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DIFFERENCES ON $[0,1]$

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ABSTRACT. Differences on the unit interval $[0,1]$ are studied. A close relationship of these differences with the nilpotent triangular conorms is shown. Consequently, a representation of differences by means of normed generators is proved. A similar representation is shown for differences on $[0,1)$, where the corresponding generator can be unbounded.

1. Introduction

Let S be a triangular conorm (t-conorm in short) and let T be a triangular norm (t-norm in short), i.e., an associative, commutative, non-decreasing binary operation on the unit interval $[0,1]$ with the usual order and with the neutral element 0 and 1, respectively. We can define the following binary operations on $[0,1]$, see, e.g., [1, 5, 7], resembling the usual difference:

$$b -_{\mathrm{T}} a = \sup(x \in [0,1];\ T(a,x) < b),$$
$$b -_{\mathrm{S}} a = \inf(x \in [0,1];\ S(a,x) > b).$$

It is easy to see that if T and S form a dual pair, i.e., $S(a,b) = 1-T(1-a,1-b)$ for all $a,b \in [0,1]$, then the following version of de Morgan law holds:

$$b -_{\mathrm{S}} a = (a' -_{\mathrm{T}} b')', \quad \text{where} \quad x' = 1-x.$$

Note that the operation $-_{\mathrm{T}}$ is often called a fuzzy implication (and then $-_{\mathrm{S}}$ is called a fuzzy complication). Due to the previous duality, we will deal with t-conorm based differences only. Directly from the definition we see, that for all $a,b,c \in [0,1]$,

$$b -_{\mathrm{S}} a \le b \quad \text{for all} \quad a,b \text{ and } S \tag{D1}$$

and

$$a \le b \quad \text{implies} \quad c -_{\mathrm{S}} b \le c -_{\mathrm{S}} a \quad \text{for all} \quad S. \tag{D2}$$

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Further note that $b -_S a = 0$ whenever $a \geq b$ and that $b -_S 0 = b$. If S is a continuous t-conorm, then the corresponding difference $-_S$ fulfills the following associativity-like property, which is often called *the exchange principle*:

$$(b -_S a) -_S c = (b -_S c) -_S a\,. \tag{D3}$$

Note that the exchange principle does not hold for a general t-conorm S. Take, e.g., a t-conorm S defined for $a, b \in [0,1]$ by

$$S(a,b) = \begin{cases} 1, & \text{if } \min(a,b) > \frac{1}{2}\,, \\ \max(a,b)\,, & \text{otherwise}\,. \end{cases}$$

Then

$$(1 -_S \tfrac{2}{3}) -_S \tfrac{1}{2} = \tfrac{1}{2} -_S \tfrac{1}{2} = 0 \neq \tfrac{1}{2} = 1 -_S \tfrac{2}{3} = (1 -_S \tfrac{1}{2}) -_S \tfrac{2}{3}$$

Recently, several general structures dealing with differences (or relative inverses, in other words) were introduced. The unit interval $[0,1]$ equipped with the difference $-_S$ is an *abelian RI-set* of Kalmbach and Riečanová [2] if and only if (D3) holds, i.e., when S is a continuous t-conorm. The same is true for the difference $-_S$ defined on the halfopen interval $[0,1)$.

If, additionally,

$$b -_S (b -_S a) = a \quad \text{if and only if} \quad 0 \leq a \leq b \leq 1 \tag{D4}$$

holds, then $([0,1], -_S)$ is a *full difference poset* of Mesiar [4].

Finally, if we define $b -_S a$ only in the case $a \leq b$, i.e., $-_S$ is a partial binary operation on $[0,1]$, and (D4) holds true whenever $b -_S a$ is defined, then $([0,1], -_S)$ is a *difference poset* of Kôpka and Chovanec [3] and $([0,1), -_S)$ is an *Abelian RI-poset* of Kalmbach and Riečanová [2].

2. Differences on $[0,1]$

The main goal of this chapter is the characterization of a general difference $\ominus$ on $[0,1]$ fulfilling properties $(D1)$–$(D4)$.

