



SCATTERED SPACES, COMPACTIFICATIONS AND AN APPLICATION TO IMAGE CLASSIFICATION PROBLEM

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ABSTRACT. In this paper, we characterize spaces such that their one-point compactification is a scattered space. We study spaces such that their Wallman compactification is scattered and we give necessary and sufficient conditions on the subspace A to get its closure scattered. An application of scattered spaces to image classification problem is also presented.

1. Introduction

A topological space X is said to be scattered if every non-empty subset S of X contains at least one point which is isolated in S (that is, X is scattered if and only if it contains no non-empty subset which is dense-in-itself).

Compact scattered spaces have found important use in analysis and topology (see for example [10] in which S. M r o w k a, M. R a j a g o p a l a n, T. S o u n d a r a r a j a n characterized compact scattered Hausdorff spaces). On the other hand, from 2004 to 2015 K. B e l a i d et al. [1], [4], [5] studied some topological spaces and compactifications.

Section 2 is devoted to a short study of digital spaces and to a characterization of the subspace A to get its closure scattered.

Section 3 deals with the problem when the one-point compactification (resp. Wallman compactification) is a scattered space.

The purpose of Section 4 is to start a study about spaces such that their one-point compactification is a SP -scattered space. Moreover, we consider some numerical applications of scattered spaces to image classification problem in this section.

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2. Scattered subspaces and digital topology

First, recall the definition of the digital line: The digital line (called also Khalimsky line) is the set of integers \mathbb{Z} equipped by the topology \mathcal{K} , generated by $\mathcal{G} = \{\{2n-1, 2n, 2n+1\} \mid n \in \mathbb{Z}\}$ [8].

PROPOSITION 2.1. *The digital line is a scattered space.*

Proof. Let S be a subset of \mathbb{Z} . If there exists an odd integer $k \in S$, then $\{k\}$ is an open set of \mathbb{Z} ; so that k is an isolated point in S . Suppose now, all elements of S are even. Hence for $k \in S$, $\{k\}$ is an open set of S . Thus k is an isolated point in S . Therefore the digital line is a scattered space. \square

Now, we recall the definition of the digital plane (also called the Khalimsky plane).

DEFINITION 2.2. The Khalimsky topology on the set \mathbb{Z}^2 is the topology \mathcal{K} generated by: For any $z = (x, y) \in \mathbb{Z}^2$

- if x, y are even, $\{(x-i, y+k) \mid i, k \in \{-1, 0, 1\}\}$,
- if x is even and y is odd $\{(x+k, y) \mid k \in \{-1, 1\}\}$,
- if x is odd and y is even $\{(x, y+k) \mid k \in \{-1, 1\}\}$,
- $\{z\}$ otherwise.

The set \mathbb{Z}^2 equipped with the Khalimsky topology is called the digital plane.

PROPOSITION 2.3. *The digital plane is a scattered space.*

Proof. Let S be a subset of \mathbb{Z}^2 . Suppose that there exists $z \in S$ such that $z = (x, y)$ where x and y are odd. Then $\{z\}$ is an open set of \mathbb{Z}^2 ; so that z is an isolated point in S . Suppose that S does not contain such an element and suppose that there exists $z \in S$ such that $z = (x, y)$ with either x odd and y even or x even and y odd. Hence $\{z\}$ is an open set of S ; so that z is an isolated point in S . Finally, if every element $z = (x, y)$ of S with x, y is even, then $\{z\}$ is an open set of S ; so that z is an isolated point in S . \square

DEFINITION 2.4. Let X be a topological space. A subset S of X is called a scattered subset of X if S considered as a topological space is scattered.

The following result is an immediate consequence of the fact that an isolated point in the closure of a subset N of a topological space X is an element of N ; so that it is an isolated point of N .

PROPOSITION 2.5. *Let X be a topological space and $N \subseteq X$. If \overline{N} is scattered, then N is scattered.*

SCATTERED SPACES

EXAMPLE 2.6. The closure of a scattered subset is not necessary scattered. Let (X_1, T_1) be the digital line and X_2 be the set of integers. Let Y be the disjoint union of X_1 and X_2 equip Y with the following topology:

$$T = \{Y\} \cup \{O \mid O \in T_1\} \cup \{X_1 \cup (\downarrow x) \mid x \in X_2\},$$

where $(\downarrow x) = \{y \in X_2 \mid y \leq x\}$.

