^f \sqsubseteq \mathcal{B}^g$ and $\mathcal{B}^g \sqsubseteq \mathcal{B}^f$.

In the next section, we review the concept of fuzzy bags and quote some results about them. For more details see [7] and [8].

3. L-fuzzy bags

In what follows, O is the set of all objects, L is a complete lattice and $\mathcal{F}_L(O) = \{A|A : O \to L\}$ is the set of all L-fuzzy subsets of O. In the case of $L = [0, 1]$, we write $\mathcal{F}(O)$. Also $i \in I_n = \{1, 2, \dots, n\}$, where $n \in \mathcal{N}$.

Definition 4 ([8]). An L-fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}}$ is a pair $(\tilde{f}, B^{\tilde{f}})$, where $\tilde{f} : P \to \mathcal{F}_L(O)$ is a function and B^{f} is the following subset of $P \times L \times \mathcal{N}_0$

$$B^{\tilde{f}} = \Big\{(p, \delta, card(O^p_\delta))\big| \, p \in P, \delta \in L\Big\},$$

where

$$O^p_\delta = \{o \in O | \tilde{f}(p)(o) = \delta\}.$$

Obviously, a bag is a particular case of the L-fuzzy bag, where for all $p \in P$, $\tilde{f}(p)$ is a crisp subset of O. Similar to bags, the L-fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}}$ consists of two parts. The first one is the function $\tilde{f}$ that can be seen as an information source about the relation between objects and properties. The second part $B^{\tilde{f}}$ is a summary of the information in $\tilde{f}$ obtained by means of the count operation $card(.)$.

Note 1 ([8]). In the case that $L = [0, 1]$, the defined L-fuzzy bag in Definition 4 is called fuzzy bag.

Here, the concept of L-fuzzy bag is illustrated by an example.

Example 2 ([8]). Let $L = [0, 1]$, $O = \{\text{Ben, Sue, Tom, John, Stan, Bill, Kim, Ana, Sara}\}$ and $P = \{\text{young, middle age, old}\}$ is the set of some linguistic descriptions of age. Let the degrees of membership of all $o \in O$ in the set of each property $p \in P$ be given as in Table 2.

Table 2. The degrees of memberships for Example 2.

p \ o	Ben	Sue	Tom	John	Stan	Bill	Kim	Ana	Sara
young	0.7	0.2	0.4	0.0	0.7	0.4	0.2	0.7	0.1
middle age	0.3	0.8	0.7	0.3	0.3	0.7	0.8	0.3	0.5
old	0.1	0.2	0.1	0.9	0.1	0.1	0.2	0.1	0.5

So, by Definition 4, we can define fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$, where

$$\tilde{f}(\text{young}) = \left\{\frac{0.7}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.4}{\text{Tom}}, \frac{0.7}{\text{Stan}}, \frac{0.4}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.7}{\text{Ana}}, \frac{0.1}{\text{Sara}}\right\},$$

$$\tilde{f}(\text{middle age}) = \left\{\frac{0.3}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.7}{\text{Tom}}, \frac{0.3}{\text{John}}, \frac{0.3}{\text{Stan}}, \frac{0.7}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.3}{\text{Ana}}, \frac{0.5}{\text{Sara}}\right\},$$

$$\tilde{f}(\text{old}) = \left\{\frac{0.1}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.1}{\text{Tom}}, \frac{0.9}{\text{John}}, \frac{0.1}{\text{Stan}}, \frac{0.1}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.1}{\text{Ana}}, \frac{0.5}{\text{Sara}}\right\},$$

and

$$
\begin{aligned}
B^{\tilde{f}} = \big\{&(\text{young}, 0.7, 3), (\text{young}, 0.4, 2), (\text{young}, 0.2, 2), (\text{young}, 0.1, 1),\\
&(\text{middle age}, 0.8, 2), (\text{middle age}, 0.7, 2), (\text{middle age}, 0.5, 1), (\text{middle age}, 0.3, 4),\\
&(\text{old}, 0.9, 1), (\text{old}, 0.5, 1), (\text{old}, 0.2, 2), (\text{old}, 0.1, 5)\big\}.
\end{aligned}
$$

Remark 1 ([8])**.** As it can be seen, the more important part of an L-fuzzy bag is information function $\tilde{f}$. Therefore, it is possible to study the properties of L-fuzzy bags just by considering their information functions.

