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SHAPE OF THE FUZZY CONCLUSION GENERATED BY LINEAR INTERPOLATION OF TRAPEZOIDAL IF ... THEN RULES

LÁSZLÓ T. KÓCZY — SZILVESZTER KOVÁCS

ABSTRACT. The method of linear rule interpolation is one of the possibilities of approximate reasoning in sparse fuzzy rule bases. (This method uses the Extension Principle and an extended definition of the Minkowski distance for comparable fuzzy terms.) In most practical applications, the shape of the membership functions of the terms in the rules is triangular or trapezoidal, so it is interesting to examine the shape of the generated conclusion in these cases. We find that in general the piecewise linear shape is lost in the conclusion, and this means more computational time for the reasoning. It is also examined if the conclusion maintains normality, as multistep interpolative reasoning requires the preservation of this property. The obtained conditions are interpreted from a semantical point of view.

1. Linear interpolation of two fuzzy rules

A fuzzy control algorithm should always be able to infer proper control action for every state of the controlled process. There are some cases when the fuzzy rule base is not complete. For example the fuzzy control rules were derived from experts' knowledge and there are some "gaps" between the rules. The rule base contains all of the most important basic rules, but there are some "filling" rules that are missing. In this case the rule base is sparse:

$$\bigcup_{i=1}^{r} \operatorname{supp}\{A_i\} \subset X,$$

where

$$\mathcal{R} = \{R_1, \dots, R_r\} \qquad \text{the fuzzy rule base},$$
$$R_i = A_i \to B_i \qquad \text{the } i\text{th fuzzy rule},$$

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$$\boldsymbol{X} = X_1 \times X_2 \times \cdots \times X_n \qquad \text{the input universe of discourse},$$
$$\boldsymbol{Y} = Y_1 \times Y_2 \cdots \times Y_m \qquad \text{the output universe of discourse}.$$

In the gaps of the sparse rule base, the use of the classical reasoning algorithms [1, 2] is impossible, because there exist observations x, which do not intersect with any of the rule antecedents:

$$\exists \boldsymbol{x} \in \boldsymbol{X}, \qquad \boldsymbol{x} \cap \bigcup_{i=1}^{r} \operatorname{supp}\{A_i\} = \emptyset .$$

The method of *linear rule interpolation* is one of the possibilities of approximate reasoning in the sparse rule bases. This method is based on the Resolution Principle, and on the Extension Principle. It decomposes the problem of fuzzy approximation into an infinite family of crisp problems, corresponding to the α-cuts of the rules and the observation. It solves the interpolation for each case independently and deduces the fuzzy solution by uniting these results into a fuzzy approximation again. For details of this method see, e.g., [3, 4, 5].
The basic form of fuzzy rule interpolation is the *linear interpolation of two fuzzy rules.* This interpolation deals only with the two rules from rule base, whose antecedents are the closest flanking antecedents to the observation:

$$\begin{aligned}
&\boldsymbol{A}_1 \prec \boldsymbol{x} \prec \boldsymbol{A}_2 , \\
&\operatorname{dist}(\boldsymbol{A}_1, \boldsymbol{x}) = \min_{i \in \{1, Ix\}} \operatorname{dist}(\boldsymbol{A}_i, \boldsymbol{x}) , \quad I_x \equiv \{ i \mid 1 \le i \le r \,\&\, \boldsymbol{A}_i \prec \boldsymbol{x} \} , \\
&\operatorname{dist}(\boldsymbol{x}, \boldsymbol{A}_2) = \min_{i \in \{1, Jx\}} \operatorname{dist}(\boldsymbol{x}, \boldsymbol{A}_i) , \quad J_x \equiv \{ i \mid 1 \le i \le r \,\&\, \boldsymbol{x} \prec \boldsymbol{A}_i \} .
\end{aligned}$$

