Inflation in Poland under State-Dependent Pricing[1]

*Paweł BARANOWSKI – Mariusz GÓRAJSKI – Maciej MALACZEWSKI – Grzegorz SZAFRAŃSKI**

Abstract

We investigate the short-term dynamics of the Polish economy by means of a small-scale DSGE model with stochastic menu costs. We compare macroeconomic evidence of price rigidity in a model with the state-dependent Phillips curve to a benchmark model with a conventional time-dependent price stickiness. With a moderate 2.3% upper boundary on menu costs the estimated state-dependent pricing model for Poland indicates a median duration of prices about 14 months, whereas the same measure of price stickiness in the time-dependent pricing model is 3 months shorter. The result from the state-dependent pricing model estimated from macro data is closer to, both, micro-price evidence, and surveys on frequency of price changes in Poland. The difference is explained by a selection effect being present in the model with state-dependent price stickiness, only. It yields more intense and impact price adjustment after a monetary policy shock.

Keywords: *state-dependent price stickiness, Bayesian estimation, menu costs, Phillips curve, New-Keynesian DSGE*

JEL Classification: C51, E31, E32, E52

1. Introduction

New-Keynesian dynamic stochastic general equilibrium (DSGE) models are prominent tools for analysing short-term deviations of the economy from its teady state (see Woodford, 2003; Smets and Wouters, 2003; Gali, 2008). The vast

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majority of DSGE models incorporates the Calvo (1983) time-dependent price stickiness. In this setting firms receive an opportunity to reset a price in every period, with a constant probability. Under this assumption a New-Keynesian Phillips curve describes a short-term relationship between inflation and output gap.

Despite its huge popularity, the Calvo price setting unrealistically assumes that the timing of the price decisions is exogenous. It results in a constant average frequency of price adjustment across firms and time, which is inconsistent with the microeconomic evidence (e.g. Dhyne et al., 2006; Klenow and Kryvtsov, 2008; Midrigan, 2010). Moreover, it is argued that the Calvo pricing alone is not able to reproduce persistent pattern in inflation which is widely observed in the data. Thus in empirical studies this DSGE framework is enhanced with backward-looking non-optimising firms (Gali and Gertler, 1999) or dynamic indexation (Christiano, Eichenbauma and Evans, 2005). These extensions of a purely forward-looking firms stand behind the derivation of a hybrid version of New-Keynesian Phillips curve (hybrid NKPC) which is a base of many empirical studies for Central European economies (e.g. Basarac, Skrabic and Soric, 2011; Vašíček, 2011).

An alternative explanation of price rigidity in a New-Keynesian paradigm relies on introducing menu costs into price-setting decisions of firms. Even relatively small menu costs discourage firms from frequent price adjustments inducing considerable price stickiness. Hence, the frequency of price adjustments is state-dependent i.e. dependent on shocks and current prices of all representative firms. The difference from the Calvo approach consists of both varying share of the firms adjusting the prices and their non-random selection. The 'selection effect' (i.e. the bigger propensity for price adjustment among firms with the prices farther away from their optimal level) in the state-dependent pricing models, considerably complicates the derivation of the Phillips curve. In some of the state-dependent pricing models the 'selection effect' may also have some consequences to the degree of money non-neutrality (e.g. Caplin and Spulber, 1987; Golosov and Lucas, 2007).

Only few papers take DSGE models with state-dependent pricing to the macroeconomic data. Usually, authors calibrate the parameters to meet the microeconomic evidence (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Landry, 2010) or they estimate hazard functions of price adjustment from macroeconomic data (Sheedy 2010). As the evidence from micro-level datasets in Poland is scarce, we focus on a closed-form aggregate inflation equation derived in the model of Dotsey, King and Wolman (1999; DKW model afterwards) by Bakhshi, Khan and Rudolf (2007). They expand state-dependent Phillips curve (henceforth: SDPC) around a positive steady-state inflation. The authors after a series of exercises on a simulated data claim that additional terms in SDPC (i.e. expected output gaps, as well as expected and lagged inflation – cf. (18) in Annex) offer empirical explanation of intrinsic persistence in inflation. The paper of Bakhshi, Khan and Rudolf (2007), although it has not been challenged with any empirical data, is closely related to our paper being a theoretical background. The theoretical contribution of our paper consists of enhancing SDPC specification with external habit persistence. The main goal of this paper is to estimate the DSGE model with Phillips curve motivated by state-dependent pricing mechanism of DKW for the Polish economy and to answer the question whether a menu cost approach is in line with the evidence of price stickiness observed from micro-level data.

To this end, we analyse short-term dynamics of Polish economy using small--scale closed-economy New-Keynesian DSGE model. We compare implications of the estimated DSGE model with a state-dependent Phillips curve to the benchmark model with time-dependent pricing i.e. with hybrid New-Keynesian Phillips curve of Gali and Gertler (1999). To replicate a short-term persistence in inflation and output both models include other sources of economic inertia (i.e. habit persistence in consumption, interest rate smoothing in a Taylor-type rule, see Taylor, 1993), which are routinely included in the empirical DSGE models for Poland (Baranowski and Szafrański, 2012; Torój and Konopczak, 2012; Krajewski, 2015). We estimate both models with Bayesian techniques and compare implied distributions of price vintages, degrees of price stickiness (expressed by average price durations), as well as parameters in Phillips curve equations, and impulse responses to macroeconomic shocks. We are interested whether the macroeconomic illustration of transmission mechanism and the assessment of microeconomic price rigidity depend on the choice of pricing mechanism. To the best of our knowledge this paper offers one of the first Bayesian estimation of the DSGE model with DKW pricing.

The structure of the paper is as follows. In section II we describe the state-dependent pricing mechanism of DKW and we specify the SDPC to be estimated on Polish data in a three-equation DSGE framework. Finally, in section III we compare the implications of the estimated state-dependent pricing model and the benchmark Calvo model in terms of: distribution of firms across price vintages, mean of price duration, parameters in Phillips curve equations, and impulse responses to macroeconomic shocks.

2. The State Dependent Pricing DSGE Model

2.1. State Dependent Pricing Mechanism

The DKW model of pricing mechanism introduces a stochastic menu cost as a source of price rigidity. As in standard DSGE models, continuum of firms indexed by $i \in [0; 1]$ produces differentiated final goods and set their prices to maximize expected discounted profits. Each firm faces different stochastic menu costs, which are calculated in terms of the amount of labour necessary to adjust a price. Menu costs are treated as if they were independent (across time and firms) realizations of a continuous random variable, ξ_t, with cumulative distribution function $G(x) = c_1 + c_2 \cdot tan(c_3 \cdot x + c_4)$, $x \in [0, B]$ (see Figure 1). Economically, *B* as an upper bound of menu cost distribution represents an opportunity cost of price adjustment which discourages firms from changing the price every period. A firm resets the price when an expected marginal revenue from changing the price exceeds a realization of menu cost (ξ_t). A fraction of firms, drawing menu costs below a given threshold, sets a new price in a period *t*, P_t^*, that maximizes its profits (see (11) in Annex). The firms with relatively high menu costs leave their prices unaltered. In a consequence, firms are assigned to different groups ('price vintages') with price changed *j* periods ago $(j = 1, 2, \ldots, J)$. The price in the vintage *j*, P_{t-j}^*, is homogenous across the vintage. In period *t* these firms change their price whenever:

$$v_{0,t} - v_{j,t} > \xi_t W_t \tag{1}$$

where $v_{0,t}$ and $v_{j,t}$ are sums of discounted expected profits conditional on events of 'setting new price' (P_t^*) and 'no price change' ($P_{(t-j)}^*$), respectively, and W_t denotes economy-wide real wage rate in period t (see formulas (10) in Annex).