THEOREM 1. *Let $\ominus$ be a difference on $[0,1]$, i.e., a binary operation on $[0,1]$ such that for all $a, b, c \in [0,1]$ it holds*

$$b \ominus a \leq b\,; \tag{D1}$$

$$a \leq b \Longrightarrow c \ominus b \leq c \ominus a\,; \tag{D2}$$

$$(b \ominus a) \ominus c = (b \ominus c) \ominus a\,; \tag{D3}$$

$$b \ominus (b \ominus a) = a \Longleftrightarrow a \leq b\,. \tag{D4}$$

Then there is a unique nilpotent t-conorm S such that $\ominus = -_S$, i.e.,

$$b \ominus a = g^{-1}\big(\max(0, g(b) - g(a))\big),$$

where g is the normed generator of S.

Recall that a continuous t-conorm S is called nilpotent if it is non-strict Archimedean, i.e., when $S(a,a) > a$ for all $a \in (0,1)$ and there is some $b \in (0,1)$ such that $S(b,b) = 1$. A nilpotent t-conorm S has a unique normed generator g, g being an increasing bijection of the unit interval $[0,1]$ onto $[0,1]$, $S(a,b) = g^{-1}\big(\min(1, g(a) + g(b))\big)$. For more details see, e.g., [6].

Before proving Theorem 1, we show the continuity of the difference $\ominus$.

LEMMA 1. *Let $\ominus$ be a difference on $[0,1]$ satisfying the suppositions of the Theorem 1. Then $\ominus$ is a continuous binary operation on $[0,1]$, i.e., if $a_n \to a$ and $b_n \to b$ then also $(b_n \ominus a_n) \to (b \ominus a)$.*

P r o o f. The proof is divided into several steps.

i) We recall first some properties of difference $\ominus$ which can be easily derived from $(D1)$–$(D4)$:

α) $b \ominus a = 0 \iff a \geq b$;

β) $b \ominus a = 0$ and $a \ominus b = 0 \iff a = b$;

γ) $a \leq b \Longrightarrow a \ominus c \leq b \ominus c$;

δ) $a \leq b \leq c \Longrightarrow (c \ominus a) \ominus (c \ominus b) = b \ominus a$ and $(c \ominus a) \ominus (b \ominus a) = c \ominus b$;

ε) $b \ominus a = c \ominus a > 0 \Longrightarrow b = c$ (cancellation law).

The property α) directly implies the continuity of $\ominus$ in the case when $a > b$.

ii) Now, we show that $(b \ominus a_n) \to (b \ominus a)$ when $a = b$. Denote

$$s_n = \sup_{m \geq n} a_m \qquad \text{and} \qquad i_n = \inf_{m \geq n} a_m .$$

If $a = b$, then $0 \leq b \ominus a_n \leq b \ominus i_n$. The sequence $\{b \ominus i_n\}$ is nonincreasing and hence

$$\inf(b \ominus i_n) = c = \lim(b \ominus i_n).$$

The property (D4) ensures $i_n = b \ominus (b \ominus i_n) \leq b \ominus c$ and consequently

$$a = \sup i_n = b \leq b \ominus c .$$

But then by (D2) and β) it is $c = 0$. It follows

$$0 \leq \liminf(b \ominus a_n) \leq \limsup(b \ominus a_n) \leq \limsup(b \ominus i_n) = 0, \quad \text{i.e.,}$$

$$(b \ominus a_n) \to 0 = (b \ominus a).$$

iii) For $a < b$, we keep the notation of ii). Similarly as above we show $a < (b \ominus c)$. On the other hand,

$$c = \inf(b \ominus i_n) \geq (b \ominus \sup i_n) = (b \ominus a).$$

Then

$$a \leq (b \ominus c) \leq b \ominus (b \ominus a) = a,$$

i.e., by δ),

$$a = (b \ominus c) \quad \text{and} \quad c = (b \ominus a),$$

whereas the sequence $\{b \ominus s\}$ is non-decreasing. Put

$$\sup(b \ominus s_n) = d = \lim(b \ominus s_n).$$

From $(b \ominus s_n) \leq (b \ominus a)$ we get immediately that $d \leq (b \ominus a)$ and hence $a \leq (b \ominus d)$. Further, it is, up to possibly some finite number of indexes, $s \leq b$ and thus

$$s_n = b \ominus (b \ominus s_n) \geq (b \ominus d).$$

But then

$$a = \inf s_n \geq (b \ominus d).$$

The last inequality for a and $(b \ominus d)$ together with the previous one leads to the equality

$$a = (b \ominus d), \qquad \text{i.e.,} \qquad d = (b \ominus a).$$

We have

$$(b \ominus a) = c = \lim(b \ominus i_n) \geq \limsup(b \ominus a_n) \geq \liminf(b \ominus a_n) \geq \lim(b \ominus s_n) =$$
$$= d = (b \ominus a),$$

which proves $(b \ominus a_n) \to (b \ominus a)$.

