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ON SOME AUTOMORPHISM GROUPS OF LOGICS

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ABSTRACT. In this paper, we deal with a group of automorphisms of logics. The results concern mainly with the logics of idempotents of rings. A binary operation with orthocomplementation on the logic $U(R)$ of all idempotents of an associative ring R with identity are defined. The binary operation is in general non associative and non commutative. The properties of the left Jordan groupoid are then studied.

1. Introduction

The study of automorphisms of logics and orthomodular lattices has its origin in explanation of the role of some symmetry groups appearing in quantum theories. The first attempt was made in the paper [2] of Emch and Piron and later (1977) for denumerable Boolean algebras by Mc Kenzie [11]. G. Kalmbach [7] has shown that for each group G there exists such logic L that the group $\mathrm{Aut}(L)$ of all automorphisms of L is isomorphic with G. A similar result for finite groups was obtained by G. Schrag [15]. Kallus and Trnková [8] exhibit a construction of logics with given automorphism groups and given atomistic sublogics. M. Navara [12] presents a construction of logics with given centers state spaces and sublogics. The paper [13] of M. Navara and I. Tkadlec contains similar results as will as a section which is devoted to the automorphism group of concrete logics. Further papers [1], [3]–[5] and [14], [16] concern also the automorphism groups of the set of all states of the logic L.

The present contribution is devoted to the study of the automorphism groups of logics of idempotents (or projectors) of the rings. The last section includes also the considering of the operations which have been introduced on the logics $U(R)$ (or $P(R)$) of all idempotents (or projectors) of the ring R.

The familiarity with such notions as logic, orthomodular lattice, Boolean algebra, block, orthoisomorphism is assumed (see also [6]).

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2. Basic notions

Let $(L, \leq, 0, 1, ')$ denote an orthoposet.

DEFINITION 1. An *automorphism* of L is a bijection $\alpha : L \to L$ such that both α and α^{-1} preserve the orthocomplements and partial ordering. (Thus they preserve also all joins and meets which exist in L.)

DEFINITION 2. Let $(L, \leq, 0, 1, ')$ be an orthoposet then two elements $p, q \in L$ are compatible if they are contained in a Boolean subalgebra of L. In this case we write $p \leftrightarrow q$. By $C(L)$ we denote the center of L, i.e., the set $C(L) = \{c \in L;\, c \leftrightarrow d \text{ for each } d \in L\}$.

DEFINITION 3 ([9]). Let $(L, \leq, 0, 1, ')$ be an orthoposet. The non-empty subset B, $0 \neq B \subset L$ is called an *M–base* if

(B1) $p \in B$, $p \leq q$ implies $q \in B$,
(B2) $\operatorname{card}\{\{p, p'\} \cap B\} = 1$ for each $p \in L$.

If $p \leq q'$ we say that p, q are orthogonal and we write then $p \perp q$.

In [9], I have shown that the M–base is identical with the maximal subset of mutually non-orthogonal elements of L and I have given also some other characterization of the M–base. In the present paper, I will consider the logics of all idempotents (or projectors) of the ring R.

All the considered rings are supposed to be associative and have identity.

DEFINITION 4. Let R be a ring. An element $e \in R$ is said to be an *idempotent* of R if $e^2 = e$. If the ring R is a $*$ring with the involution $*$, then the element $e \in R$ is said to be a *projector* if $e = e^2 = e^*$.

Now the symbol $U(R)$ stands for the set of all idempotents of R. In case that R is a $*$ring, let $P(R)$ be the set of all projections of R. It is clear that $P(R) \subset U(R)$.

When R is the field or integral domain, then $U(R) = \{0, 1\}$ and if R is a Boolean ring, then $U(R) = R$. It is well known [10] that the set of all idempotents of a ring R (or the set $P(R)$ of all projectors of $*$ring) is an orthocomplemented orthomodular poset with respect to the order $p_1 \leq p_2 \Leftrightarrow p_1 p_2 = p_2 p_1 = p_1$, and the orthocomplement $p' = 1 - p$, where $p_1, p_2, p \in R$.

Now we will consider the logics of all idempotents (or projectors) of the $*$ring R. First we introduce some necessary lemmas.