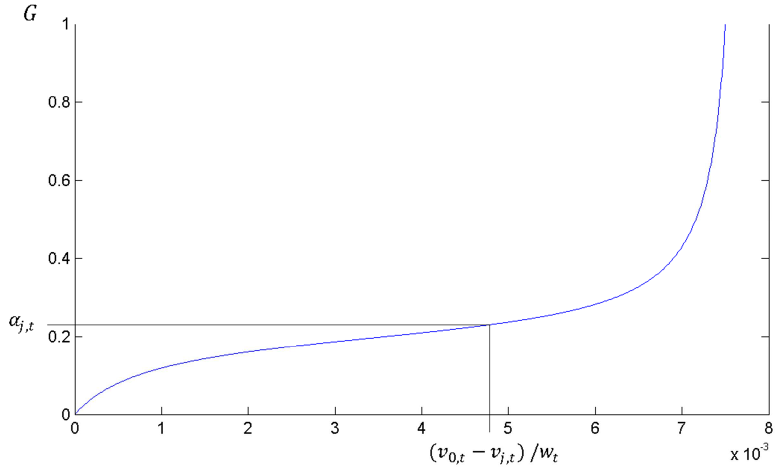
Let, at the beginning of period *t*, $\omega_{j-1,t-1}$, $j = 1, 2, ..., J$, denotes a fraction of all firms belonging to a price vintage *j*. In a period *t* a portion of the firms, $\alpha_{j,t} = G\left((v_{0,t} - v_{j,t}) / W_t\right)$, from vintage *j* with relatively low menu costs, sets a new price P_t^*. Next period they move to the first vintage (*j* = 1). The rest of the firms from vintage *j* does not change the price, hence they migrate to a vintage *j* + 1. In the last vintage *J* the benefits from resetting the price are bigger than the upper bound *B* of menu cost, consequently all firms reset the price and migrate to the first vintage. Due to strictly positive steady-state inflation $\Pi > 0$ and bounded support of menu cost distribution, there exists a finite number of price vintages, *J*. The number of non-empty vintages depends on current shocks, the following model parameters: steady-state inflation Π and price elasticity of demand ε, and the shape of *G* (see Figure 1). The dynamic relationship between $\omega_{j,t}$ and $\alpha_{j,t}$ are determined by:

$$\omega_{j,t} = \left(1 - \alpha_{j,t}\right)\omega_{j-1,\, t-1}, \;\; j = 1,\; 2,\; \ldots,\; J-1 \tag{2}$$

$$\omega_{0,t} = \sum_{j=1}^{J} \alpha_{j,t}\omega_{j-1,\, t-1} \tag{3}$$

Laws of motion given by equations (2) and (3) govern changes in a distribution of firms across price vintages.

Figure 1
Cumulative Distribution Function *G* of Menu Cost and the Fraction of Firms $\alpha_{j,t}$ Resetting the Price



Note: The function $G(x) = c_1 + c_2 \cdot tan(c_3 \cdot x + c_4)$, where $c_1 = 0.1964$, $c_2 = 0.0625$, and $c_3 = 2.7558 / B$, $c_4 = 1.2626$, $B = 0.0075$.
Source: cf. Dotsey, King and Wolman (1999).

As a consequence of equations (2) and (3) the conditional probabilities of resetting the price $\alpha_{1,t}$, $\alpha_{2,t}$, …, $\alpha_{J,t}$ for a given t are increasing functions of j called hazard functions. In a consequence, fractions of firms in consecutive price vintages $\omega_{0,t}$, $\omega_{1,t}$, …, $\omega_{J-1,t}$ are decreasing with *j*. Under positive steady-state inflation and state-dependent pricing the later the firm resets its price the bigger is the probability of a price adjustment. This phenomenon known as a selection effect is not present in the pricing mechanism of Calvo (1983). Here, the timing of price changes is exogenous and random. Moreover, in the Calvo pricing the number of firms declines with vintages at the geometric rate: $\omega_{j,t}^{Calvo} = (1-\theta)\theta^j$, where $0 < \theta < 1$ is the Calvo price-rigidity parameter, and the hazard ratio is constant $\alpha_{j,t}^{Calvo} = 1 - \theta$.

The firm optimization problem in DKW model, described in details in Annex, and the price aggregation of the Dixit-Stiglitz type lead to the state-dependent Phillips curve expanded around non-zero steady-state inflation. In Annex we also introduce necessary modifications to the original SDPC enhancing the model with external habit persistence in consumption (see Abel, 1990).

2.2. DSGE Model

In the empirical part we consider a simple three-equational DSGE framework consisting of a dynamic IS curve, Taylor rule and Phillips curve in two alternative versions with: time-dependent and state-dependent pricing.[2] The IS curve with habit persistence explains dynamics of output gap x_t:

$$x_t = \gamma E_t\left(x_{t+1}\right) + \left(1-\gamma\right)x_{t-1} - \sigma_1\left(\tilde{i}_t - E_t\tilde{\pi}_{t+1}\right) + v_t^x \tag{4}$$

where the parameters $\gamma = \dfrac{\sigma}{\sigma + h\left(\sigma - 1\right)}$, $\sigma_1 = \dfrac{1}{\sigma + h\left(\sigma - 1\right)}$ depend on a constant consumer relative risk aversion $\sigma > 0$, a measure of external habit persistence is $0 < h < 1$ (Abel, 1990), and the error term, $v_t^x = \rho_x v_{t-1}^x + \epsilon_t^x$, which is a stationary autoregressive shock in preferences, $\epsilon_t^x \sim NID(0,\ \sigma^x)$.

The following Taylor rule describes the detrended interest rate dynamics $\tilde{i}_t$ induced by monetary policy under interest rate smoothing:

$$\tilde{i}_t = \lambda\tilde{i}_{t-1} + \left(1-\lambda\right)(\phi_\pi\tilde{\pi}_t + \phi_x x_t) + \epsilon_t^i \tag{5}$$

where $\lambda,\ \phi_\pi,\ \phi_x$ are parameters of central bank reactions, and ϵ_t^i is a white noise monetary policy shock: $\epsilon_t^i \sim NID(0,\ \sigma^i)$.