iv) Immediately we obtain the following:

$$(1 \ominus a_n) \to (1 \ominus a);$$

$$(a_n \ominus b) = (1 \ominus b) \ominus (1 \ominus a_n) \to (1 \ominus b) \ominus (1 \ominus a) = (a \ominus b).$$

v) Let s_n and i_n be defined as in ii) and put

$$S = \sup_{m \geq n} b_m \qquad \text{and} \qquad I_n = \inf_{m \geq n} b_m.$$

Suppose that $a = b$ (recall that the case $a > b$ is obvious, see i)).
Then $0 \leq (b \ominus a) \leq (S_n \ominus i_n)$. The sequence $\{S_n \ominus i_n\}$ is nonincreasing and thus $\inf(S_n \ominus i_n) = e = \lim(S_n \ominus i_n)$.

Then $i_n = S_n \ominus (S_n \ominus i_n) \le (S_n \ominus e)$ together with iv) ensure

$$a = \sup i_n \le (S_n \ominus e) \to (b \ominus e) = (a \ominus e),$$

and consequently $e = 0$. As far as $(b_n \ominus a_n) \le (S_n \ominus i_n)$, it follows

$$(b_n \ominus a_n) \to 0 = (b \ominus a).$$

vi) Finally, let $b > a$. Using the previous notation, we see that

$$(I_n \ominus s_n) \le (b_n \ominus a_n) \le (S_n \ominus i_n).$$

By iv), $S_n \ominus (b \ominus a) \to b \ominus (b \ominus a) = a$, and then by v) we have

$$(S_n \ominus (b \ominus a)) \ominus i_n \to 0 \quad \text{(recall that } i_n \to a\text{)}.$$

By the exchange principle (D3), it is

$$\big(S_n \ominus (b \ominus a)\big) \ominus i_n = (S_n \ominus i_n) \ominus (b \ominus a).$$

Recall that $\lim(S_n \ominus i_n) = e$ and thus by iv) it is

$$(S_n \ominus i_n) \ominus (b \ominus a) \longrightarrow e \ominus (b \ominus a),$$

and consequently $e \ominus (b \ominus a) = 0$. The inequalities $S_n \ge b$ and $i_n < a$ ensure $e \ge (b \ominus a)$ and hence by α) it is $e = (b \ominus a)$.
On the other hand, the sequence $\{I_n \ominus s_n\}$ is non-decreasing. Put

$$\sup(I_n \ominus s_n) = f = \lim(I_n \ominus s_n).$$

Then due to iv), $f \ge \lim(I_n \ominus s_k) = (b \ominus s_k)$ for all $k = 1, 2, \dots$ and thus $f \ge \lim_k (b - s_k) = (b \ominus a)$. Then

$$(b \ominus a) \le f = \lim(I_n \ominus s_n) \le \liminf (b_n \ominus a_n) \le \limsup (b_n \ominus a_n) \le$$
$$\le \lim(S_n \ominus i_n) = e = (b \ominus a),$$

i.e., $(b_n \ominus a_n) \longrightarrow (b \ominus a)$. □

P r o o f o f T h e o r e m 1. Let $\ominus$ be a difference on $[0, 1]$ fulfilling $(D1)$–$(D4)$. For $a, b \in [0, 1]$, put

$$S(a, b) = 1 \ominus \big((1 \ominus a) \ominus b\big).$$

Due to the exchange principle $(D3)$, S is a commutative binary operation on $[0, 1]$. By $(D2)$, S is non-decreasing. Further,

$$S(a, 0) = 1 \ominus \big((1 \ominus a) \ominus 0\big) = 1 \ominus (1 \ominus a) = a \quad \text{(by (D4))},$$

i.e., 0 is the neutral element of S. We show the associativity of S:

$$\begin{aligned} S(S(a,b),c) &= 1 \ominus \big((1 \ominus S(a,b)) \ominus c\big) \\ &= 1 \ominus \big((1 \ominus (1 \ominus ((1 \ominus a) \ominus b))) \ominus c\big) \\ &= 1 \ominus \big(((1 \ominus a) \ominus b) \ominus c\big) \\ &= 1 \ominus \big(((1 \ominus b) \ominus a) \ominus c\big) \\ &= 1 \ominus \big(((1 \ominus b) \ominus c) \ominus a\big) \\ &= S\big(S(b,c),a\big) = S\big(a,S(b,c)\big) . \end{aligned}$$