LEMMA 1. *Let R be an associative ring with identity and let $e \in R$ be an idempotent of R. Then the element $a = 1 - 2e$ is regular and $a = a^{-1}$.*

Proof. It is clear that $a^{-1} = (1 - 2e)(1 - 2e) = 1 - 4e + 4e^2 = 1$. □

Definition 5. Let R be an associative ring with identity. The bijective mapping $\alpha : R \to R$ is said to be an automorphism of R if

(i) $\alpha(1) = 1$,
(ii) $\alpha(a+b) = \alpha(a) + \alpha(b)$, $a, b \in R$,
(iii) $\alpha(ab) = \alpha(a)\,\alpha(b)$, $a, b \in R$. If the ring R is a $*$ring, then we moreover require,
(iv) $\alpha(a^*) = \big[\alpha(a)\big]^*$.

Lemma 2. *Let R be an associative ring with identity and let $a \in R$ be a regular element of R. Then the mapping $\alpha_a : R \to R$ defined by the identity*

$$\alpha_a(x) = a^{-1}xa, \quad x \in R, \tag{1}$$

is an automorphism of the ring R.

P r o o f. The proof is simple and clear. □

Lemma 3. *Let R be an associative ring with identity, further let $p \in U(R)$ and α be an automorphism of the ring R. Then $\alpha(p) \in U(R)$. In the case when a is a regular, selfadjoint element of the $*$ring R, such that $a^2 = 1$, then $\alpha_a(p) \in P(R)$ provided, that $p \in P(R)$.*

P r o o f. The proof is clear. □

Let α be an automorphism of the ring R. We denote by $\alpha|U$ the restriction of α onto $U(R)$.

Lemma 4. *Let R be an associative ring with identity. If α is a ring automorphism of R, then $\alpha|U$ is an automorphism of the logic $\big(U(R), \leq, 0, 1, '\big)$ onto itself.*

P r o o f. It is clear that $(\alpha|U)(1) = 1$ and $(\alpha|U)(0) = 0$. Let $p, q \in U(R)$ and suppose that $p \perp q$. Then according to [10], it follows that $pq = qp = 0$, and we have $\alpha(pq) = \alpha(qp) = \alpha(p)\alpha(q) = \alpha(q)\alpha(p) = \alpha(0) = 0$. Therefore $\alpha(p) \perp \alpha(q)$ and $(\alpha|U)(p \vee q) = (\alpha|U)(p+q) = (\alpha|U)(p) + (\alpha|U)(q)$ (See [10]).

Let $p \in U(R)$, then $p^\perp = 1 - p \in U(R)$, and we have $(\alpha|U)(p^\perp) = (\alpha|U)(1-p) = (\alpha|U)(1) - (\alpha|U)(p) = 1 - (\alpha|U)(p) = [\alpha(p)]^\perp$. □

Similar proposition turns out to be valid also for the logic $\big(P(R), \leq, 0, 1, '\big)$ of projections of a $*$ring R. We denote now by $\mathrm{Aut}\big(U(R)\big)$ the group of all automorphisms of the logic $U(R)$ (or $\mathrm{Aut}\big(P(R)\big)$ the group of all automorphisms of the logic $P(R)$ of a $*$ring R). The subgroup $\mathrm{Aut}_i\big(U(R)\big)$ of the group $\mathrm{Aut}\big(U(R)\big)$ is defined by setting $\mathrm{Aut}_i\big(U(R)\big) = \{\alpha_a;\ a$ is a regular element of

R, and the identity (1) holds}. It is obvious that $\mathrm{Aut}_i(U(R)) \subseteq \mathrm{Aut}(U(R))$. The subgroup $\mathrm{Aut}_i(U(R))$ is called the subgroup of all inner automorphisms of $U(R)$. The following example shows that it may happen that $\mathrm{Aut}_i(U(R)) \subsetneq \mathrm{Aut}(U(R))$.

Example 1. Let F_2 be a field consisting only of two elements 0 and 1. One can show that the set

$$L = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0, \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I, \quad \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = P, \right.$$
$$\left. \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = P^{\perp}, \quad \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = Q \quad \text{and} \quad \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = Q^{\perp} \right\}$$

is a lattice of idempotents of the ring $M_{22}(F_2)$ of all (2,2) matrices over F_2. The set $\mathrm{Aut}_i(U(M_{22}))$ contains only two elements: the identity and α_a, where

$$a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

But the group $\mathrm{Aut}(U(M_{22}))$ has also such element β that $\beta(0) = 0$, $\beta(I) = I$, $\beta(P) = P^{\perp}$, $\beta(Q) = Q^{\perp}$. This shows that $\beta \in \mathrm{Aut}(U(M_{22})) \setminus \mathrm{Aut}_i(U(M_{22}))$.