The $\operatorname{dist}(\boldsymbol{F}, \boldsymbol{G})$ denotes the fuzzy distance between the fuzzy sets $\boldsymbol{F}$ and $\boldsymbol{G}$. The complete information on the fuzzy distance is two extended "fuzzy sets", $d_L^\alpha(\boldsymbol{F}, \boldsymbol{G})$ and $d_U^\alpha(\boldsymbol{F}, \boldsymbol{G})$ which are two families of distances (corresponding to the α-cuts) between $\inf\{\boldsymbol{F}_\alpha\}$, $\inf\{\boldsymbol{G}_\alpha\}$ and $\sup\{\boldsymbol{F}_\alpha\}$, $\sup\{\boldsymbol{G}_\alpha\}$ (e.g., Fig. 1). If the universe of discourse of the fuzzy sets $\boldsymbol{F}, \boldsymbol{G}$ is multidimensional, the distances between $\inf\{\boldsymbol{F}_\alpha\}$, $\inf\{\boldsymbol{G}_\alpha\}$ and $\sup\{\boldsymbol{F}_\alpha\}$, $\sup\{\boldsymbol{G}_\alpha\}$ can be defined in the Minkowski sense:

$$d_L^\alpha(\boldsymbol{F}, \boldsymbol{G}) = \Big(\sum_{i=1}^{k} d_L^\alpha(F_i, G_i)^w \Big)^{1/w} ,$$
$$d_U^\alpha(\boldsymbol{F}, \boldsymbol{G}) = \Big(\sum_{i=1}^{k} d_U^\alpha(F_i, G_i)^w \Big)^{1/w} ,$$

where

$$\boldsymbol{F}, \boldsymbol{G} \in X_1 \times X_2 \times \cdots \times X_k \qquad \text{the universe of discourse of the fuzzy sets}.$$

There are certain necessary conditions for defining fuzzy distances between fuzzy sets. One is the existence of full ordering in every component of the universe of discourse of the fuzzy sets, and as a consequence, the existence of a partial ordering $\langle$ in the universe of discourse (graduality of the components). The other one is the existence of distances in every component of the universe of discourse of the fuzzy sets.

A further important restriction is that all the comparable fuzzy sets should be convex and normal, otherwise some α-cuts are not connected or do not exist at all, which makes the distance corresponding to these α-cuts meaningless.

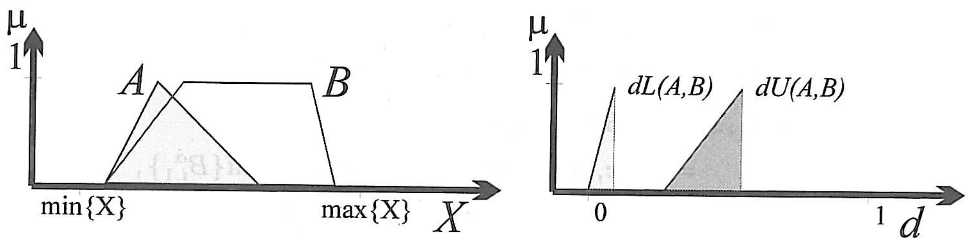

FIG. 1. Normalized fuzzy distance between the fuzzy sets $\boldsymbol{A}$ and $\boldsymbol{B}$; $d_L^\alpha(\boldsymbol{A},\boldsymbol{B})$, $d_U^\alpha(\boldsymbol{A},\boldsymbol{B})$.

The Fundamental Equation of the Linear Interpolation of Two Fuzzy Rules is:

$$\mathrm{dist}(\boldsymbol{A}_1,\boldsymbol{x}) : \mathrm{dist}(\boldsymbol{x},\boldsymbol{A}_2) = \mathrm{dist}(\boldsymbol{B}_1,\boldsymbol{y}) : \mathrm{dist}(\boldsymbol{y},\boldsymbol{B}_2)\,,$$

where

$$\boldsymbol{A}_1 \langle \boldsymbol{x} \langle \boldsymbol{A}_2 \quad \text{and} \quad \boldsymbol{B}_1 \langle \boldsymbol{B}_2\,,$$

$$R_i = \boldsymbol{A}_i \to \boldsymbol{B}_i\,, \quad i \in [1,2]\,, \qquad \text{the fuzzy rules flanks the observation } \boldsymbol{x}\,.$$

The partial ordering $\langle$ as a precedence "$<$" or "$>$" can have a different meaning on the side of antecedents and consequents. The only important thing is the flanking of the observation on the antecedent side and the existence of the ordering on the consequent side (the consequents of the two rules must be comparable in the sense of the $\langle$ ordering).