Let $N = X_1$. Then N is a scattered set. It is immediate that $\overline{N} = Y$. Let $S = X_2$. Since X_2 has no isolated points, \overline{N} is not scattered.

PROPOSITION 2.7. *Let X be a T_0 -space and N be a subset of X . Then \overline{N} is scattered if and only if N and $\overline{N} - N$ are scattered.*

Proof. It is clear that if \overline{N} is scattered then N and $\overline{N} - N$ are scattered.

Suppose now that N and $\overline{N} - N$ are scattered and let S be a subset of \overline{N} . We consider two cases:

- Case 1: $S \subseteq N$ or $S \subseteq \overline{N}$. Then there exists an element z of S and an open set O of X such that $O \cap S = \{z\}$.
- Case 2: $S \cap N \neq \emptyset$ and $S \cap \overline{N} \neq \emptyset$. Since N is scattered and $S \cap N \neq \emptyset$, there exists an element z of $S \cap N$ and an open set O of X such that $O \cap (S \cap N) = \{z\}$. Suppose that $(O \cap S) \cap (\overline{N} - N) \neq \emptyset$. Since $\overline{N} - N$ is scattered, there exists an element x of $(O \cap S) \cap (\overline{N} - N)$ and an open set U of X such that $U \cap ((O \cap S) \cap (\overline{N} - N)) = \{x\}$. Hence either $S \cap U = \{x\}$ or $S \cap U = \{x, z\}$. It is immediate that if $S \cap U = \{x\}$ then x is an isolated point of S . Suppose that $S \cap U = \{x, z\}$. Since X is a T_0 -space and $x \in \overline{N} - N$, there exists an open set V of X such that $z \in V$ and $x \notin V$. Hence $(V \cap O \cap U) \cap S = \{z\}$. Thus S is scattered. \square

PROPOSITION 2.8. *Let X be a T_0 -space and S_1, S_2 be two scattered subspaces of X . Then the following statements hold:*

- (1) $S_1 \cap S_2$ is a scattered subset of X .
- (2) $S_1 \cup S_2$ is a scattered subset of X .

Proof. We have,

- (1) Straightforward.
- (2) Since S_1 is a scattered subspace of X , there exists $x \in S_1$ and an open neighborhood O of x such that $S_1 \cap O = \{x\}$. If $O \cap S_2 = \emptyset$, then $O \cap (S_1 \cup S_2) = \{x\}$; so that x is an isolated point of $S_1 \cup S_2$.

Suppose that $O \cap S_2 \neq \emptyset$. Since $O \cap S_2$ is a subset of the scattered subspace S_2 of X , there exists $y \in O \cap S_2$ and an open neighborhood V of y such that $(O \cap S_2) \cap V = \{y\}$. Hence $O \cap V = \{y\}$ or $O \cap V = \{x, y\}$. If $O \cap V = \{y\}$, then y is an isolated point of $S_1 \cup S_2$. Since X is a T_0 -space,

there exists an open set W of X such that $x \in W$ and $y \notin W$ or $y \in W$ and $x \notin W$. Hence $(O \cap V \cap W) \cap (S_1 \cup S_2) = \{x\}$ or $(O \cap V \cap W) \cap (S_1 \cup S_2) = \{y\}$. Thus $S_1 \cup S_2$ has an isolated point. Therefore $S_1 \cup S_2$ is a scattered subspace of X . \square

COROLLARY 2.9. *Let X be a topological space and $\{S_i \mid i \in I\}$ be a finite collection of scattered subspaces of X . Then $\cup_{i \in I} S_i$ is a scattered subspace of X .*

EXAMPLE 2.10. The assumption that the collection is finite in the Corollary 2.9 is essential, in fact, the rational topological space \mathbb{Q} is an infinite union of singletons; so that \mathbb{Q} is an infinite union of scattered subspaces. It is immediate that \mathbb{Q} is not a scattered space.

