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INTERPOLATION OF THE CONTROL SURFACE PREPARED BY THE LINGUISTICALLY ORIENTED DESIGN

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ABSTRACT. The main topic of this article is interpolation of the control surface designed by linguistically oriented approach. The problem of linguistically oriented design is solved by various methods based on the fuzzy sets. Their main principle consists in definition of fuzzy sets of linguistic terms which interpret the input and output values and in the design of linguistic description by specification of the relation between the input and output using words of natural language. However, when the number of fuzzy variables and their possible linguistic values increase then the number of rules to be specified increases very rapidly. A possible solution of this problem may be the design of some isolated points or areas in control surface having a key role in control and then to interpolate the "holes" in between by some kind of interpolation method.

1. Definition of the problem

The controller is a device giving specific value of the output variable on the basis of the concretely defined input variables and control surface is a function

$$y = f(\mathbf{x}), \tag{1}$$

where

$$\mathbf{x} = (x_1, x_2, \dots, x_n) \tag{2}$$

is a vector of input variables in n-dimensional space.

Given m points $Y_1, \dots, Y_m$ of the control surface

$$Y_j = f(\mathbf{X}_j), \qquad j = 1, \dots m,$$

we seek the interpolation function $y(\mathbf{x})$ which fulfils the condition

$$\lim_{\mathbf{x} \to \mathbf{X}_j} y(\mathbf{x}) = Y_j, \qquad j = 1, \dots m. \tag{3}$$

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2. Solution of the problem

Taking into account that the function we are seeking is an unknown control surface we may expect that the influence of the known points Y_j will decrease if the variable (function) $R_j(\mathbf{x})$ defined by

$$R_j(\mathbf{x}) = \sqrt{\sum_{i=1,\dots,n} (x - x_{ji})^2},$$

where n is the dimension of the vector $\mathbf{x}$, will increase. Then we can define an interpolation function by

$$y(\mathbf{x}) = \frac{\sum_{j=1,\dots,m} \Big(Y_j \cdot G_j\big(R_j(\mathbf{x})\big)\Big)}{\sum_{k=1,\dots,m} \Big(G_k\big(R_k(\mathbf{x})\big)\Big)}, \tag{4}$$

where the functions $G_k\big(R_k(\mathbf{x})\big)$, $\forall k = 1,\dots,m$ must fulfil some conditions specified later. Finally, put

$$H_j\big(R_j(\mathbf{x})\big) = \frac{G_j\big(R_j(\mathbf{x})\big)}{\sum_{k=1,\dots,m} \Big(G_k\big(R_k(\mathbf{x})\big)\Big)}. \tag{5}$$

The conditions for the functions $G_k\big(R_k(\mathbf{x})\big)$ defined for the solution of the interpolation problem can be transformed to conditions for the functions $H_j\big(R_j(\mathbf{x})\big)$. To fulfill (3) we can impose conditions

$$\lim_{\mathbf{x} \to \mathbf{X}_k} H_j\big(R_j(\mathbf{x})\big) = 0, \tag{6}$$

for $k, j = 1,\dots,m$ and $k \neq j$ and

$$\lim_{\mathbf{x} \to \mathbf{X}_k} H_j\big(R_j(\mathbf{x})\big) = 1, \tag{7}$$

for $k, j = 1,\dots,m$ and $k = j$.

If $H_j\big(R_j(\mathbf{x})\big)$ are not negative, the condition for decreasing of the defined points' contribution can be transformed to

$$H_j\big(R_j(\mathbf{X}_a)\big) > H_j\big(R_j(\mathbf{X}_b)\big) \qquad \text{if} \qquad R_j(\mathbf{X}_a) < R_j(\mathbf{X}_b). \tag{8}$$

The functions $H_j(\mathbf{x})$ can be understood to be multidimensional membership functions, or membership functions of the multidimensional variable $R_j(\mathbf{x})$. The interpolation function $y(\mathbf{x})$ is a result of the simplified fuzzy reasoning.