As a time-dependent benchmark in a three-equational DSGE model we consider the hybrid NKPC of Gali and Gertler (1999):

$$\tilde{\pi}_t = \beta_f E_t\left(\tilde{\pi}_{t+1}\right) + \beta_b\tilde{\pi}_{t-1} + \chi_0 x_t + \chi_1 x_{t-1} + \epsilon_t^\pi \tag{6}$$

where $\epsilon_t^\pi \sim NID(0,\ \sigma^\pi)$ and β_f, β_b, χ_0, χ_1 are structural parameters that depend on 'deep' parameters (cf. Annex) of time-dependent pricing (including Calvo parameter, θ, and fraction of backward-looking firms, τ), consumer utility function ($h,\ \sigma,\ \varphi$), production function (α), and the demand function (ε):

$$\chi_0 = \frac{\varphi + \alpha + \sigma\left(1-\alpha\right)}{1-\alpha}\frac{\left(1-\tau\right)\left(1-\theta\right)\left(1-\beta\theta\right)}{\theta + \tau\left(1-\theta\left(1-\beta\right)\right)}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$$

$$\chi_1 = h\left(1-\sigma\right)\frac{\left(1-\tau\right)\left(1-\theta\right)\left(1-\beta\theta\right)}{\theta + \tau\left(1-\theta\left(1-\beta\right)\right)}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon},\quad \beta_f = \frac{\beta\theta}{\theta + \tau\left(1-\theta\left(1-\beta\right)\right)}$$

$$\text{and}\quad \beta_b = \frac{\tau}{\theta + \tau\left(1-\theta\left(1-\beta\right)\right)}.$$

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[2] The microfoundations of three-equational DSGE model with state-dependent pricing mechanism are presented in Annex.

The SDPC described by equation (18) in Annex in comparison to the hybrid NKPC, cf. equation (6), includes additional leads of inflation and output gap and infinite number of lagged inflation terms. Moreover in SDPC, there are unobserved characteristics of price vintages: $\left(\widetilde{\omega}_{j,t+j}\right)_{j=0,1,\ldots,J-1}$ and $\left(\widehat{\Omega}_{t-l}\right)_{l=0,1,\ldots}$ that depend on the history of transition of firms between price vintages. We apply necessary modifications to SDPC equation (18), that facilitate the Bayesian estimation. The simplified version of SDPC is of the form:

$$\widetilde{\pi}_t = E_t \sum_{j=1}^{11} \delta_j^{'} \widetilde{\pi}_{t+j} + \vartheta x_{t-1} + E_t \sum_{j=0}^{11} \widehat{\psi}_j^{'} x_{t+j} + \sum_{l=1}^{11} \mu_l^{'} \widetilde{\pi}_{t-l} + \epsilon_t^{\pi} \tag{7}$$

where error term $\epsilon_t^{\pi} \sim NID(0, \sigma^{\pi})$ represents an adverse technological shock in the economy, and the formulas for parameters $\delta_j^{'}$, ϑ, $\widehat{\psi}_j^{'}$, $\mu_l^{'}$ are presented in Annex below formulas (17) and (18).

Compared to fully fledged SDPC (18), in the equation (7) we omit the unobserved components $\left(\widetilde{\omega}_{j,t+j}\right)_{j\,=\,0,1,\ldots,J-1}$, and $\left(\widehat{\Omega}_{t-l}\right)_{l\,=\,0,1,\ldots}$. This simplification is motivated by the results of Bakhshi, Khan and Rudolf (2006). They show that under DKW-DSGE with interest rate smoothing the contribution of the omitted terms is only substantial for the instantaneous and one-quarter lagged responses to a policy shock (see Figure 8 of their paper). Moreover, estimation of these terms within a Bayesian techniques would be a matter of serious numerical complication. Overcoming these difficulties we still take into consideration the steady-state fractions of firms in *J* price vintages $\omega_0, \omega_1, \ldots, \omega_{J-1}$. They are related to the deep parameters of the model through (2) and (3) (see also equation (12) and (13) in Annex). A constant steady-state inflation $\Pi = 3\%$ p.a. (i.e. average annual inflation in the sample) is assumed which implies the maximal number of price vintages, $J = 12$. Moreover, the infinite number of lagged inflation terms is approximated by the lag distribution with 11 significant terms.

Most of the empirical studies for Poland take small open economy perspective (e.g. Kolasa, 2009; Hałka and Szafranek, 2016; Dąbrowski and Wróblewska, 2016). Some of them show that Polish inflation is only moderately affected by foreign shocks (Brada et al., 2015) or exchange rate movements (Hałka and Kotłowski, 2014). We argue that open economy model is feasible only for Calvo pricing. Introducing open economy in the state-dependent pricing setup would make the firm optimisation problem far more complicated. For this reason the vast majority of DKW applications including Bakhshi, Khan and Rudolf (2007) are performed in a closed economy.[3] Hence, in order not to obscure the picture of the paper we decided to consider closed economy version, leaving open economy issue for further research.

3. Bayesian Estimation and Discussion of Results

3.1. Data and Methods

Both DSGE models with time-dependent and state dependent pricing are estimated on quarterly data from the Polish economy for the period 1997:1 – 2016:3. Inflation is measured by quarterly change of seasonally adjusted Consumption Price Index (CPI), interest rate is a short-term interest rate on 1 month interbank deposits (WIBOR 1m). Because a disinflation process is a dominating long-term component in the first 5 years of the sample (see Internet Appendix A on the data[4]), we perform the estimations on inflation and interest rates detrended with Hodrick-Prescott filter. The output gap is calculated as a percentage deviation of seasonally adjusted GDP from its HP trend, which is a standard approach in determining steady-state level of output in most of the DSGE studies.

To learn about the parameters of DSGE models from the data we perform a Bayesian estimations in Dynare (see Adjemian et al., 2011). Instead of performing themaximum likelihood calculations (which are inefficient with so many hidden variables in DKW pricing and not so many observations) or calibrating the model to match the empirical moments we use Bayesian inference. It is very useful in the specific case when the researcher is fairly confident on the values of some parameters and he wants to estimate the others, which are more important from his point of view.

Recall that distribution of price vintages in DKW model, $\omega_0, \omega_1, \ldots, \omega_{J-1}$, depends in a non-trivial way on deep parameters of the model: Π, m, B (see Annex (12) and (13)). To employ these relationships in Markov Chain Monte Carlo methods we construct an exponential polynomial of a markup, $m \equiv \dfrac{\varepsilon}{\varepsilon - 1}$, and an upper bound of menu cost, *B*, that interpolates ω_j reasonably well. The grid which also includes interaction terms is built on a joint domain of $m \in [1.1; 1.35]$ and $B \in (0.0075; 0.05)$.

We start from formulating fairly diffuse priors for the parameters in both models (except for monetary policy reactions) and we keep them comparable between the models wherever it is possible (see details on prior distribution in Internet Appendix B[5]). Next, we calculate joint posterior distribution in Metropolis random-walk algorithm by simulating 2 million realization of a Markov chain and

[3] One notable exception includes a DKW model calibrated by Landry (2010).

[4] Internet appendix A is available at:
<http://www.katek.uni.lodz.pl/sites/default/files/info_files/SDPC_Appendix_A.pdf>.