If R is a ring, then we remember that $C(U(R))$ denotes the center of the logic $U(R)$ of all idempotents of R. The set $Z(R) = \{c \in R;\ ac = ca,\ a \in R\}$ will be called the center of R. Evidently $Z(R) \cap U(R) \subset C(U(R))$. The following assertion gives a sufficient condition for $Z(R) \cap U(R) = C(U(R))$.

Proposition 1. *Let R be an associative ring with the identity. If the set $U(R)$ is the set of all generators of R, then $Z(R) \cap U(R) = C(U(R))$.*

Proof. Let $p \in C(U(R))$, then $q \leftrightarrow p$ for each $q \in U(R)$. Therefore $qp = pq$. From the condition of this proposition it follows that $ap = pa$ for each $a \in R$. But this implies that $p \in Z(R) \cap U(R)$. □

Corollary. *Let $M_{nn}(K)$ and $U(M_{nn}(K))$ be the ring of all (n,n)-matrices over the field K, respectively and the set of all idempotents of $M_{nn}(K)$. The Proposition 1 implies that $C(U(M_{nn}(K))) = \{0,1\}$, i.e., the logic $U(M_{nn}(K))$ is irreducible. The same result is obtained for the logic $P(H)$ of all closed projections of the Hilbert space H.*

DEFINITION 6. Let R be an associative ring with identity. The ideal I of R is said to be a *completely prime ideal*, if $ab \in I$, $a, b \in R$ implies that $a \in I$ or $b \in I$. The set $\mathrm{Spec}(R)$ of all completely prime ideals of R is called a completely spectrum of R.

Now we can characterize some of the M–bases of the logic $U(R)$.

PROPOSITION 2. *Let R be an associative ring with identity and let I be a completely prime ideal of R. Then the set $B = (R \setminus I) \cap U(R)$ is an M–base of the logic $U(R)$.*

Proof.

(i) Clearly $1 \in B$.

(ii) Let $p \in U(R)$. Because $p(1-p) = 0 \in I$ and I is a completely prime ideal, we must have $p \in I$ or $1 - p \in I$. But at the same time it cannot be $p \in I$, $1 - p \in I$ because we would then have $p + (1-p) = 1 \in I$. From this it follows that for each $p \in U(R)$ $\mathrm{card}\big(\{p, 1-p\} \cap B\big) = 1$.

(iii) Let $p_1 \in B$, $p_2 \in U(R)$ and let $p_1 \leq p_2$. Then it holds that $p_1 p_2 = p_2 p_1 = p_1$ and we have $1 - p_1 \in I$. Therefore $p_2 - p_1 = p_2(1 - p_1) \in I$ and we obtain $1 - p_2 = (1 - p_1) - (p_2 - p_1) \in I$. But this means that $p_2 \notin I$ and $p_2 \in (R \setminus I) \cap U(R) = B$ and B is an M–base. □

COROLLARY. *According to Proposition 2, there is a mapping s from the set* $\mathrm{Spec}(R)$ *of the ring R into the set $B\big(U(R)\big)$ of all M–bases of the logic $U(R)$. A mapping $s\colon$* $\mathrm{Spec}(R) \to B\big(U(R)\big)$ *is defined by setting $s(I) = (R \setminus I) \cap U(R)$, for each $I \in$* $\mathrm{Spec}(R)$.

PROPOSITION 3. *Let $(L, \leq, 0, 1, ')$ be a logic and let α be the automorphism of this logic. Then the following assertions turn out to be valid:*

(i) *If $p, q \in L$, $p \perp q$ then $\alpha(p) \perp \alpha(q)$.*

(ii) *If B is an M–base of P, then $\alpha(B)$ is also an M–base of P.*

(iii) *If $M(L)$ is the set of all M–bases of L, then the mapping $\alpha^- : M(L) \to M(L)$ defined by setting $\alpha^-(B) = \alpha(B)$, $B \in M(L)$ is a homeomorphism of the topological space $M(L)$.*

(iv) *If $p \leftrightarrow q, p, q \in L$, then $\alpha(p) \leftrightarrow \alpha(q)$. If $C(L)$ is the center of L, then $\alpha(C) = C$. If A is a block of the logic L, then $\alpha(A)$ is also a block of L.*

Proof. The proof is straightforward. □

3. Some remarks concerning the direct products

Given a family L_i, $i \in I$, of logics, then the set L of all sequences $p = \{p_i, i \in I\}$ where $p_i \in L_i$ for every $i \in I$, represents the Cartesian product of the logics L_i. We will then write $L = \prod\limits_{i \in I} L_i$. Now if we define the ordering and orthocomplementation in L pointwise and if $I = \{I_i\}$ and $0 = \{0_i\}$, where I_i and 0_i are the unity and the null-element in L_i, respectively, then L is also a logic.