We have just shown that S is a t-conorm. Due to Lemma 1, the difference $\ominus$ is continuous and consequently also our t-conorm S is continuous. Fix an element $a \in (0,1)$. Then also $(1 \ominus a) \in (0,1)$ and $(1 \ominus a) \geq (1 \ominus a) \ominus a$. It follows that

$$a = 1 \ominus (1 \ominus a) < 1 \ominus \big((1 \ominus a) \ominus a\big) = S(a,a) ,$$

i.e., S is an Archimedean continuous t-conorm. The continuity of $\ominus$ ensures the existence of an element $b \in (0,1)$ such that $b = 1 \ominus b$. Then

$$S(b,b) = S\big(b,(1 \ominus b)\big) = 1 \ominus \big((1 \ominus b) \ominus (1 \ominus b)\big) = 1 \ominus 0 = 1 ,$$

and thus S is a nilpotent t-conorm. On the other hand, $b -_{\mathbf{S}} a = 0 = b \ominus a$ if and only if $a \geq b$ and for $a < b$ it is

$$\begin{aligned} b -_{\mathbf{S}} a &= \inf\big(x \in [0,1] ;\ S(a,x) \geq b\big) \\ &= \inf\big(x \in [0,1];\ 1 \ominus ((1 \ominus a) \ominus x) \geq b\big) \\ &= \inf\big(x \in [0,1] ;\ 1 \ominus b \geq (1 \ominus a) \ominus x\big) \\ &= \inf\big(x \in [0,(1 \ominus a)];\ x \geq (1 \ominus a) \ominus (1 \ominus b) = (b \ominus a)\big) \\ &= (b \ominus a) . \end{aligned}$$

Let g be (the only one) normed generator generating nilpotent t-conorm S. Then if $a < b$, it is

$$\begin{aligned} b \ominus a = b -_{\mathbf{S}} a = \inf\big(x \in [0,1] ;\ g^{-1}\big(\min\big(1, g(a) + g(x)\big)\big) \geq b\big) = \\ = \inf\big(x \in [0,1] ;\ \min\big(1, g(a) + g(x)\big) \geq g(b)\big) = \\ = g^{-1}(g(b) - g(a) , \end{aligned}$$

what proves

$$b \ominus a = g^{-1}\big(\max\big(0, g(b) - g(a)\big)\big) \quad \text{for all} \quad a,b \in [0,1] .$$

□

We have just shown that a nilpotent t-conorm can be characterized by means of a difference on $[0,1]$. Properties $(D1)$–$(D4)$ of the difference $\ominus$ ensures all desired properties of the corresponding binary operation S to be a nilpotent t-conorm (commutativity, associativity, monotonicity, boundary conditions, continuity, Archimedean property, nilpotency). If S is not a nilpotent t-conorm, then the difference $-_S$ breaks some of the properties $(D1)$–$(D4)$ on $[0,1]$. However, if S is a strict t-conorm (i.e., S is continuous and strictly monotone on $(0,1)^2$), then $-_S$ fulfills $(D1)$–$(D4)$ on $[0,1)$, but not on $[0,1]$. Note that in the last case, $(1-_S a) = 1$ for all $a \in [0,1)$ and $(1-_S 1) = 0$, and hence (D4) cannot be true for $b = 1$ and $a < 1$. Recall that $([0,1), -_S)$ forms an Abelian RI-poset [2] for each continuous Archimedean t-conorm S, i.e., for strict and nilpotent t-conorms. In the following section we show that there is one-to-one correspondence between Abelian RI-posets on $[0,1)$ and continuous Archimedean t-conorms.

3. Differences on $[0,1)$

An Abelian RI-poset on $[0,1)$ is based on a difference $\ominus$ fulfilling $(D1)$–$(D4)$ on $[0,1)$ restricted to the pairs $b \geq a$.