With the extended fuzzy sets of fuzzy distances the fundamental equation is written as:

$$d_L^\alpha(\boldsymbol{A}_1,\boldsymbol{x})\colon d_L^\alpha(\boldsymbol{x},\boldsymbol{A}_2) = d_L^\alpha(\boldsymbol{B}_1,\boldsymbol{y})\colon d_L^\alpha(\boldsymbol{y},\boldsymbol{B}_2)\,,$$

$$d_U^\alpha(\boldsymbol{A}_1,\boldsymbol{x})\colon d_U^\alpha(\boldsymbol{x},\boldsymbol{A}_2) = d_U^\alpha(\boldsymbol{B}_1,\boldsymbol{y})\colon d_U^\alpha(\boldsymbol{y},\boldsymbol{B}_2)\,, \quad \forall\alpha \in (\Lambda_{A_1} \cup \Lambda_{A_2} \cup \Lambda_{B_1} \cup \Lambda_{B_2})\,,$$

where

$\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{x}$: convex and normal fuzzy sets,

d_L: lower fuzzy distance of the α-cuts,

d_U: upper fuzzy distance of the α-cuts.

It means, that for the interpolative reasoning of two rules $2 \cdot |\Lambda_{A_1} \cup \Lambda_{A_2} \cup \Lambda_{B_1} \cup \Lambda_{B_2}|$ equations must be separately solved (where Λ_F denotes the corresponding level set of the fuzzy set F).

If the distance in the consequence universe can be calculated as the difference of the coordinates, for all i conclusion variables, the consequence is:

$$d_L^\alpha(B_{i,1}, y_i) = \inf\{y_i^\alpha\} - \inf\{B_{i,1}^\alpha\}, \qquad d_L^\alpha(y_i, B_{i,2}) = \inf\{B_{i,2}^\alpha\} - \inf\{y_i^\alpha\},$$
$$d_U^\alpha(B_{i,1}, y_i) = \sup\{y_i^\alpha\} - \sup\{B_{i,1}^\alpha\}, \qquad d_U^\alpha(y_i, B_{i,2}) = \sup\{B_{i,2}^\alpha\} - \sup\{y_i^\alpha\},$$

$$d_L^\alpha(\boldsymbol{A}_1, \boldsymbol{x}) \cdot d_L^\alpha(y_i, B_{i,2}) = d_L^\alpha(B_{i,1}, y_i) \cdot d_L^\alpha(\boldsymbol{x}, \boldsymbol{A}_2),$$
$$d_U^\alpha(\boldsymbol{A}_1, \boldsymbol{x}) \cdot d_U^\alpha(y_i, B_{i,2}) = d_U^\alpha(B_{i,1}, y_i) \cdot d_U^\alpha(\boldsymbol{x}, \boldsymbol{A}_2),$$
$$d_L^\alpha(\boldsymbol{A}_1, \boldsymbol{x}) \cdot \inf\{B_{i,2}^\alpha\} - d_L^\alpha(\boldsymbol{A}_1, \boldsymbol{x}) \cdot \inf\{y_i^\alpha\} =$$
$$= d_L^\alpha(\boldsymbol{x}, \boldsymbol{A}_2) \cdot \inf\{y_i^\alpha\} - d_L^\alpha(\boldsymbol{x}, \boldsymbol{A}_2) \cdot \inf\{B_{i,1}^\alpha\},$$

$$\inf\{y_i^\alpha\} = \frac{\boldsymbol{w}_{1L}^\alpha \inf\{B_{i,1}^\alpha\} + \boldsymbol{w}_{2L}^\alpha \inf\{B_{i,2}^\alpha\}}{\boldsymbol{w}_{1L}^\alpha + \boldsymbol{w}_{2L}^\alpha},$$

where

$$\boldsymbol{w}_{1L}^\alpha = \frac{1}{d_L^\alpha(\boldsymbol{A}_1, \boldsymbol{x}}, \qquad \boldsymbol{w}_{2L}^\alpha = \frac{1}{d_L^\alpha(\boldsymbol{x}, \boldsymbol{A}_2)},$$