3. Scattered spaces and compactifications

First, recall that a compactification of a topological space X is a compact topological space $K(X)$ such that the natural embedding $i : X \rightarrow K(X)$ is continuous and the subspace $i(X)$ is a dense subspace of $K(X)$. Hence the topological space X can be identified with $i(X)$. Thus X is considered as a dense subset of $K(X)$.

Remarks 3.1. The following remarks are an immediate consequence of the definition of scattered spaces and the construction of compactification:

- (1) Let $K(X)$ be a compactification of a topological space X . If $K(X)$ is scattered then X is also a scattered space.
- (2) Let X be a scattered topological space and A be a subset of X . Then the subspace A is also scattered.

The following proposition is an immediate consequence of the Proposition 2.7.

PROPOSITION 3.2. *Let X be a topological space such that its compactification $K(X)$ is a T_0 -space. Then $K(X)$ is scattered if and only if X and $K(X) - X$ are scattered.*

The most known compactification of a topological space is the one-point compactification (also called, the Alexandroff compactification). The one-point compactification is defined as follows: let (X, T) be a non-compact topological space and $\infty \notin X$. Set $\tilde{X} = X \cup \{\infty\}$. We equip \tilde{X} with the topology \tilde{T} whose elements are open subsets of X and subsets U of \tilde{X} such that $\tilde{X} - U$ is a closed compact subset of X . The space with underlying set \tilde{X} and topology \tilde{T} is called the one-point compactification of X . Remark that if X is a compact topological space then \tilde{X} is not a compactification of the space X and it is called the Alexandroff extension of X [7].

The following result gives a complete characterization of a space such that its one-point compactification is a scattered space. It is an immediate consequence of Proposition 3.2.

PROPOSITION 3.3. *Let X be a non-compact T_0 -space. Then the one-point compactification \tilde{X} of X is a scattered space if and only if X is a scattered space.*

Now let us recall, the Wallman compactification construction introduced in 1938 for T_1 -spaces [11]: Let (X, T) be a T_1 -topological space.

A closed filter on X is a collection \mathcal{F} of non-empty closed sets with properties:

- (1) \mathcal{F} is closed under finite intersections,
- (2) $P_1 \in \mathcal{F}, P_1 \subseteq P_2$ implies $P_2 \in \mathcal{F}$.

Let wX be the collection of maximal closed filters on X . Let D be a closed set of X . Set $D^* = \{\mathcal{F} \in wX \mid D \in \mathcal{F}\}$. Then the collection $\mathcal{C} = \{D^* \mid D \text{ is a closed set of } X\}$ is a base for the closed sets of a topology T_w on wX .

For an open set U of X , we define $U^* = \{\mathcal{F} \in wX \mid F \subseteq U \text{ for some } F \in \mathcal{F}\}$. It is easily seen that the collection $\{U^* \mid U \text{ is an open set of } X\}$ is a base for open sets of the topology T_w . The space with underlying set wX and topology T_w is called the Wallman compactification of X .

Recall that Kovar [9] has proved that the Wallman compactification remainder (that is, $wX - X$) is finite if and only if there exists $n \in \mathbb{N}$ such that the supremum of cardinalities of every collection of disjoint closed non-compact sets is lower than n .

PROPOSITION 3.4. *Let X be a T_1 -space such that its Wallman compactification remainder is finite. Then the Wallman compactification wX is a scattered space if and only if X is a scattered space.*

Proof. We have,

- (1) Necessary condition: immediate.
- (2) Sufficient condition: Since $wX - X$ is finite and wX is a T_1 -space, $wX - X$ is a discrete space. So that $wX - X$ is scattered. That wX is a scattered space follows immediately from Proposition 2.7. □

EXAMPLE 3.5. The assumption that the Wallman compactification remainder is finite in the Proposition 3.4 is essential. In fact: Let \mathbb{N} be the discrete space of natural numbers. It is immediate that \mathbb{N} is a scattered space and $w\mathbb{N} - \mathbb{N}$ is infinite. Now, since \mathbb{N} is normal, $w\mathbb{N}$ coincides with the Stone Cech compactification $\beta\mathbb{N}$ of \mathbb{N} . Hence $w\mathbb{N}$ is not a scattered space.

The following lemma is an immediate consequence of the Wallman compactification construction.