[5] Internet appendix B is available at:
<http://www.katek.uni.lodz.pl/sites/default/files/info_files/SDPC_Appendix_B.pdf>.

dropping initial 50% of them as burn-in cycles. We compare the statistics from the simulated posterior distributions of SDPC and hybrid NKPC, and the steady-state fraction of firms in price vintages $\left(\omega_j\right)_{j=0,1,\ldots,J-1}$. From the realisations of Markov chains we generate impulse response functions, which describe an estimated reaction of inflation, output gap and interest rate to an unanticipated shock.

3.2. Posteriors and Distribution of Price Vintages

The estimation results of the equivalent deep parameters in time-dependent and state-dependent pricing model do not differ significantly (see Table 1 and Internet Appendix B). The means of posterior distributions of time-dependent model are always inside 90% HPD intervals of state-dependent case. The results show that our knowledge on the deep parameters is considerably updated by the data, except for the coefficients at inflation in the Taylor rule (ϕ_π). All of the three estimated parameters (σ, h, λ) of posterior distributions are very close to the time-dependent pricing case. A posterior mean of σ, which explains consumption relative risk aversion, takes a relatively big number about 6. There is an evidence of strong inertial behaviour in a monetary policy reaction function and of a strong habit formation, with means of λ and h close to 0.85 in both models. The only significant differences can be found in the shock characteristics. In comparison to the time-dependent pricing model the variances of technological (σ^π) and preference shocks (σ^x) are bigger in the state-dependent pricing model, and the persistence of preference shock (ρ_x) is lower in the latter model.

From the posterior mean of B we conclude that to replicate a degree of price stickiness observed in the seasonally adjusted data one needs an upper bound of menu costs at 2.3% in terms of real wages which is ca. 1.8% as measured in terms of real output. In the estimated DSGE with hybrid NKPC the mean of the Calvo parameter, θ, informs that a relatively big fraction of firms, 72%, do not change their prices in a given quarter. There is also a low fraction of backward-looking non-optimising firms (22% on average). These parameters would be hard to compare with the state-dependent pricing model without producing the posterior distribution of firms in price vintages (see Figure 2), conditional probabilities of changing the price (see Figure 3), and average price duration statistics in both models.

The estimated steady-state fractions of firms resetting their prices $j = 0, 1, \ldots, J$ quarters ago are denoted by ω_j in SDPC (see Figure 2). In the SDPC the maximum number of price vintages is equal to J = 12. In the hybrid NKPC model there is an infinite number of vintages and we calculate the corresponding fractions from the general formula $\omega_j^{Calvo} = \left(1-\theta\right)\theta^j$. From Figure 2 it is important to note that the fractions, ω_j^{Calvo}, in NKPC model decay faster with price vintages than in SDPC. In the state-dependent pricing model it is an outcome of an increase in the fraction of firms resetting the price ($\alpha_{j,t}$) for the consecutive price vintages (due to the selection effect) – see Figure 3. Contrarily, in the time-dependent pricing model the probability of price adjustment ($1-\theta$) is constant.

T a b l e 1

Posterior Distributions in the Time-dependent and State-dependent Pricing Model

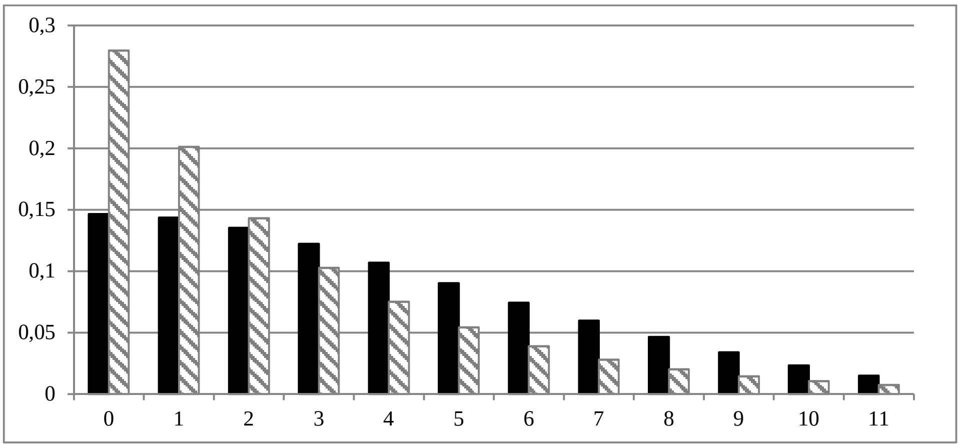
Parameters	Time-dependent model		State-dependent model	
	Mean	St. dev.	Mean	HPD interval (90%)
θ	0.72	0.08	NA	NA
τ	0.22	0.08	NA	NA
B	NA	NA	**0.023**	(0.007; 0.042)
m	1.26	0.15	1.23	(1.04; 1.52)
h	0.84	0.13	0.88	(0.76; 1.00)
φ	2.81	1.62	1.50	Calibrated
σ	5.94	1.66	6.00	(3.86; 8.14)
ϕ_π	2.00	0.05	2.00	(1.83; 2.16)
ϕ_x	0.00	0.01	0.02	(–0.06; 0.10)
λ	0.84	0.02	0.85	(0.82; 0.88)
ρ_x	0.48	0.11	0.37	(0.21; 0.53)
σ^π	0.36	0.04	0.45	(0.39; 0.51)
σ^x	0.21	0.03	0.26	(0.20; 0.31)
σ^i	0.25	0.02	0.25	(0.21; 0.28)

Note: St. dev. stands for standard deviation of posterior distribution, NA (not applicable) for the parameters that are not included in one of the models, HPD means the highest posterior density.

Source: Own calculations with Dynare 4.3 (Adjemian et al., 2011).

F i g u r e 2

Posterior Means of Fraction of Firms in Price Vintages ω_j at Steady State in the Estimated Models with State-dependent (black) and Time-dependent (shaded) Pricing

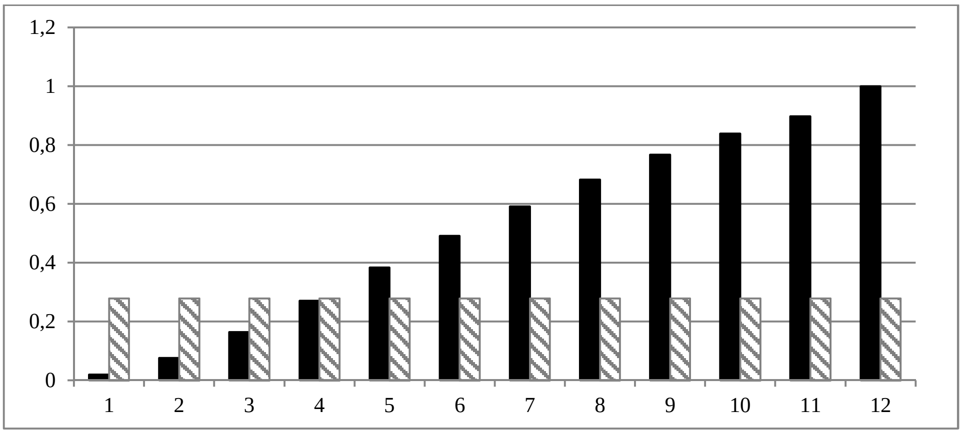


Note: In the time-dependent model $\omega_j^{Calvo} = (1-\theta)\theta^j$.