and

$$\sup\{y_i^\alpha\} = \frac{\boldsymbol{w}_{1U}^\alpha \sup\{B_{i,1}^\alpha\} + \boldsymbol{w}_{2U}^\alpha \sup\{B_{i,2}\}}{\boldsymbol{w}_{1U}^\alpha + \boldsymbol{w}_{2u}^\alpha},$$

where

$$\boldsymbol{w}_{1U}^\alpha = \frac{1}{d_U^\alpha(\boldsymbol{A}_1, \boldsymbol{x})}, \qquad \boldsymbol{w}_{2u}^\alpha = \frac{1}{d_U^\alpha(\boldsymbol{x}, \boldsymbol{A}_2)}.$$

Solving these equations we get the result in the form of level set equations (using the Representation Theorem):

$$y^\alpha = \left[\inf\{y^\alpha\}, \sup\{y^\alpha\}\right],$$
$$\boldsymbol{y} = \cup_\alpha \alpha \cdot \boldsymbol{y}^\alpha, \qquad \forall \alpha \in (\Lambda_{A_1} \cup \Lambda_{A_2} \cup \Lambda_{B_1} \cup \Lambda_{B-2}).$$

2. The shape of the fuzzy conclusion generated by linear interpolation in fuzzy rule base which contains only trapezoidal and triangular terms

There are a lot of practical implementations of the fuzzy logic control where the membership functions of the terms are restricted to triangular or trapezoidal (or at least piecewise linear). Therefore it is interesting to examine the shape of the generated conclusion in these special cases.

Let us now examine the shape of the conclusion generated by the linear interpolation from trapezoidal terms. A trapezoidal membership function can be described by four points, the infima and suprema of its support and core:

$$\inf\{\operatorname{supp} F\} = F_L^S\,, \qquad \sup\{\operatorname{supp} F\} = F_U^S\,,$$
$$\inf\{\operatorname{core} F\} = F_L^C\,, \qquad \sup\{\operatorname{core} F\} = F_U^C\,.$$

The rules to be interpolated will be denoted by:

"If x is A_1 then y is B_1" and

"If x is A_2 then y is B_2",

where x denotes the observation and y is the conclusion.

The observation x, the antecedents A_i and the consequents B_i are trapezoidal. Then the equations of the left and right flanks of x, A_i and B_i are:

$$x_L^\alpha = \alpha(x_L^C - x_L^S) + x_L^S\,, \qquad x^\alpha U = \alpha(x_U^C - x_U^S) + x_U^S\,,$$
$$A_{iL}^\alpha = \alpha(A_{iL}^C - A_{iL}^S) + A_{iL}^S\,, \qquad A_{iU}^\alpha = \alpha(A_{iU}^C - A_{iU}^S) + A_{iU}^S\,,$$
$$B_{iL}^\alpha = \alpha(B_{iL}^C - B_{iL}^S) + B_{iL}^S = \alpha s_{iL} + k_{iL}\,,$$
$$B_{iU}^\alpha = \alpha(B_{iU}^C - B_{iU}^S) + B_{iU}^S = \alpha s_{iU} + k_{iU}\,, \qquad i \in [1,2]\,.$$

With the equation of these flanks the distances are:

$$d_L^\alpha(A_1, x) = \alpha(x_L^C - x_L^S - A_{1L}^C + A_{1L}^S) + x_L^S - A_{1L}^S = \alpha \cdot m_{1L} + c_{1L}\,,$$
$$d_L^\alpha(x, A_2) = \alpha(A_{2L}^C - A_{2L}^S - x_L^C + x_L^S) + A_{2L}^S - x_L^S = \alpha m_{2L} + c_{2L}\,,$$
$$d_U^\alpha(A_1, x) = \alpha(x_U^C - x_U^S - A_{1U}^C + A_{1U}^S) + x_U^S - A_{1U}^S = \alpha m_{1U} + c_{1U}\,,$$
$$d_U^\alpha(x, A_2) = \alpha(A_{2U}^C - A_{2U}^S - x_U^C + x_U^S) + A_{2U}^S - x_U^S = \alpha m_{2U} + c_{2U}\,.$$