After the substitution of (5) to (4) we obtain :

$$y(\mathbf{x}) = \sum_{j=1,\dots,m} \Big(Y_j \cdot H_j\big(R_j(\mathbf{x})\big)\Big). \tag{9}$$

3. Examples of the functions $H_j(R_j(\mathbf{x}))$

Because of the form of function (5) conditions (6) – (8) can be fulfilled for instance by the class of monotone functions $G_j(R_j(\mathbf{x}))$ for which

$$\lim_{R_j(\mathbf{x})\to 0} G_j(R_j(\mathbf{x})) = \infty \quad \text{and} \quad \lim_{R_j(\mathbf{x})\to\infty} G_j(R_j(\mathbf{x})) = 0$$

holds.

We have made experiments with functions $G_j(R_j(\boldsymbol{x}))$ based on the form

$$G_j(R_j(\mathbf{x})) = \frac{1}{R_j^n(\mathbf{x})}\,.$$

This function gives very good results for $n = 2$. After the substitution to (5) we obtain

$$H_j(R_j(\mathbf{x})) = \frac{\left(R_j(\mathbf{x})\right)^{-2}}{\sum\limits_{k=1,\dots,m}\left(\left(R_k(\mathbf{x})\right)^{-2}\right)}\,. \tag{10}$$

This function is used in the next section.

4. Examples of the control surfaces

Because of the access to the LFLC-edu 1.5 computer program ([1]), it was natural to use it for the purposes of the demonstration of the interpolation capabilities of the considered function.

For the examples PD control member surfaces for the inverted pendulum problem have been chosen. We consider the simplest version which is a mathematical pendulum (i.e., a mass point being fastened in a non-mass fixed hinge), in its non-stable inverted position. The considered system with parameters: $g = 10\ \text{ms}^{-2}$; $l = 1$ m; $m = 1$ kg has one degree of freedom and is described by the differential equation :

$$y'' - 10\sin y = u\,,$$

where y is an angle between the pendulum and vertical direction, u is the control action we can imagine as a moment of a force acting in the axis of rotation of the pendulum.

The PD control member is described by the function :

$$u_t = C_{PD}(E_t, \Delta E_t)\,,$$

where variable $E_t = y_t - v$ is the *error* in time t, $\Delta E_t = E_t - E_{t-1}$ is the *change of error*, v is the set-point value (0 in our case), y_t, u_t are the *process output* and *control action* at time t respectively.

Context values of the input and output values are:

$$E \in \left\langle -\frac{\pi}{2}, +\frac{\pi}{2} \right\rangle; \quad \Delta E \in \langle -0.5, 0.5 \rangle; \quad u \in \langle -90, 90 \rangle .$$

Time sample is 0.004 s.

The figures show control surfaces in context values, the axes run from the bottom towards the E variable, from the left to the right for the ΔE variable and from the bottom to the top for the u variable. The view on the system of coordinates is in the 3-dimensional perspective from the azimuth 30° and elevation 30°.

The coordinates x of the vector $\boldsymbol{x}$ in the equations before are standardized to the context values above.