Now we introduce two propositions.

PROPOSITION 4. *Let $U(R_i)$, $i \in I$, be the logic of idempotents of the ring R_i, and let $G_i = \mathrm{Aut}\big(U(R_i)\big)$ be the automorphism group of the logic $U(R_i)$. Then $\prod\limits_{i \in I} G_i$ is a group of automorphisms of the logic $\prod\limits_{i \in I} U(R_i)$.*

P r o o f . The proof is easy. $\square$

Let $\prod\limits_{i \in I} U(R_i)$ be the logic of idempotents of the ring $\prod\limits_{i \in i} R_i$. Put $E_i' = \{p = \{p_i\}_{i \in I}\,;\ p_i \in U(R_i)\,,\ p_j = 0_j\,, \text{ if } j \neq i\}$, $i \in I$.

Now we have the following proposition:

PROPOSITION 5. *Let $\prod\limits_{i \in I} U(R_i)$ be the logic of idempotents of the ring $R = \prod\limits_{i \in I} R_i$. The sets E_i', $i \in I$, are logics of idempotents of the ring $\prod\limits_{i \in I} R_i$. If $p \in E_i'$, $q \in E_j'$, $i \neq j$, then $p \perp q$. The sets E_i', $i \in I$, need not be the sublogics of $\prod\limits_{i \in I} U(R_i)$. If α is an automorphism of $\prod\limits_{i \in K} U(R_i)$ and K is a finite set of indices, then each $p \in \prod\limits_{i \in K} R_i$ has the form $p = \bigvee\limits_{i \in K} p_i^-$, $p_i^- \in E_i'$, and $\alpha(p) = \sum\limits_{i \in K} \alpha(p_i^-)$.*

P r o o f . It is clear that the elements $p \in E_i'$, $q \in E_j'$, $i \neq j$ are mutually orthogonal, and that the sets E_i, $i \in I$, are the logics. Now, let $p \in \prod\limits_{i \in I} U(R_i)$. Then p can be written as $p = \bigvee\limits_{i \in I} p_i^-$, $p_i^- \in E_i$, $i \in I$. Let α be an automorphism of $\prod\limits_{i \in K} U(R_i)$, then $\alpha(p) = \alpha\Big(\bigvee\limits_{i \in K} p_i^-\Big) = \sum\limits_{i \in K} \alpha(p_i^-)$. $\square$

We remark only that $\alpha|\,E_i'$ need not be an automorphism of E_i'. Therefore, if $\alpha \in \mathrm{Aut}\Big(\prod\limits_{i \in K} U(R_i)\Big)$, then α need not have the form $\alpha = \{\alpha_i\}$, where α_i is an automorphism of $U(R_i)$.

4. Operations on $U(R)$ and $P(R)$

Let R be an associative $*$ring with identity and let $U(R)$ and $P(R)$ be the logics of all idempotents and of all projectors of R, respectively.

By Lemmas 1 and 2 of the second section it is possible to define the inner automorphism $\alpha_p : R \to R$, $p \in U(R)$ setting

$$\alpha_p(x) = (1-2p)x(1-2p), \quad x \in R.$$

This ring automorphism induces on $U(R)$ an automorphism $\alpha^\wedge : U(R) \to U(R)$.
Indeed. If $q \in U(R)$, $p \in U(R)$, then we have

$$\alpha_q^{\wedge 2}(p) = \alpha_q^2(p) = [(1-2q)\,p\,(1-2q)]^2 = [(1-2q)\,p\,(1-2q)(1-2q)\,p\,(1-2q)] = \\ = (1-2q)\,p^2(1-2q) = (1-2q)\,p\,(1-2q) = \alpha_q^\wedge(p).$$

Therefore $\alpha_q^\wedge(p)$ is an idempotent. If p, q are the projectors of the $*$ring R, then $\alpha_q^\wedge(p)$ is also a projector.