Statement 1. *The equation of the left and right slope of the conclusion calculated from the linear interpolation of two rules $A_1 \to B_1$ and $A_2 \to B_2$ and the observation x if the terms are trapezoidal is*

$$\inf\{y^\alpha\} = \\ = \frac{\alpha^2(m_{2l}s_{1L}+m_{1L}s_{2L})+\alpha(m_{2L}k_{1L}+c_{2L}s_{1L}+m_{1L}k_{2L}+c_{1L}s_{2L})+c_{2L}k_{1L}+c_{1L}k_{2L}}{\alpha(m_{1L}+m_{2L}+c_{1L}+c_{2L}}\,,$$

$$\sup\{y^\alpha\} = \\ = \frac{\alpha^2(m_{2U}s_{1U}+m_{1U}s_{2U})+\alpha(m_{2U}k_{1U}+c_{2U}s_{1U}+m_{1U}k_{2U}+c_{1U}s_{2U})+c_{2U}k_{1U}+c_{1U}k_{2U}}{\alpha(m_{1U}+m_{2U})+c_{1U}+c_{2U}}\,,$$

where

$$m_{1L} = x_L^C - x_L^S - A_{1L}^C + A_{1L}^S\,, \qquad c_{1L} = x_L^S - A_{1L}^S\,,$$
$$m_{2L} = A_{2L}^C - A_{2L}^S - x_L^C + x_L^S\,, \qquad c_{2L} = A_{2L}^S - x_L^S\,,$$
$$m_{1U} = x_U^C - x_U^S - A_{1U}^C + A_{1U}^S\,, \qquad c_{1U} = x_U^S - A_{1U}^S\,,$$
$$m_{2U} = A_{2U}^C - A_{2U}^S - x_U^C + x_U^S\,, \qquad c_{2U} = A_{2U}^S - x_U^S\,,$$
$$s_{iL} = B_{iL}^C - B_{iL}^S\,, \qquad k_{iL} = B_{iL}^S\,,$$
$$s_{iU} = B_{iU}^C - B_{iU}^S\,, \qquad k_{iU} = B_{iU}^S\,, \qquad i \in [1,2]\,.$$

The result of the Statement presents that the linearity of the membership functions of the terms (observation, antecedents and consequents) is not preserved in the conclusion (e.g., Fig. 2).

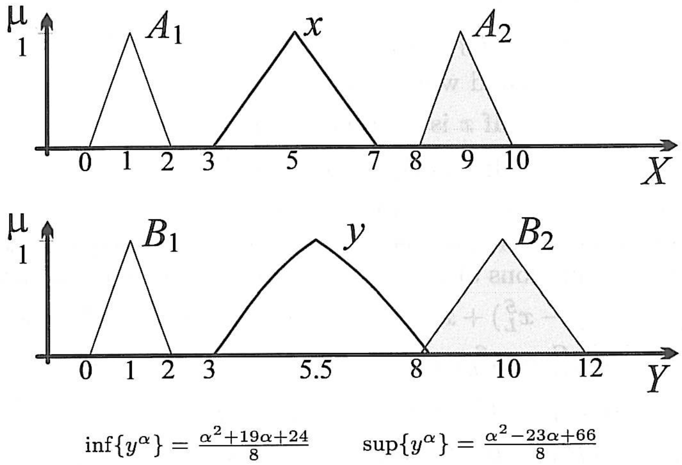

Fig. 2 Linearity of the membership functions of the terms is not preserved in the conclusion

COROLLARY 1. *The slopes of the conclusion y are linear if either both the two antecedents or the two consequents of the rules have the same slope pairwise on each side.*