LEMMA 3.6. *Let X be a T_1 -space and $\mathcal{F} \in wX$. If C is a closed set of X such that $F \cap C \neq \emptyset$, for each $F \in \mathcal{F}$, then $C \in \mathcal{F}$.*

PROPOSITION 3.7. *Let X be a T_1 -space such that its Wallman compactification is a scattered space. If \mathcal{C} is a collection of disjoint closed non-compact sets of X , then there exists an open set O of X and $F \in \mathcal{C}$ such that:*

- (1) *There exists a non-compact closed set G of X such that $G \subseteq F \cap O$.*
- (2) *For each closed non-compact set $K \subseteq C$ with $C \in \mathcal{C} - \{F\}$, $K \not\subseteq O$.*

Proof. We have,

- (1) Let $S = \{\mathcal{F} \in wX \mid \exists C \in \mathcal{C}, C \in \mathcal{F}\}$. Since wX is scattered, there exists $\mathcal{F} \in S$ such that \mathcal{F} is an isolated point in S . Hence there exists an open set O of X such that $O^* \cap S = \{\mathcal{F}\}$. Thus $\mathcal{F} \in O^*$; so that there exists $G \in \mathcal{F}$ such that $G \subseteq O$. Since $\mathcal{F} \in S$, let $F \in \mathcal{C}$ such that $F \in \mathcal{F}$. Then $G \subseteq F \cap O$.
- (2) Let $C \in \mathcal{C} - \{F\}$ and K be a closed non-compact set of X such that $K \subseteq C$. Then there exists $\mathcal{K} \in S$ such that $K \in \mathcal{K}$. It is immediate that $\mathcal{K} \neq \mathcal{F}$, since elements of \mathcal{C} are disjoint; so that $\mathcal{K} \notin O^*$. Therefore $K \not\subseteq O$. \square

4. Applications of scattered spaces to image classification

The image classification problem, which involves assigning a label to each pixel of the observed image, is considered as a basic problem in mathematical imaging and is very useful in medical image processing. The image classification problem is to find a partition of the treated image into some regions such that each partition represents a class, that means a set of pixels with the same label. In this work, the label is chosen to be the grey level intensity. Moreover, assuming that both the number and parameters of these classes are known, we solve the problem of a supervised classification by grouping pixels having similar grey level intensity. The solution we obtain is a compound of some regions separated by boundaries. The classes are considered as phases separated by interfaces boundaries.

In this part, we are concerned with the problem of classifying a given original image I_0 according to n predefined grey level intensities $C_i, 1 \leq i \leq n$. On the other hand, we mention that if X is the domain on which I_0 is defined (an open bounded domain of \mathbb{R}^2), then I_0 is represented by an observed data function $I_0 : X \rightarrow \mathbb{R}$ (the grey level intensity). The mathematical formulation of classifying I_0 using n predefined classes $C_i, 1 \leq i \leq n$, such that the grey level intensity is chosen as a classifier can be written as: Find a family of open sets $\{O_i \mid i = 1, \dots, n\}$ verifying $O_i \cap_{i \neq j} O_j = \emptyset$ and $X = \cup_{i=1}^n O_i \cup \Gamma_i$, where

$$\Gamma_i = \partial O_i \cap X. \quad (4.1)$$

SCATTERED SPACES

Let us note that Γ_i is the boundary of the region O_i except the common points with ∂X . Moreover, $\Gamma_i = \cup_{i \neq j} O_{ij}$, where O_{ij} represents the interface between O_i and O_j . We have,

$$O_{ij} = O_{ji} = \Gamma_i \cap \Gamma_j \cap X, \quad \text{for all } i \neq j. \quad (4.2)$$

To more clarify graphical representations of equations (4.1) and (4.2), we give an example of a partition of X in Fig. 1.

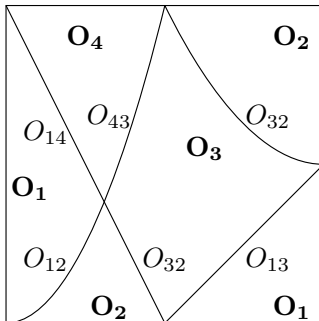


FIGURE 1. An example of a partition of X .