Source: Own calculations.

F i g u r e 3

Posterior Means of Hazard Functions in the Estimated Models with State-dependent (black) and Time-dependent (shaded) Pricing



Note: In the time-dependent model $\alpha_j^{Calvo} = (1-\theta)$.
Source: Own calculations.

As a result there is a difference in the estimated mean durations of price between the models, which is about 4.6 quarters in DKW economy compared to about 3.6 quarters in hybrid NKPC. The average price duration obtained in the microeconomic studies for the Polish economy is somewhere between both of these estimates. According to the survey data Polish firms adjust price every four quarters (Jankiewicz and Kołodziejczyk, 2008). Similar results hold for other European countries (see Alvarez, 2008, Table 3). Macias and Makarski (2013) using micro-price data for the Polish economy (2004 – 2008) show that mean implied price duration is about 3.6 quarters. However, these calculations take into account price changes from promotions and seasonality. These short-term components of price dynamics observed in many consumption goods including food are eliminated from our dataset because of data processing (seasonal adjustment and price aggregation). Hence, price rigidity calculated from macro data should be far above evidence from micro data as it is in SDPC case. Moreover, the propensity of adjusting the price (hazard ratio) is bigger in the Calvo model for the first four firm's vintages, j = 0, 1, 2, 3, and becomes smaller for firms which have not updated the price for more than 4 quarters – see Figure 3. This leads to the conclusion that the probability of large price adjustment is relatively bigger in DKW model.[6] Thus, in DKW model the time between price adjustment is longer, but the probability of intense price changes is higher than in hybrid NKPC.

[6] Notice that in DSGE-DKW model, since we assume positive steady-state inflation, we expect firms in higher price vintages to have larger price adjustment.

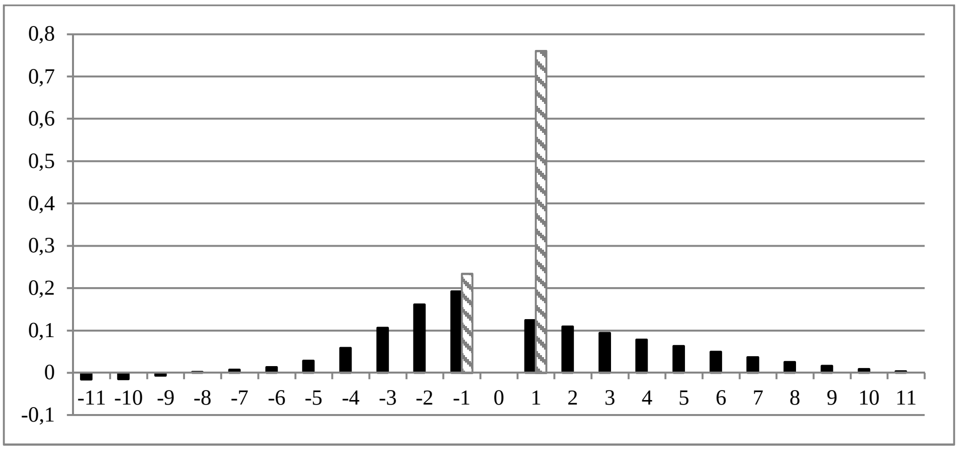
3.3. Phillips Curves: SDPC vs. Hybrid NKPC

In the next step we analyse the mean parameters of the both estimated Phillips curves to explain whether Polish inflation is mainly driven by expectations on inflation and output or by their intrinsic persistence (inertia). The results show that one-quarter ahead inflation expectation in SDPC are of much lower magnitude than in time-dependent counterpart (NKPC), which is mostly forward--looking (see Figure 4). The impact of inflation expectations in SDPC is more prolonged in time and it decreases with a time horizon. Still the sum of inflation expectation terms in SDPC is about 81% of one-period expectation parameter in hybrid NKPC which means that the estimated SDPC is less forward-looking in inflation than the estimated NKPC.

On the other hand, SDPC gives an appealing explanation of internal (menu cost) inflation persistence. The sum of the estimated parameters at lagged inflation in SDPC is greater than 0.5 which is more than 2 times bigger than in hybrid NKPC. Summing up, from the perspective of SDPC intrinsic persistence is a prevailing force of inflation determination in the Polish economy contrary to the conclusions from the estimated hybrid NKPC which is mainly forward-looking.

Figure 4

Median of Posterior Distributions of the Parameters at Lagged (–) and Expected (+) Inflation in SDPC (in black) and Hybrid NKPC (shaded)

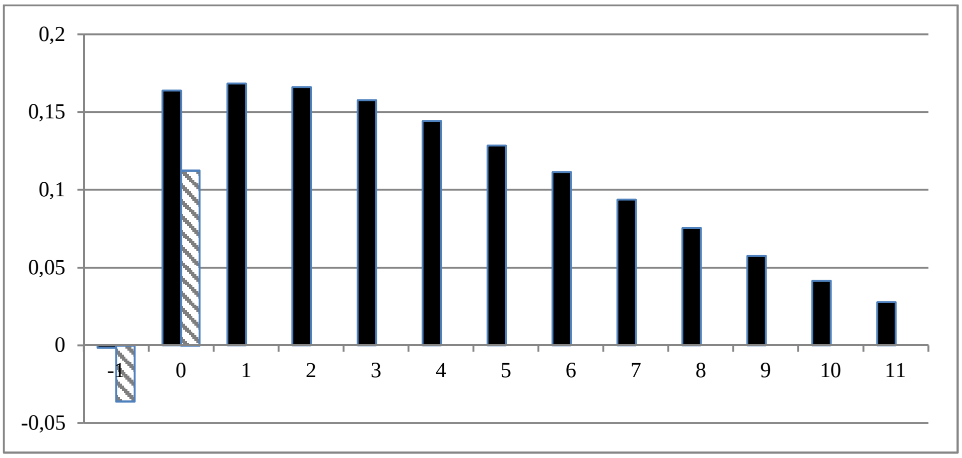


Note: in SDPC the parameters at lagged terms $\tilde{\pi}_{t-l}$ (for $-l = -11, \dots, -1$) are denoted by μ_l', and at lead terms $\tilde{\pi}_{t+j}$ by δ_j' (for $j = 1, \dots, 11$). In hybrid NKPC they are denoted by β_b and β_f, respectively.
Source: Own calculations.

In terms of the output gap the SDPC is potentially more forward looking than hybrid NKPC because of the price mechanism of menu cost involved. The effect of current output gap on inflation is comparable across the models. In SDPC there is, however, an additional influence of output gap expectations on inflation, which by its construction is absent from NKPC (see Figure 5). This impact of output gap expectations on inflation is a medium-term phenomenon lasting up to several quarters ahead. It is also stronger than the contemporaneous impact up to 6 quarters ahead. A maximum effect of output gap expectations is located at the two-quarter lead, and then it slowly decays. In result of those medium-term output gap expectations and despite similar strength of habit persistence the lagged effect of output gap in SDPC is also stronger than in hybrid NKPC. In summary, firms from the perspective of state-dependent price stickiness are on average more forward looking in determining aggregate inflation to conjuncture than their counterparts in the conventional model with NKPC of Gali and Gertler (1999).