P r o o f . If either both the two antecedents and the two consequents of the rules have the same slope on its each side pairwise

$$A^C_{1L} - A^S_{1L} = A^C_{2L} - A^S_{2L} \Longrightarrow m_{1L} + m_{2L} = 0\,,$$
$$A^C_{1U} - A^S_{1U} = A^C_{2U} - A^S_{2U} \Longrightarrow m_{1U} + m_{2U} = 0\,,$$
$$B^C_{1L} - B^S_{1L} = B^C_{2L} - B^S_{2L} \Longrightarrow s_{1L} = s_{2L}\,,$$
$$B^C_{1U} - B^S_{1U} = B^C_{2U} - B^S_{2U} \Longrightarrow s_{1U} = s_{2U}\,,$$

then

$$m_{1L} \cdot s_{2L} + m_{2L} \cdot s_{1L} = 0\,,$$
$$m_{1U} \cdot s_{2U} + m_{2U} \cdot s_{1U} = 0\,,$$

$$\inf\{y^\alpha\} = \frac{\alpha(m_{2L}k_{1L} + c_{2L}s_{1L} + m_{1L}k_{2L} + c_{1L}s_{2L}) + c_{2L}k_{1L} + c_{1L}k_{2L}}{c_{1L} + c_{2L}}\,,$$
$$\sup\{y^\alpha\} = \frac{\alpha(m_{2U}k_{1U} + c_{2U}s_{1U} + m_{1U}k_{2U} + c_{1U}s_{2U}) + c_{2U}k_{1U} + c_{1U}k_{2U}}{c_{1U} + c_{2U}}\,,$$

the slopes of the conclusion y are linear. □

These are very strong restrictions for the shape of the terms, but they are satisfied in many practical cases as, e.g., when the state variables are covered by "equidistant" terms.

3. The convexity and normality of the fuzzy conclusion

The terms and the observation of the classical reasoning algorithms [1, 2], and the linear rule interpolation are restricted to convex and normal fuzzy sets. Using multiple level reasoning it is important to check the normality of the generated results between the reasoning levels. Therefore, it is interesting to examine:

— if the conclusion generated by the linear rule interpolation is suitable for further reasoning steps;

— if it keeps normality or not.

Remark 1. The conclusion generated by the linear rule interpolation is always convex.

Proof. A fuzzy set is convex if all the α-cuts of the set are connected. The α-cuts of the conclusion generated by the linear rule interpolation are always connected, because the conclusion is derived from a union of intervals (resolution form):

$$y = \cup_\alpha \alpha \cdot \left[\inf\{y^\alpha\}, \sup\{y^\alpha\}\right], \qquad \forall \alpha \in (\Lambda_{A1} \cup \Lambda_{A2} \cup \Lambda_{B1} \cup \Lambda_{B2}).$$

The *normality of the conclusion* is depending on its hight. The fuzzy conclusion is normal, if its membership function assumes all values in the interval $[0,1]$.

The α-cuts of the conclusion are intervals. For checking the normality, we have to examine the order of the end points of the intervals. These points should always be ordered (in sense of $\langle\ \rangle$ in proper way:

$$\inf\{y^\alpha\} \le \sup\{y^\alpha\}, \qquad \forall \alpha.$$

If the above requirement is not fulfilled, the membership function is not real, it "wraps around itself" (Fig. 3).

The conclusion y is normal, if

$$\text{height}(y) = \max_{[0,1]} \{\alpha \mid \inf\{y^\alpha\} \sup\{y^\alpha\}\} = 1.$$

Satisfying that every α-cut is contained in every lower α-cut, it is enough to examine the upper and lower core points of the conclusion $\left(\inf\{y^1\}, \sup\{y^1\}\right)$ for checking it's normality:

$$\inf\{y^1\} = y_L^C = \frac{(A_{2L}^C - c_L^C)B_{1L}^C + (x_L^C - A_{1L}^C)B_{2L}^C}{A_{2L}^C - {}_{1L}^C},$$

$$\sup\{y^1\} = y_U^C = \frac{(A_{2U}^C - x_U^C)B_{1U}^C + (x_U^C - A_{1U}^C)B_{2U}^C}{A_{2U}^C - A_{1U}^C}.$$