Finally, by the concept of a partition of X given above, the image classification process consists in finding a partition $\{O_i \mid i = 1, \dots, n\}$ of X such that I_0 is close to c_i in O_i . The classified image I will be defined by

$$I(x) = c_i \quad \text{for all } x \in O_i. \quad (4.3)$$

A very simple way to realize such a classification is to apply the closest class algorithm that consists of creating clusters where elements within each cluster are similar, according to a predefined classifier. This classification process allows to select samples for each chosen class. Suppose for example that we aim to classify the given image I_0 according to the grey level intensity, by considering only two classes: $c_1 = 0$ (black) and $c_2 = 255$ (white). Then, the classified image is given by

$$I(x) = c, \quad x \in X, \quad (4.4)$$

such that

$$c(x) = \begin{cases} c_1, & O_1 \\ c_2, & X - \overline{O_1} \end{cases} \quad (4.5)$$

To better illustrate this process, we give in Fig. 2(a) an initial scanner image of a human head. Fig. 2(b) shows the classified image using the closest class algorithm.

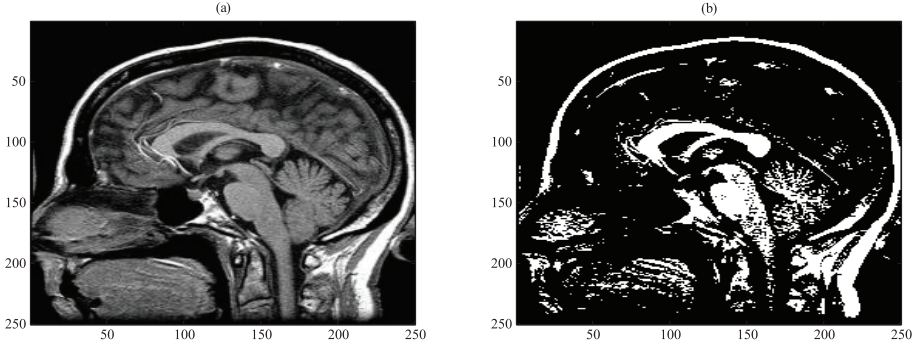


FIGURE 2. The initial image (a), the classified image using 2 classes $c_1 = 0$, $c_2 = 255$.

In order to generalize this process to n classes with $n > 2$, one can easily consider the equation (4.4), such that

$$c(x) = \begin{cases} c_1, & O_1, \\ \vdots & \\ c_{n-1}, & O_{n-1}, \\ c_n, & X - \cup_{i=1}^{n-1} \overline{O_i}, \end{cases} \quad (4.6)$$

where O_i represents the subset of pixels that should be reassigned to the class c_i . The unknowns are then the open subsets O_i that constitute the partition of X . The obtained result shows a classified image I that is close to the original one I_0 .

From a numerical point of view, the image I_0 is defined by a finite element formulation. The domain is considered as a square and it is decomposed in a grid of pixels. The classification process is interpreted by an iterative process as follows:

- Input: initial image I_0 .
- Compute isolated points of $I_0 \implies \mathcal{I}(I_0)$.
- For $i = 1, \dots, k$.
 - Set $I_i = I_{i-1} - \mathcal{I}(I_{i-1})$.
 - Compute the isolated point of $I_i \implies \mathcal{I}(I_i)$.
 - Set $i = i + 1$.
- Output: classified image $I = \cup_i I_i$.

One should note that if X is scattered then this iterative process reaches the empty set. This final issue in this case gives an over-classification problem which clearly implies a non-regularized classification result. This represents a major drawback. In fact, an acceptable classification result is based on a regular and

homogeneous partition. A regular partition means that the union of all interfaces between two different subsets: $\Gamma_i = \cup_{i \neq j} O_{ij}$ is of minimal length. An homogeneous partition means that in each set O_i the grey level intensity is close to c_i in the L^2 norm sense. To overcome this inconvenience, regularization techniques should be used. To better visualize this increasing dispersion of classes, we give in Fig. 3 numerical tests showing the classification process by considering respectively the classes $\{0, 50, 100, 150, 200, 255\}$ and then 256 classes such that all the grey level intensities from the black color ($c_0 = 0$) to the white color ($c_{255} = 1$) are considered. We see that Fig. 3(b) for which all intensities are considered is similar to the original image I_0 . This implies naturally an over-classification result and it is not accepted as a classification result. This over-classification is an immediate consequence of the fact that X is a scattered space.