Notation 2 ([8])**.** We set $\tilde{\mathbf{B}}_L(P, O)$ as the set of all L-fuzzy bags $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$. Where, $\tilde{f} : P \to \mathcal{F}_L(O)$ and $B^{\tilde{f}}$ are as defined in Definition 4. Also we set $\tilde{\mathbf{B}}(P, O)$ as the set of all fuzzy bags. Clearly, $\mathbf{B}(P, O) \subseteq \tilde{\mathbf{B}}(P, O) \subseteq \tilde{\mathbf{B}}_L(P, O)$.

Here, we define intersection and union of L-fuzzy bags.

Definition 5. Let $\tilde{\mathcal{B}}^{\tilde{f}_i} \in \tilde{\mathbf{B}}_L(P_i, O_i)$ for all $i \in I_n$ be given L-fuzzy bags, $\overline{O} = \cap_{i\in I_n} O_i$ and $\overline{P} = \cap_{i\in I_n} P_i$. Then, their intersection is L-fuzzy bag

$$
\cap_{i\in I_n} \tilde{\mathcal{B}}^{\tilde{f}_i} = \Big(\cap_{i\in I_n} \tilde{f}_i, B^{\cap_{i\in I_n}\tilde{f}_i}\Big), \tag{1}
$$

where

$$
\cap_{i\in I_n} \tilde{f}_i : \overline{P} \to \mathcal{F}_L(\overline{O}) \quad \text{such that} \quad (\cap_{i\in I_n} \tilde{f}_i)(p) = \cap_{i\in I_n} \tilde{f}_i(p).
$$

Also

$$
B^{\cap_{i\in I_n}\tilde{f}_i} = \Big\{\big(p, \delta, card(O^p_\delta)\big) \,\big|\, p \in \overline{P}, \delta \in L\Big\},
$$

where

$$
O^p_\delta = \Big\{o \in \overline{O} | (\cap_{i\in I_n} \tilde{f}_i)(p)(o) = \delta\Big\}.
$$

Note that by Definition 4, $\cap_{i\in I_n} \tilde{\mathcal{B}}^{\tilde{f}_i} = \tilde{\mathcal{B}}^{\cap_{i\in I_n}\tilde{f}_i}$.

Definition 6. Let $\tilde{\mathcal{B}}^{\tilde{f}_i} \in \tilde{\mathbf{B}}_L(P_i, O_i)$ for all $i \in I_n$ be given L-fuzzy bags, $\overline{O} = \cup_{i\in I_n} O_i$ and $\overline{P} = \cup_{i\in I_n} P_i$. Then, their union is L-fuzzy bag

$$
\cup_{i\in I_n} \tilde{\mathcal{B}}^{\tilde{f}_i} = \Big(\cup_{i\in I_n} \tilde{f}_i, B^{\cup_{i\in I_n}\tilde{f}_i}\Big), \tag{2}
$$

where

$$
\cup_{i\in I_n} \tilde{f}_i : \overline{P} \to \mathcal{F}_L(\overline{O}) \quad \text{such that} \quad (\cup_{i\in I_n} \tilde{f}_i)(p) = \cup_{i\in I_n} \tilde{f}_i(p).
$$

Also

$$
B^{\cup_{i\in I_n}\tilde{f}_i} = \Big\{\big(p, \delta, card(O^p_\delta)\big) \,\big|\, p \in \overline{P}, \delta \in L\Big\},
$$

where

$$
O^p_\delta = \Big\{o \in \overline{O} \,| (\cup_{i\in I_n} \tilde{f}_i)(p)(o) = \delta\Big\}.
$$

Note that by Definition 4, $\cup_{i\in I_n} \tilde{\mathcal{B}}^{\tilde{f}_i} = \tilde{\mathcal{B}}^{\cup_{i\in I_n}\tilde{f}_i}$.

The following definition equipes the set of all L-fuzzy bags with an order.