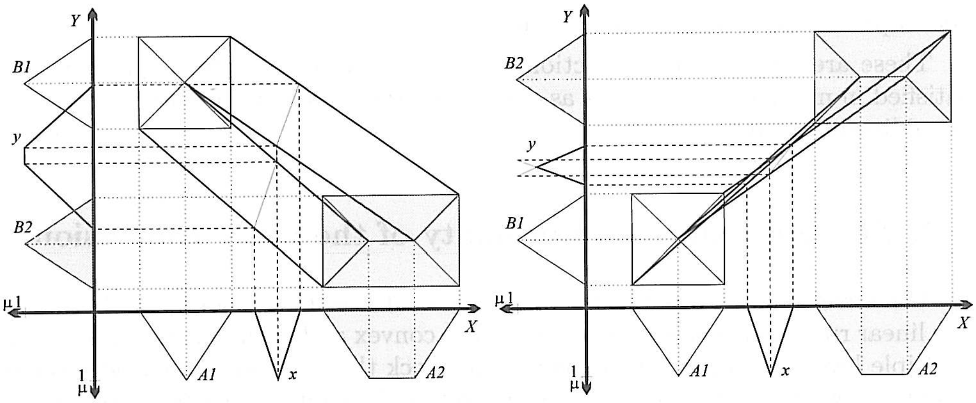

FIG. 3. Normality of the conclusion generated by linear interpolation is not always satisfied

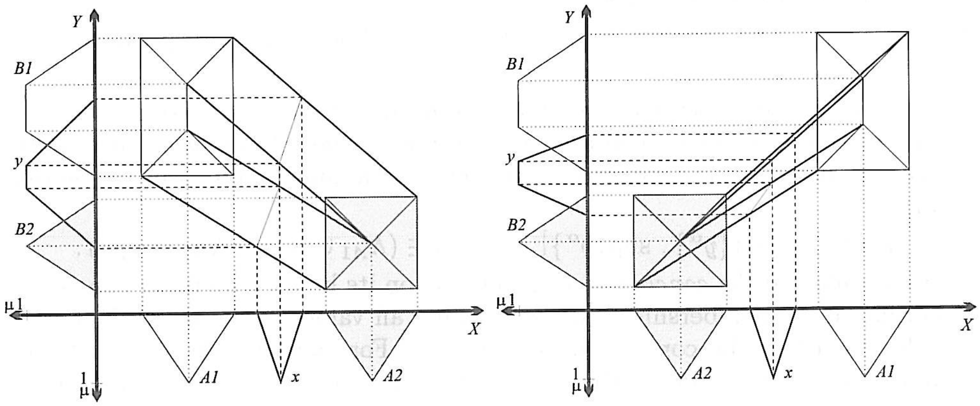

FIG. 4. If the antecedents and the observation are triangular, the conclusion is normal

STATEMENT 2. *The conclusion y calculated from trapezoidal terms with linear interpolation of two rules is normal if and only if*

$$\inf\{y^1\} = \sup\{y^1\},$$

$$\frac{(A^C_{2L} - x^C_L)B^C_{1L} + (x^C_L - A^C_{1L})B^C_{2L}}{A^C_{2L} - A^C_{1L}} \le \frac{(A^C_{2U} - x^C_U)B^C_{1U} + (x^C_U - A^C_{1U})B^C_{2U}}{A^C_{2U} - A^C_{1U}}.$$

Generally this premise is not fulfilled, so the normality of the conclusion is not necessarily true (e.g., Fig. 3).

Moreover there are cases, when no conclusion exists at all (e.g., Fig. 5).