THEOREM 2. *A binary operation $\ominus$ is a difference on $[0,1)$ fulfilling $(D1)$–$(D4)$ if and only if there is a continuous Archimedean t-conorm S such that $\ominus = -_S$, what means that there is generator g, $g\colon [0,1) \to [0,\infty)$, g is continuous strictly increasing, $g(0) = 0$, so that*

$$b \ominus a = g^{-1}\big(\max(0, g(b) - g(a))\big), \quad a, b \in [0,1).$$

P r o o f. It is enough to show the "if"part. Lemma 1 ensures the continuity of the difference $\ominus$ on each closed square $[0,d]^2$, $d \in (0,1)$, and consequently $\ominus$ is continuous on the whole halfopen square $[0,1)^2$.

For $a = 0$, we put obviously $S(a,b) = b$ for each $b \in [0,1]$.

For $a, b \in (0,1]$, put $S(a,b) = c$ if $a, b, c \in (0,1)$ and $(c \ominus b) = a$ and $S(a,b) = 1$ otherwise (i.e., if either $a = 1$, or $b = 1$, or there is no $c \in (0,1)$ such that $(c \ominus b) = a$). Note that the previous definition of S is equivalent to the following one:

$$S(a,b) = \sup\big(x \in [0,1)\,;\,(x \ominus a) \leq b\big), \quad a, b \in [0,1),$$

together with the boundary condition $S(a,1) = S(1,b) = 1$ for all $a, b \in [0,1]$. The binary operation S is well defined: if $(c \ominus b) = (d \ominus b) = a > 0$, then by the cancellation law ε) it is $c = d$. It is easy to see that the binary operation S is commutative, non-decreasing and that 0 is its neutral element. The continuity of S on $[0,1)^2$ follows from the continuity of $\ominus$ on $[0,1)^2$. The continuity of S on the whole square $[0,1]^2$ is ensured (due to the commutativity and monotonicity of S on $[0,1]^2$) by the following continuity:

$$a_n \to 1 \Longrightarrow S(a_n, b) \to 1 \quad \text{for all} \quad b \in [0,1).$$

Taking into account $1 \geq S(a_n, b) \geq S(a_n, 0) = a_n \to 1$, the last claim is obvious.

We show the associativity of S. Suppose first that for some given $a, b, c \in [0,1]$, it is $S\big(S(a,b), c\big) = d < 1$. Then $(d \ominus c) = S(a,b)$, i.e., $(d \ominus c) \ominus b = a$. But then $b = (d \ominus c) \ominus a = (d \ominus a) \ominus c$ and consequently $d = S\big(S(b,c), a\big) = S\big(a, S(b,c)\big)$ proving the associativity of S in this case.
Now let $S\big(S(a,b), c\big) = 1$. If $\max(a,b,c) = 1$, then obviously also $S\big(a, S(b,c)\big) = 1$ and hence $S\big(S(a,b), c\big) = S\big(a, S(b,c)\big)$. Suppose that each element a, b and c is less than 1, i.e., $\max(a,b,c) < 1$. If $S(a,b) = 1$, then the inequality $S(b,c) \geq b$ ensures $S\big(a, S(b,c)\big) \geq S(a,b) = 1$ and consequently $S\big(S(a,b), c\big) = 1 = S\big(a, S(b,c)\big)$. The same can be easily shown in the case $S(b,c) = 1$. We have to examine the last case when $S(a,b) = e < 1$ and $S(b,c) = f < 1$. Then $(e \ominus b) = a$, $(f \ominus c) = b$ and $S(S(a,b), c) = S(e,c) = 1$, what implies $(d \ominus c) \leq e$ for all $d \in [0,1)$. Recall that if for some d it is $(d \ominus c) \geq e$, then the continuity of the difference $\ominus$ together with the equality $(c \ominus c) = 0$ ensures the existence of an element $d^* \in [0,1)$ such that $(d^* \ominus c) = e$ and hence $S(e,c) = d^* < 1$, a contradiction. Hence for all $d \in [0,1)$ we have:

$$(e \ominus b) = a, \quad (f \ominus c) = b \quad \text{and} \quad (d \ominus c) < e.$$

But then

$$\begin{aligned}(d \ominus a) &= d \ominus (e \ominus b) = d \ominus \big(e \ominus (f \ominus c)\big) \\ &= (d \ominus c) \ominus \big((e \ominus (f \ominus c)) \ominus c\big) \\ &= (d \ominus c) \ominus \big((e \ominus c) \ominus (f \ominus c)\big) \\ &= (d \ominus c) \ominus (e \ominus f) \\ &< e \ominus (e \ominus f) = f,\end{aligned}$$

which is equivalent to $S(a,f) = S(a, S(b,c) = 1 = S\big(S(a,b), c\big)$, proving the associativity of S. We have just shown that S is a continuous t-conorm.