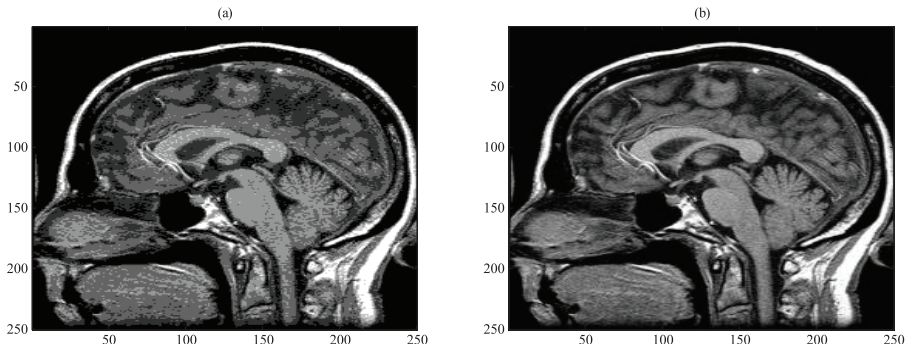


FIGURE 3. The classified image with the classes $\{0, 50, 100, 150, 200, 255\}$ (a), the final result of classification using the concept of a scattered space (b).

To avoid the result of an over-classification process obtained using the concept of scattered spaces, we can consider two different options:

- The first option is to regularize the classification process. The main goal of this regularization is to obtain smoother contours. Various methods are proposed in the literature in this context. The classical techniques and the most used ones are based on variational approaches. We refer the reader for example to [2], [12] for the use of such techniques. The principle is to minimize the following cost function

$$g(O) = \sum_{i=1}^n \int_{O_i} (I_0(x) - c_i)^2 dx, \quad (4.7)$$

where $O = (O_1, O_2, \dots, O_n)$ is a partition of X . This function g measures the root mean square difference between the original image and the classified image. The principle of minimization consists in finding for each $x \in X$, the argument of the minimum $i_x = \operatorname{argmin}\{|I_0(x) - c_i| : i = 1, \dots, n\}$ and add x to the subset O_{i_x} . In that way, each pixel of the original image is assigned in the classified image to its closest class. In order to obtain regularized results for the classification problem, other techniques based on the topological gradient have been used, [3].

- Since scattered spaces reaches automatically to an over-classification result, then we propose to study the case of more general spaces called the SP -scattered spaces, [6]. Let us first define such spaces.

DEFINITION 4.1. Let X be a topological space. An element x of X is said to be a P point if the set of open neighborhoods of x is closed under countable intersection.

DEFINITION 4.2. Let A be non-empty subset of X and let $\mathcal{P}(A)$ be the set of P points of A . X is said to be SP -scattered if the interior of $\mathcal{P}(A)$ is non-empty for all $A \subseteq X$.

In fact, given a set of data points chosen according to their grey level intensity as a classifier, one should group clusters of points such that points in each cluster set are similar to each other. This similarity reaches to an automatic regularization of the classification result in comparison with that obtained by the scattered space concept. The numerical results of such regularization method are shown in Fig. 4. One should compare between the non-regularized image given by the Fig. 4(a) and that shown by Fig. 4(b) which presents smoother contours.

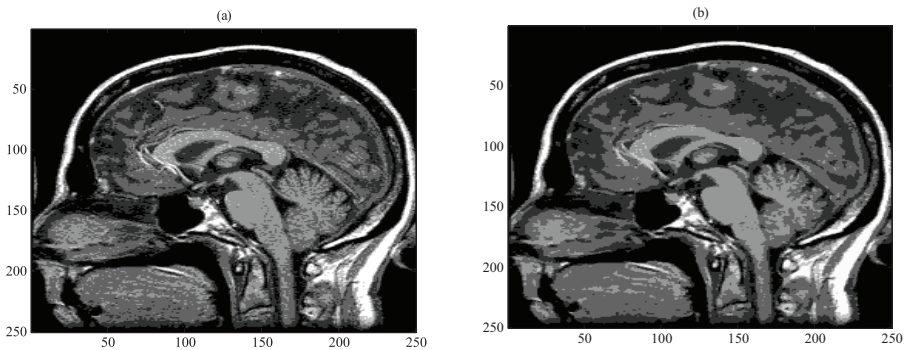


FIGURE 4. The non-regularized classified image (a), the final regularized result of classification (b).