COROLLARY 2. *If the terms are triangular, the conclusion is always normal.*

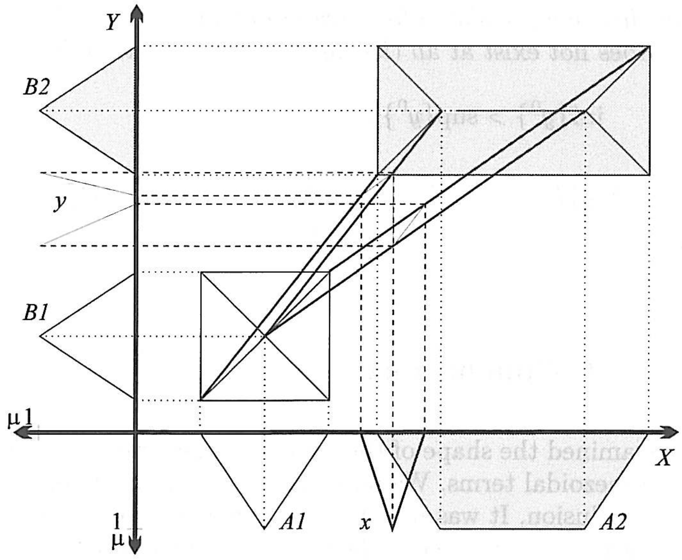

FIG. 5. A case when the conclusion does not exist at all

P r o o f. If the terms are triangular, the lower and upper points of the cores of the membership functions are equal ($A_{iL}^C = A_{iU}^C$, $B_{iL}^C = B_{iU}^C$, $x_L^C = x_U^C$). Because of this $\inf\{y^1\} = \sup\{y^1\}$, so the conclusion is normal (Statement 2) (e.g., Fig. 2). □

COROLLARY 3. *If the terms of the antecedents and the observation are triangular, the conclusion is always normal (e.g., Fig. 4).*

P r o o f. If the terms of the antecedents and the observation are triangular, the lower and upper points of the cores of the membership functions are equal:

$$A_{iL}^C = A_{iU}^C = A_i^C, \qquad x_L^C = x_U^C = x^C.$$

Because of this we can rewrite the expression of Statement 2 into:

$$\frac{(A_2^C - x^C)B_{1L}^C + (x^C - A_1^C)B_{2L}^C}{A_2^C - A_1^C} \le \frac{(A_2^C - x^C)B_{1U}^C + (x^C - A_1^C)B_{2U}^C}{A_2^C - A_1^C}.$$

Solving this inequality:

$$\begin{array}{lll} \text{if } A_1^C \le x^C \le A_2^C & \text{then} & 0 \le (A_2^C - x^C)(B_{1U}^C - B_{1L}^C) + (x^C - A_1^C)(B_{2U}^C - B_{2L}^C), \\ \text{if } A_1^C \ge x^C \ge A_2^C & \text{then} & 0 \ge (A_2^C - x^C)(B_{1U}^C - B_{1L}^C) + (x^C - A_1^C)(B_{2U}^C - B_{2L}^C). \end{array}$$

These conditions are always fulfilled ($B_{iL}^C \le B_{iU}^C$), so the conclusion is normal.

STATEMENT 3. *The conclusion y calculated from trapezoidal terms with linear interpolation of two rules does not exist at all (its hight is zero) if and only if*

$$\inf\{y^0\} > \sup\{y^0\},$$

$$\frac{(A_{2L}^S - x_L^S)B_{1L}^S + (x_l^S - A_{1L}^S)B_{2L}^S}{A_{2L}^S - A_{1L}^S} > \frac{(A_{2U}^S - x_U^S)B_{1U}^S + (x_u^S - A_{1U}^S)B_{2U}^S}{A_{2U}^S - A_{1U}^S}.$$

4. Conclusions

In this paper we have examined the shape of the conclusion generated by the linear interpolation from trapezoidal terms. We have shown that in general, the linear shape is lost in the conclusion. It was investigated under what conditions the linear shape is kept. It was also examined if the conclusion maintained convexity and normality. Convexity automatically follows from the algorithm, but normality is not always true. Moreover there are cases — because of the non-normality — when the conclusion does not exist at all. Conditions for normality in general (with some special cases) and for the reality of the generated conclusion were also determined. It would be a very interesting question to examine, if the generated (non-linear) conclusion could be substituted by a linear one, to reduce the computational time of the reasoning algorithm. A more general analysis of the behaviour of the conclusion for arbitrarily shaped piecewise linear antecedents and consequents will follow in [6, 7].

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Dept. of Telecommunication and Telematics
Technical University of Budapest
Stoczek u.2
Budapest H–1111
HUNGARY

Computer Centre
University of Miskolc
Miskolc–Egyetemváros
Miskolc, H–3515
HUNGARY

E-mail: szkszilv@gold.uni-miskolc.hu