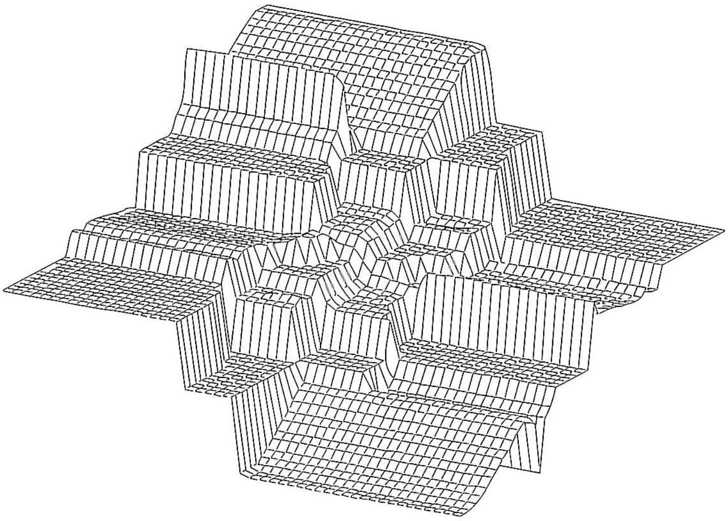

FIGURE 1. The control surface prepared by the linguistically oriented approach. 37 rules used.

Fig. 1 shows the control surface created using the LFLC-edu 1.5 computer program. The original LFLC Inference and Defuzzification mechanism with predefined linguistic values of input and output variables was used. There have been used global values for the ΔE variable which is represented by the left-right orientation of the parts of surface which have one level of output variable. 37 rules were used for the design of the description.

In Fig. 2 the surface created by interpolation of the 101 points is shown which are samples from the control surface in Fig. 1 for the input values typical of used rules. In the interpolation function (10) $m = 6$ points with the smallest values of $R_j(\boldsymbol{x})$ have been used.

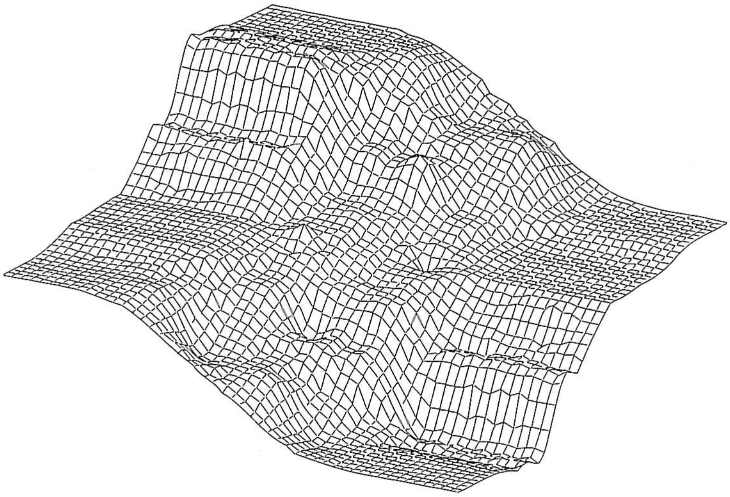

FIGURE 2. The Control surface prepared by interpolation function ([10]). 101 typical points from the control surface prepared by linguistically oriented approach, $m = 6$ have been used.

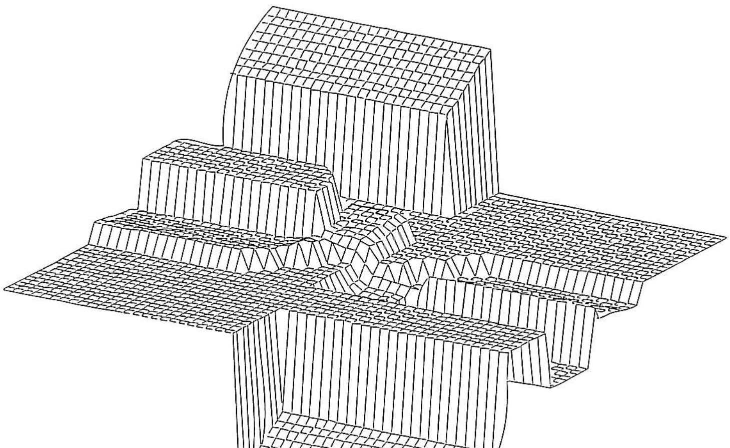

FIGURE 3. The Control surface prepared by the linguistically oriented approach. 23 rules used.