It remains to show $S(a,a) > a$ for all $a \in (0,1)$. If $S(a,a) = 1$ for some $a < 1$, then obviously $S(a,a) = 1 > a$. If $S(a,a) < 1$ then there is $b \in [0,1)$ such that $(b \ominus a) = a > 0$ and thus $S(a,a) = b > a$ (if not, i.e., if $b \leq a$, then $(b \ominus a) = 0$, a contradiction). This together with the continuity of S proves the Archimedean property of the t-conorm S.

The coincidence of the differences $\ominus$ and $-_{\mathrm{S}}$ follows directly from the definition of S. Let g be an additive generator of S (note that g is unique up to a positive multiplicative constant). Similarly as in Theorem 1, we can show that

$$(b \ominus a) = (b -_{\mathrm{S}} a) = g^{-1}\big(\max\big(0, g(b) - g(a)\big)\big), \quad a, b \in [0,1).$$

□

Due to Theorem 2, continuous Archimedean t-conorms can be characterized by means of differences on $[0,1)$ fulfilling $(D1)$–$(D4)$. We have to decide only whether the corresponding t-conorm S would be a nilpotent or a strict one. By Theorem 1, the nilpotent case allows to extend the difference $\ominus$ from the halfopen interval $[0,1)$ to the closed interval $[0,1]$, preserving $(D1)$–$(D4)$. A similar extension in the strict case is impossible (dropping the axiom $(D4)$). However, the following classification can be easily verified.

COROLLARY 1. *Let $\ominus$ be a difference on $[0,1)$ fulfilling $(D1)$–$(D4)$ and let S be the corresponding t-conorm. Then:*

1) *S is strict t-conorm if and only if for all $a \in [0,1)$ there is an element $c \in [0,1)$ such that $(c \ominus a) = a$;*
2) *S is a nilpotent t-conorm if and only if there is some $a \in [0,1)$ such that $(c \ominus a) < a$ for all $c \in [0,1)$.*

Coming back to the representation of a difference $\ominus$ (fulfilling $(D1)$–$(D4)$) on $[0,1)$ by means of an additive generator g, we see that g is either bounded (in the nilpotent case—then there is a unique normed generator g) or it is unbounded (in the strict case), see [6]. Consequently, a difference $\ominus$ on $[0,1)$ is either isomorphic to the ordinary difference $-$ on $[0,1)$ or to the ordinary difference $-$ on $[0,\infty)$. Note that the ordinary difference $-$ on an interval $[0,d)$, $d \in (0,\infty]$, is defined by $(b-a) = \max(0, b-a)$.

4. Concluding remarks

i) For a given nilpotent t-conorm S, or, equivalently, for a given difference $\ominus$ on $[0,1]$ fulfilling $(D1)$–$(D4)$, we can introduce a strong negation n (order-reversing involution on $[0,1]$), $n(a) = (1 \ominus a)$. Let T be an n-dual t-norm to S, i.e., for all $a, b \in [0,1]$ it is

$$T(a,b) = n\big(S(n(a), n(b))\big).$$

Then

$$(b \ominus a) = (b -_{\mathbf{S}} a) = \big(n(a) -_{\mathbf{S}} n(b)\big)$$

and

$$(b -_{\mathbf{T}} a) = \big(n(a) -_{\mathbf{T}} n(b)\big) = n(b -_{\mathbf{S}} a) = n(b \ominus a).$$

ii) Miyakoshi and Shimbo [5] obtained results similar to Theorems 1 and 2 with respect to the difference $-_{\mathbf{T}}$, where left-continuous t-norms were assumed. However, the right-continuity in the second argument for the differences had to be supposed in [5].

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