Figure 5

Median of Posterior Distribution of Parameters at Lagged (–), Current (*j* = 0) and Expected (+) Output Gap in SDPC (black) and Hybrid NKPC (shaded)



Note: In SDPC: ϑ for $-l = -1$, $\hat{\psi}_l^{'}$ for $j = 0, \ldots, 9$, and in hybrid NKPC: χ_1 and χ_0.

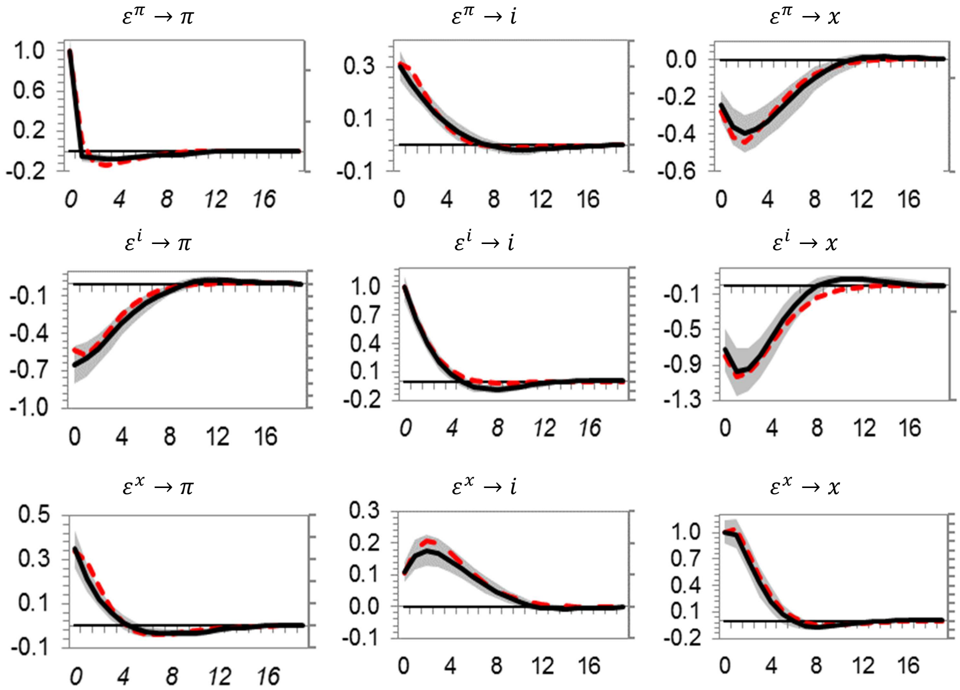
Source: Own calculations.

3.4. Impulse Response Functions

In the last part we analyse impulse response functions (IRFs) in both models using posterior distributions of the estimated parameters. The impulse response functions in both time- and state-dependent pricing models (cf. Figure 6) are economically plausible and they exhibit a similar hump-shaped pattern of reaction, frequently reported in other DSGE studies. In the first row of Figure 6 there are effects of one-percentage-point (1 p.p.) adverse technology shocks, which lowers firms productivity making the re-optimizing firms to set higher prices. Consequently, lower production opens the negative output gap, which is dampened by central bank response to a higher inflation. In the second row there are effects of monetary policy shock. An 'extra' raise of central bank interest rate by 1 p.p. (above the level consistent with the Taylor rule) make households to postpone a part of their current consumption, which generates negative output gap and lowers inflation. Third row describes the effects of a 1 p.p. preference shock, which raises the weight of current utility in the lifetime utility path. This makes current consumption more valuable leading to a positive output gap and higher inflation. At last, central bank raises interest rate in response to higher economic activity.

Figure 6

Impulse Response Functions of Inflation π, Interest Rate i, and Output gap x to One-percentage-point Shocks to Technology ε^{π} (adverse), Monetary Policy ε^{i} and Preferences ε^{x}



Note: Black line and a shaded area denote, respectively, posterior medians of IRFs and their 90% HPD intervals from the state-dependent pricing model. Dashed lines are means of posterior IRFs from the time-dependent pricing model.

Source: Own calculations with Dynare 4.3.

Although the estimated DSGE model with state-dependent pricing identifies less persistent preference shocks IRFs from both models estimated for the Polish economy are generally hard to distinguish.

The differences are negligible in economic terms, and statistically insignificant. No wonder, both, state-dependent and time-dependent models are estimated from the same macroeconomic data with many parameters of dynamic IS and policy reactions generated from very similar posterior distributions. What is a puzzle in these empirical results that similar short-term adjustment to shocks are derived from completely different Phillips curves. The estimated SDPC is more forward looking in terms of output gap with an assumption of strong intrinsic inflation persistence and weaker dependence on inflation expectations. Contrarily, the hybrid NKPC is mainly driven by inflation expectations and not very forward looking in terms of output gap.

Conclusions

The estimated state-dependent pricing model with moderate upper bound of menu costs (2.3% in terms of labour costs) indicates a 3-month longer price duration in the Polish economy than the time-dependent counterpart. The price stickiness evidence from the model with menu costs is closer to both micro-price data and surveys on Polish inflation. The DSGE models different in terms of price setting mechanism but comparable in all other respects generate impulse response functions, which are hard to distinguish. There are however considerable differences in estimated Phillips curves. The state dependent Phillips curve is based on strong inflation persistence and forward-lookingness in respect to output. The estimated hybrid NKPC is mainly driven by inflation expectations. The differences are explained by a selection effect being present in the state-dependent pricing model only, which yields more intense and impact price adjustment after a policy shock than in the time-dependent model.

We conclude that it may have important consequences in reconciling the apparently contradictory results of microeconomic research on price stickiness and macroeconomic evidence obtained from traditional DSGE models with Calvo pricing. The estimated short-term response of output and inflation to the monetary policy shock in the state-dependent pricing model would be stronger if one fully accounts for the transitory short-term dynamics in fractions of firms across price vintages. The obvious limitation of our study is the closed economy setup. The level of openness (including exchange rate flexibility) of Polish economy was changing dramatically over the sample which limits the application of the simple DSGE models we consider.

Nevertheless, the conclusions from the IRF analysis should be valid if small shocks are considered and long-term inflation is close to the average over the estimated sample.

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A n n e x: DSGE Model Structure and SDPC Derivation

Households

We assume competitive labour market with firms renting labour (N_t) at an economy-wide real wage rate W_t. The household decisions are also subject to standard budget constraint. We consider representative households maximizing intertemporal utility from their consumption ($C_t(i)$), and disutility of labour (N_t):

$$U\left(C_t\left(i\right),N_t\right)=e^{v_t^p}\left(\frac{\left[C_t(i)/\left(\overline{C}_{t-1}\right)^h\right]^{1-\sigma}}{1-\sigma}-\frac{N_t^{\,1+\varphi}}{1+\varphi}\right) \tag{8}$$

where $\sigma > 0$ is a constant relative risk aversion, $\varphi > 0$ is the inverse of Frisch elasticity of labour, and $0 < h < 1$ is a measure of external habit persistence (Abel, 1990), which is measured in relation to the average consumption (across all households) from the previous period $\left(\overline{C}_{t-1}\right)^h$, and v_t^p is an AR(1) process, which we interpret as a preference shock in period *t*. Using the necessary condition for maximizing consumer utility and market clearing condition one can derive the dynamic IS curve with habit persistence (4), where $v_t^x=\sigma v_t^p$.