Fig. 3 – 5 demonstrate the interpolation used for covering the "holes" in the designed surface. The surface created by LFLC-edu using 23 rules is in Fig. 3, in Fig. 4 is the same source with the parts not "covered" by rules used from

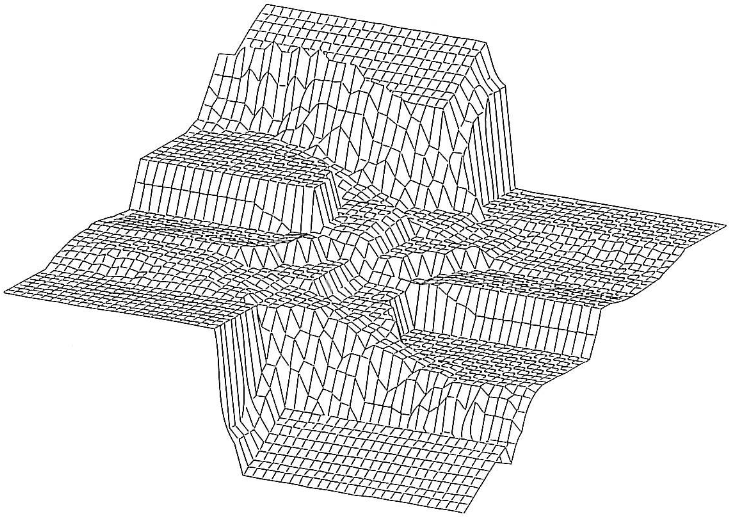

FIGURE 4. The Control surface prepared by the linguistically oriented approach. 23 rules used. In the areas without rules interpolation ([10]) from 67 points, $m = 6$ was used.

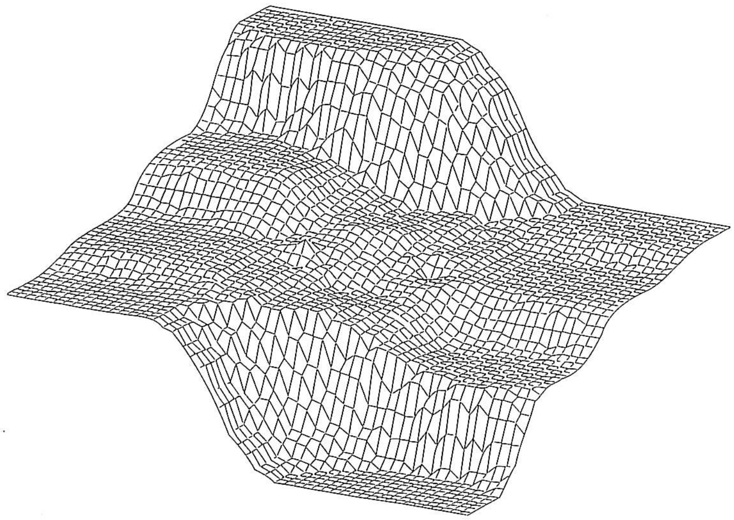

FIGURE 5. The Control surface prepared by interpolation function ([10]). Used 67 typical points from the control surface described by 23 linguistic rules, $m = 6$.

the surface on the Fig. 5 created like the source in Fig. 2 from 67 sample points taken from surface on Fig. 3.

5. Conclusion

From the experiments illustrated by the figures above we may deduce, that this interpolation method is useful in the case of manual designing and tuning of the linguistically oriented descriptions of the control members. It is recommended for the phase of designing because there the speed of inference is not critical. It will be better for the specific application to use some kind of quick inference method with automatic generation rules from the known control surface, look up table method or the quick interpolation method. Some examples may be found in references [2] and [3].

It is possible to influence the shape of the multidimensional membership function (5) by adding the multiplicative coefficients to the coordinates of the vector $\boldsymbol{x}$.

It is also possible to add weight coefficient for the function $H_j(\boldsymbol{x})$ in (9) to influence "contribution" of some points in the control source.

These and some other improvements are to be the topic of further research.

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