Definition 7 ([8])**.** An L-fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}}$ is an L-fuzzy sub bag of $\tilde{\mathcal{B}}^{\tilde{g}}$, denoted by $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ if and only if $\tilde{f}(p) \tilde{\subseteq} \tilde{g}(p)$ for all $p \in P$. That means $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ if and only if for all $p \in P$, $\tilde{f}(p)$ be an L-fuzzy subset of $\tilde{g}(p)$.

Definition 8 ([8])**.** Two L-fuzzy bags $\tilde{\mathcal{B}}^{\tilde{f}}$ and $\tilde{\mathcal{B}}^{\tilde{g}}$ are equal, denoted by $\tilde{\mathcal{B}}^{\tilde{f}} \cong \tilde{\mathcal{B}}^{\tilde{g}}$ if $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ and $\tilde{\mathcal{B}}^{\tilde{g}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{f}}$ that means if $\tilde{f} = \tilde{g}$.

The next theorem gives some useful results about L-fuzzy bags.

Theorem 1 ([8])**.** *Operations $\cup$ and $\cap$ in $\tilde{\boldsymbol{B}}_L(P, O)$ satisfy the laws of idempotency, commutativity, associativity and distributivity. Moreover, $\mathcal{B}^0$ is neutral for operation $\cup$ and $\mathcal{B}^1$ is neutral for operation $\cap$.*

In the following definition, we introduce the concept of complement of an L-fuzzy bag.

Definition 9 ([8])**.** Let $\eta\colon L \to L$ be a fixed strong negation [1], this means an involutive decreasing bijection. Consider $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$. Then, the η-complement of $\tilde{\mathcal{B}}^{\tilde{f}}$ is the L-fuzzy bag $(\tilde{\mathcal{B}}^{\tilde{f}})^c = (\tilde{f}^c, B^{\tilde{f}^c})$, where $\tilde{f}^c\colon P \to \mathcal{F}_L(O)$ such that $\tilde{f}^c(p)(o) = \eta\big(\tilde{f}(p)(o)\big)$ for all $p \in P$ and $o \in O$.

Note that by Definition 4, $(\tilde{\mathcal{B}}^{\tilde{f}})^c = \tilde{\mathcal{B}}^{\tilde{f}^c}$.

Note 2 ([8])**.** In Definition 9, if $L = [0, 1]$ and η is the standard negation, $\eta(x) = 1 - x$ for all $x \in [0, 1]$ [1], then $\tilde{\mathcal{B}}^{\tilde{f}^c}$ is called complement of $\tilde{\mathcal{B}}^{\tilde{f}}$.

Example 3 ([7]). Consider the fuzzy bag of Example 2. The complement of this fuzzy bag is $\tilde{\mathcal{B}}^{\tilde{f}^c} = (\tilde{f}^c, B^{\tilde{f}^c})$, where

$$\begin{aligned}
\tilde{f}^c(\text{young}) &= \left\{\frac{0.3}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.6}{\text{Tom}}, \frac{1.0}{\text{John}}, \frac{0.3}{\text{Stan}}, \frac{0.6}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.3}{\text{Ana}}, \frac{0.9}{\text{Sara}}\right\},\\
\tilde{f}^c(\text{middle age}) &= \left\{\frac{0.7}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.3}{\text{Tom}}, \frac{0.7}{\text{John}}, \frac{0.7}{\text{Stan}}, \frac{0.3}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.7}{\text{Ana}}, \frac{0.5}{\text{Sara}}\right\},\\
\tilde{f}^c(\text{old}) &= \left\{\frac{0.9}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.9}{\text{Tom}}, \frac{0.1}{\text{John}}, \frac{0.9}{\text{Stan}}, \frac{0.9}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.9}{\text{Ana}}, \frac{0.5}{\text{Sara}}\right\},
\end{aligned}$$

and

$$\begin{aligned}
B^{\tilde{f}^c} = \big\{&(\text{young}, 1.0, 1), (\text{young}, 0.9, 1), (\text{young}, 0.8, 2), (\text{young}, 0.6, 2), (\text{young}, 0.3, 3),\\
&(\text{middle age}, 0.7, 4), (\text{middle age}, 0.5, 1), (\text{middle age}, 0.3, 2), (\text{middle age}, 0.2, 2),\\
&(\text{old}, 0.9, 5), (\text{old}, 0.8, 2), (\text{old}, 0.5, 1), (\text{old}, 0.1, 1)\big\}.
\end{aligned}$$

4. Expected value of an L-fuzzy bag event

In this section, we determine the "*size*" of a (L-fuzzy) bag via the definition of (L-fuzzy) bag expected value. To have some background of expected values, see [3]. Also please c.f. [6].