Firms

Firms indexed with $j \in (0, 1)$ transform labour to products, $Y_t(j)$, given initial technology level A_0, and aggregate technology shocks v_t^m:

$$Y_t\left(j\right)=A_0 e^{v_t^m}N_t^{1-\alpha} \tag{9}$$

where v_t^m is a stationary AR(1) stochastic process, and $0<(1-\alpha)<1$ is a labour share.

Recall that under Dixit and Stiglitz (1977) aggregation scheme the demand function is of the form $C_t\left(j\right)=\left(\dfrac{P_t\left(j\right)}{P_t}\right)^{-\varepsilon}C_t$, where $\varepsilon > 1$ is constant elasticity of substitution between goods or price elasticity of consumption.

In period t the pricing decisions require the calculation of real values of a firm conditional on the event of price adjustment ($v_{0,t}$) and price stickiness ($v_{j,t}$) for ($j=1,\ 2,\ \dots,\ J-1$):

$$\begin{aligned} v_{0,t}&=\max_P\left\{z_{0,t}(P)+E_tQ_{t,t+1}\ .\ \left(1-\alpha_{1,t+1}\right)\ .\ v_{1,t+1}+E_tQ_{t,t+1}\alpha_{1,t+1}\ .\ (v_{0,t+1}-\mathrm{W}_{t+1}\ .\ \mathrm{K}_{1,t+1})\right\} \\ v_{j,t}&=z_{j,t}\left(P_{t-j}^*\right)+E_tQ_{t,t+1}\ .\ \left(1-\alpha_{j,t+1}\right)\ .\ v_{j+1,t+1}+E_tQ_{t,t+1}\alpha_{j,t+1}\ .\ \left(v_{0,t+1}-\mathrm{W}_{t+1}\ .\ \mathrm{K}_{j,t+1}\right)\end{aligned} \tag{10}$$

where $z_{j,t}(P^*_{t-j}) = \left(\frac{P^*_{t-j}}{P_t}\right)^{-\varepsilon} \cdot Y_t \cdot \frac{P^*_{t-j}}{P_t} - \Psi_{t,j}$ is the firm's current period real profit if its nominal price is P^*_{t-j}, $\mathrm{K}_{j,t+1}$ is the average menu cost in vintage *j* and the term $Q_{t,t+k} = \beta^k U'(C_{t+k}) / U'(C_t)$ represents stochastic discount factor for the future real profits (see Campbell, 1999).

The solution to the problem of maximization the firm's real value $v_{0,t}$ is given by (cf. Dotsey, King and Wolman, 1999):

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j \cdot E_t Q_{t,t+j} \cdot \frac{\omega_{j,t+j}}{\omega_{0,t}} \cdot MC_{t+j} \cdot P^{\varepsilon}_{t+j} \cdot Y_{t+j}}{\sum_{j=0}^{J-1} \beta^j \cdot E_t Q_{t,t+j} \cdot \frac{\omega_{j,t+j}}{\omega_{0,t}} \cdot P^{\varepsilon-1}_{t+j} \cdot Y_{t+j}} \tag{11}$$

where MC_t is the real marginal cost and $\frac{\omega_{j,t+j}}{\omega_{0,t}}$ is the probability of non-adjustment of the price from period *t* to period *t* + *j*. In the case of flexible prices (*B* = 0) the formula (11) can be rewritten as $P_t^* = \frac{\varepsilon}{\varepsilon-1} MC_t P_t$. Hence, the term $\frac{\varepsilon}{\varepsilon-1}$ can be interpreted as a monopolistic markup over a nominal marginal cost.

Steady state

The steady state of the economy is defined as the constant level of inflation $\Pi > 0$, total production *Y* and stationary distribution of prices $\omega_0, \omega_1, \ldots, \omega_{J-1}$. Moreover, denote by $RP_t^* = \frac{P_t^*}{P_t}$ relative optimal price in period *t* and notice that the steady state value of RP_t^* is constant in time and given by $RP^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{j=0}^{J-1} \beta^j \cdot \omega_j \Pi^{j\varepsilon}}{\sum_{j=0}^{J-1} \beta^j \cdot \Pi^{j(\varepsilon-1)}} MC$.

Moreover, DKW pricing mechanism in the steady state is described by time-homogenous stationary Markov chain with states 1, 2, …, *J* denoting the price vintages and with transition matrices, *M*:

$$M = \begin{bmatrix} \alpha_1 & 1-\alpha_1 & 0 & \ldots & 0 \\ \alpha_2 & 0 & 1-\alpha_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{J-1} & 0 & 0 & \ldots & 1-\alpha_{J-1} \\ 1 & 0 & 0 & \ldots & 0 \end{bmatrix} \tag{12}$$

Let G be the cdf of menu cost (see Figure 1).The hazard ratios $\alpha_j = G((v_0 - v_j)/\mathrm{W})$, can be found by solving the optimization problem with the following constraints:

$$\begin{aligned} v_0 &= \max_{RP}\left\{ z_0(RP) + (1-\alpha_1)\ .\ v_1 + \alpha_1\ .\ (v_0 - \mathrm{W}\ .\ \mathrm{K}_1) \right\} \\ v_j &= z_j\left(RP^*\right) + \left(1-\alpha_{j,t+1}\right)\ .\ v_{j+1} + \alpha_j\ .\ \left(v_0 - \mathrm{W}\ .\ \mathrm{K}_j\right) \end{aligned} \tag{13}$$

where $j = 1,\ 2,\ \ldots,\ J-1$.

Here, $z_j\left(RP^*\right) = \left(RP^*\right)^{1-\varepsilon} \cdot Y \cdot \Pi^{j(\varepsilon-1)} - MC \cdot Y$. In consequence, the sums of conditional discounted current and future profits v_0, v_j are constant over time. The steady-state fractions of firms in vintages $\omega_0,\ \omega_1,\ \ldots,\ \omega_{J-1}$ form a stationary distribution of the Markov chain with transition matrix (12).