As the SP -scattered spaces represent a remarkable advantage in classification and their use can be an alternative way for the regularization process, therefore

we propose in this last part to explore some properties of these spaces. More precisely, we give in the following proposition some properties of spaces such that their one-point compactification is the SP -scattered space.

PROPOSITION 4.3. *Let X be a non-compact topological space and \tilde{X} its one-point compactification. Then the following statements are equivalent:*

- (1) $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq \tilde{X}$, with $A \neq \emptyset$.
- (2) X has the following properties:
 - $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq X$, with $A \neq \emptyset$.
 - The collection of compact closed sets of X is closed under countable union.

Proof. We have,

(1) \implies (2): That $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq X$, is straightforward. Now, let \mathcal{C} be countable collection of closed sets X . Set $A = \tilde{X} - \cup_{C \in \mathcal{C}} C$. Since $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq \tilde{X}$, there exists $a \in A$ such that a is a P point. The set $\mathcal{U} = \{\tilde{X} - C \mid C \in \mathcal{C}\}$. Then \mathcal{U} is a countable collection of open neighborhoods of a . Hence $A = \cap_{C \in \mathcal{C}} (\tilde{X} - C)$ is an open set of \tilde{X} that contains ∞ . Thus $\cup_{C \in \mathcal{C}} C$ is a compact closed set of X .

(2) \implies (1): Let A be a subset of \tilde{X} . We consider two cases:

- Case 1: $A = \{\infty\}$. Since the collection of compact closed sets of X is closed under countable union, ∞ is a P point. So that $\mathcal{P}(\{\infty\}) \neq \emptyset$.
- Case 2: $A - \{\infty\} \neq \emptyset$. Hence $A - \{\infty\} \subseteq X$. Since $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq X$, $\mathcal{P}(A - \{\infty\}) \neq \emptyset$. Thus there exists $a \in A - \{\infty\}$ such that the set of open neighborhoods of a in X is closed under finite countable intersection. Let \mathcal{U} be a countable collection of open neighborhoods of a in \tilde{X} . We have two cases:

Case i: There exists $U \in \mathcal{U}$ such that $\infty \notin U$. Then $\mathcal{U}_1 = \{U - \{\infty\} \mid U \in \mathcal{U}\}$ is a countable collection of neighborhoods of a in X . Since $\mathcal{P}(A) \neq \emptyset$, for all $A \subseteq X$, $\cap_{U \in \mathcal{U}_1} (U - \{\infty\})$ is an open neighborhood of a . So that $\cap(U : U \in \mathcal{U})$ is an open neighborhood of a , since $\cap(U : U \in \mathcal{U}) = \cap(U - \{\infty\} : U \in \mathcal{U})$.

Case ii: For every $U \in \mathcal{U}$, $\infty \in U$. Hence $\mathcal{U}_1 = \{U - \{\infty\} \mid U \in \mathcal{U}\}$ is a countable collection of neighborhoods of a in X . Since a is a P point, $\cap_{U \in \mathcal{U}_1} (U - \{\infty\})$ is an open neighborhood of a . Since $X - \cap_{U \in \mathcal{U}_1} (U - \{\infty\}) = \cup_{U \in \mathcal{U}_1} (X - (U - \{\infty\}))$ and $\{(X - (U - \{\infty\})) \mid U \in \mathcal{U}_1\}$ is countable collection of compact closed sets of X , $\cup_{U \in \mathcal{U}_1} (X - (U - \{\infty\}))$ is a compact closed set of X . Thus $\cap_{U \in \mathcal{U}_1} (U - \{\infty\}) = \tilde{X} - \cup_{U \in \mathcal{U}_1} (X - (U - \{\infty\}))$ is an open neighborhood of a . □

The previous result incites us to ask the following questions.

Questions. Let X be a space.

- (1) When is the one-point compactification \tilde{X} of X , SP -scattered?
- (2) When is the Wallman compactification wX of X , SP -scattered?

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