Definition 10. The bag expected value is the function $\mathrm{BEV}\colon \mathbf{B}(P,O) \to [0,1]$ with the following properties:

(i) $\mathrm{BEV}(\mathcal{B}^0) = 0$ and $\mathrm{BEV}(\mathcal{B}^1) = 1$; (boundary conditions).

(ii) $\mathrm{BEV}(\mathcal{B}^f) \leq \mathrm{BEV}(\mathcal{B}^g)$ whenever $\mathcal{B}^f \sqsubseteq \mathcal{B}^g$; (monotone non-decreasing).

Similar to the crisp case, we can have the fuzzy version.

Definition 11. An L-fuzzy bag expected value is the function LFBEV : $\tilde{\mathbf{B}}_L(P,O) \to [0,1]$ which satisfies the following conditions

(i) $\mathrm{LFBEV}(\mathcal{B}^0) = 0$ and $\mathrm{LFBEV}(\mathcal{B}^1) = 1$; (boundary conditions).

(ii) $\mathrm{LFBEV}(\tilde{\mathcal{B}}^{\tilde{f}}) \leq \mathrm{LFBEV}(\tilde{\mathcal{B}}^{\tilde{g}})$ whenever $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$; (monotone non-decreasing).

In the case of $L = [0,1]$, we have fuzzy bag expected value and we write FBEV.

Remark 2. The restriction of an L-fuzzy bag expected value to the crisp bags is bag expected value, i.e., $\mathrm{LFBEV}\,|_{\mathbf{B}(P,O)} = \mathrm{BEV}$.

Employing different measures, one can have various (L-fuzzy) bag expected values as it can be seen in the following examples.

Example 4. Considering probability measure Pr on $P \times O$, we can define the following fuzzy bag expected value

$$\mathrm{FBEV}_{Pr}(\tilde{\mathcal{B}}^{\tilde{f}}) = \sum_{\substack{p\in P\\ o\in O}} Pr\Big(\big\{(p,o)\big\}\Big) \times \tilde{f}(p)(o). \tag{3}$$

Note that equation (3) is the Lebegue integral based expected value. One may compare it to expectation of a fuzzy set as defined in [10]. In the case of crisp bag, we have

$$\mathrm{BEV}_{Pr}(\mathcal{B}^f) = \sum_{\substack{p\in P\\ o\in f(p)}} Pr\Big(\big\{(p,o)\big\}\Big). \tag{4}$$

Example 5. Considering possibility measure Π on $P \times O$, we can define the following L-fuzzy bag expected value

$$\mathrm{LFBEV}_{\Pi}(\tilde{\mathcal{B}}^{\tilde{f}}) = \bigvee_{\substack{p\in P\\ o\in O}} \Pi\Big(\big\{(p,o)\big\}\Big) \wedge \tilde{f}(p)(o). \tag{5}$$

Note that equation (5) is the Sugeno integral based expected value. In the case of crisp bag, we have

$$\mathrm{BEV}_{\Pi}(\mathcal{B}^f) = \bigvee_{\substack{p\in P\\ o\in f(p)}} \Pi\Big(\{(p,o)\}\Big).$$

Remark 3. Given a monotone measure μ on $P\times O$, one can consider any universal integral I acting on $[0,1]$, see [4], to define a fuzzy expectation $\mathrm{FBEV}_{I,\mu}(\tilde{\mathcal{B}}^{\tilde{f}}) = I(\mu, \tilde{f})$. Then, (5) is based on the Sugeno integral, but we can consider the Choquet, the Shilkret and other integrals, too.

5. Conclusion

Employing the proposed definition of bags by Delgado et al., which is improved version of Yager's one, a new definition of L-fuzzy bags has been given. Then, the new concept of L-fuzzy bag expected value has been introduced. When considering $L = [0,1]$, several integral-based fuzzy bag expected values were introduced, too.

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Received July 15, 2016

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