SDPC with habit persistence

To derive the Phillips Curve one has to solve the optimal problem for firms and aggregate the price distribution. The Dixit and Stiglitz (1977) price aggregation entails: $P_t^{1-\varepsilon} = \sum_{j=0}^{J-1} \omega_{j,t}\left(P_{t-j}^*\right)^{1-\varepsilon}$. Then, substituting the formula for relative optimal prices RP_t into the above equation, one obtains $1 = \sum_{j=0}^{J-1} \omega_{j,t}\left(RP_{t-j}^* \dfrac{P_{t-j}}{P_t}\right)^{1-\varepsilon}$. Log-linearization around steady-state leads to:

$$\widetilde{rp}_t^* = \frac{1}{\omega_0}\left[\sum_{j=0}^{J-2} \tilde{\pi}_{t-j} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\varepsilon-1)} - \sum_{j=1}^{J-1} \omega_j \Pi^{j(\varepsilon-1)} \widetilde{rp}_{t-j}^* + \frac{1}{\varepsilon-1} \sum_{j=0}^{J-1} \omega_j \tilde{\omega}_{j,t} \Pi^{j(\varepsilon-1)} \right] \tag{14}$$

where variables with a tilde are deviations from steady-state: $\tilde{\omega}_{j,t} = ln\,\omega_{j,t} - ln\,\omega_j$ $\tilde{\pi}_{t-j} = ln\,\Pi_{t-j} - ln\,\Pi$, $\widetilde{rp}_{t-j}^* = ln\left(RP_t^*\right) - ln\left(RP^*\right)$, $j = 0,\ 1,\ \ldots,\ J-1$.

After log-linearization of formula for relative optimal price (see Appendix A to Bakhshi, Khan and Rudolf, 2006) and assuming $\tilde{q}_{t,t+j} = 0$, we obtain:

$$\begin{aligned} \widetilde{rp}_t^* = E_t \sum_{j=0}^{J-1} \left[\tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t} + \widetilde{mc}_{t+j} + \varepsilon \sum_{k=1}^{j} \tilde{\pi}_{t+k} + x_{t+j} \right] \rho_j - \\ - E_t \sum_{j=0}^{J-1} \left[\tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t} + (\varepsilon - 1) \sum_{k=1}^{j} \tilde{\pi}_{t+\mathrm{k}} + x_{t+j} \right] \delta_j \end{aligned} \tag{15}$$

where x_t is an output gap, $\delta_j = \dfrac{\beta^j \omega_j \Pi^{j(\varepsilon-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\varepsilon-1)}}$ and $\rho_j = \dfrac{\beta^j \omega_j \Pi^{j\varepsilon}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\varepsilon}}$.

Consumer's habit persistence leads to the following relationship between percentage deviation of real marginal cost from its steady-state value and output gap:

$$\widetilde{mc}_t = \kappa_1 x_t + \kappa_2 x_{t-1} + \frac{\phi+1}{1-\alpha} d_t + \frac{\phi+1}{\alpha-1} v_t^m$$

where $\kappa_1 = \frac{\phi+\alpha+(1-\alpha)\sigma}{1-\alpha}$, $\kappa_2 = h(1-\sigma)$ and d_t is a measure of price dispersion (see Gali (2008, pp. 62 – 63), v_t^m is a shock on technology in a one-factor (n_t) production function $y_t = a_0 + v_t^m + (1-\alpha)n_t - d_t$ with $0<\alpha<1$. Hereinafter for a sake of simplicity we take $d_t = 0$ for all *t*. Substituting this equation together with $E_t v_{t+j}^m = 0$ for $j = 1, 2, \ldots, J-1$ into (15) gives:

$$\widetilde{rp}_t^* = E_t \sum_{j=1}^{J-1} \widetilde{\pi}_{t+j} \sum_{k=j}^{J-1} (\varepsilon\rho_k - (1-\varepsilon)\delta_k) + E_t \sum_{j=0}^{J-1} (\rho_j - \delta_j)(\widetilde{\omega}_{j,t+j} - \widetilde{\omega}_{0,t}) + \\ + \kappa_2 \rho_0 x_{t-1} + E_t \sum_{l=0}^{J-1} \widehat{\psi}_l x_{t+l} + \frac{\phi+1}{\alpha-1} v_t^m \tag{16}$$

where
$$\widehat{\psi}_l = \begin{cases} \kappa_1\rho_j + \rho_j - \delta_j + \kappa_2\rho_{j+1} & l = 0, 1, \ldots, J-2 \\ \kappa_1\rho_j + \rho_j - \delta_j & l = J-1 \end{cases}$$

Comparing (14) and (16) we obtain:

$$\widetilde{\pi}_t = E_t \sum_{j=1}^{J-1} \delta_j' \widetilde{\pi}_{t+j} + E_t \sum_{j=0}^{J-1} \gamma_j (\widetilde{\omega}_{j,t+j} - \widetilde{\omega}_{0,t}) + \frac{1}{\mu_0} \kappa_2 \rho_0 x_{t-1} + \\ + E_t \sum_{l=0}^{J-1} \frac{1}{\mu_0} \widehat{\psi}_l x_{t+l} - \sum_{k=1}^{J-2} \frac{\mu_l}{\mu_0} \widetilde{\pi}_{t-k} + \sum_{j=1}^{J-1} \frac{\omega_j}{\mu_0 \omega_0} \Pi^{j(\varepsilon-1)} \widetilde{rp}_{t-j}^* + \frac{1}{1-\varepsilon} \widehat{\Omega}_t + + v_t^\pi \tag{17}$$

where

$\delta_j' = \frac{1}{\mu_0} \sum_{k=j}^{J-1} (\varepsilon\rho_k - (1-\varepsilon)\delta_k)$, for $j = 1, \ldots, J-1$, $\gamma_j = \frac{1}{\mu_0}(\rho_j - \delta_j)$ for $j = 0, 2, \ldots, J-1$, $\mu_k = \frac{1}{\omega_0} \sum_{i=k+1}^{J-1} \omega_i \Pi^{i(\varepsilon-1)}$ for $k = 0, 1, 2, \ldots, J-2$, $\widehat{\Omega}_t = \sum_{j=0}^{J-1} \frac{1}{\omega_0} \Pi^{j(\varepsilon-1)} \omega_j \widetilde{\omega}_{j,t}$, and $v_t^\pi = -\frac{\phi+1}{1-\alpha} v_t^m$.

To derive the State Dependent Phillips Curve from equation (17) one needs to recurrently substitute $\widetilde{rp}_{t-1}^*, \widetilde{rp}_{t-2}^*, \ldots$, formulas from (16). Finally, we obtain the State Dependent Phillips Curve:

$$\widetilde{\pi}_t = E_t \sum_{j=1}^{J-1} \delta_j' \widetilde{\pi}_{t+j} + \sum_{l=1}^{\infty} \mu_l' \widetilde{\pi}_{t-l} + E_t \sum_{j=0}^{J-1} \widehat{\psi}_j' x_{t+j} + \vartheta x_{t-1} + \\ + \sum_{l=0}^{\infty} \eta_l \widehat{\Omega}_{t-l} + E_t \sum_{j=0}^{J-1} \gamma_j (\widetilde{\omega}_{j,t+j} - \widetilde{\omega}_{0,t}) + v_t^\pi \tag{18}$$

where $\widehat{\psi}_j' = \frac{\widehat{\psi}_j}{\mu_0}$, for $j = 0, 1, \ldots, J-1$, $\vartheta = \frac{\kappa_2\rho_0}{\mu_0}$, and matrix formulas on μ_l', η_l for $l = 1, 2, \ldots$ are given in Appendix B to Bakhshi, Khan and Rudolf (2006).