R e m a r k . Let $p, q \in U(R)$ then it is possible to define on $U(R)$ the following binary operations $\circ_1$ and $\circ_2$: We put

$$p \circ_1 q = (1-2q)\,p\,(1-2q) = p - 2pq - 2qp + 4qpq,$$
$$p \circ_2 q = (1-2p)\,q\,(1-2p) = q - 2pq - 2qp + 4pqp.$$

Now we can formalize the whole situation in the following definition:

Definition 7. The non-empty set $X \neq 0$ will be called a *left Jordan groupoid*, if on X two operations are defined: a binary operation $\circ : X \times X \to X$ and a unary operation $' : X \to X$ so that

(i) $p \circ p = p$ if $p \in X$,
(ii) $(p \circ q) \circ p = p \circ (q \circ p)$, $p, q \in X$,
(iii) $(p \circ q) \circ q = p$ if $p, q \in X$,
(iv) $(p')' = p$, $p \in X$,
(v) $(p \circ q)' = p' \circ q'$, $p, q \in X$,
(vi) $p \circ q' = p \circ q$, $p, q \in X$,
(vii) X has elements $0 \in X$ and $1 \in X$ such that $p \circ 1 = p$, $1 \circ p = 1$, $p \circ 0 = p$, $0 \circ p = 0$ and $0' = 1$.

R e m a r k . From (i) and (iii) of Definition 4 it follows that $p^2 \circ (q \circ p) = [(p \circ p) \circ q] \circ p$, if $p, q \in X$. In general, the left Jordan groupoid is non-commutative and also non-associative.

Example 2.

1. Logics $U(R)$ and $P(R)$ with respect to the operations $\circ_1$ and orthocomplementation $'$ are the left Jordan groupoids.

Indeed. The operations $\circ$ and $'$ are defined in Proposition 9 (see the last page).

2. The set $\exp X$, $X \neq 0$, where $A \circ B = A$, $A, B \in \exp X$ and $' : \exp X \to \exp X$ is the set-theoretical complementation in X.
3. Let $B = \{0,1\}$ and let $X = B^T$, T is a non-empty set. We define $f \circ g = f$ and $f' = 1 - f$ if $f, g \in X$ and let 1 be a function such that $1(t) = 1$ for each $t \in T$ and 0 be a function such that $0(t) = 0$, $t \in T$, then X is a left Jordan groupoid.

Definition 8. Let $(X, \circ, ')$ be a left Jordan groupoid and let Y be a non-empty subset of X. If $(Y, \circ, ')$ is also a left Jordan groupoid with the same operations as X, then Y is called the *Jordan subgroupoid* of X.

Example 3. Let $U(R)$, $P(R)$ be left Jordan groupoids, and $P(R) \subset U(R)$. Then it is clear that $P(R)$ is a subgroupoid of $U(R)$ according to the operations $\circ_1$ and orthocomplementation $'$ in $U(R)$.

Proposition 6. *Let X_τ, be a left Jordan groupoid for each $\tau \in T$. Then $\prod\limits_{\tau \in T} X_\tau$ is a left Jordan groupoid, if the binary operation $\circ$ and the unary operation $'$ are defined coordinatewise. Let R_1 and R_2 be the rings (associative and with identities) and let $h : R_1 \to R_2$ be a homomorphism of R_1 on R_2, then $h(U(R_1)) \subset U(R_2)$, but it may happen that $h(U(R_1)) \neq U(R_2)$.*

Proposition 7. *Let $(X, \circ, ')$ be a left Jordan groupoid, then G is an associative Jordan groupoid iff $p \circ q = p$ for each $p, q \in X$*

Proof.

a) If $p \circ q = p$ for $p, q \in X$, then $(X, \circ, ')$ is associative.

b) Let $(X, \circ, ')$ be associative, then we have, according to (iii) of Definition 4, $p = (p \circ q) \circ q = p \circ (q \circ q) = p \circ q$. □

Proposition 8. *Let $(X, \circ, ')$ be a commutative left Jordan groupoid. If $p, q \in X$, then $p \circ (p \circ q) = q$.*

Proof. $p \circ (p \circ q) = p \circ (q \circ p) = (p \circ q) \circ p = (q \circ p) \circ p = q$. □

Corollary. *Let $(X, \circ, ')$ be a left Jordan groupoid, then X need not be commutative, and if it is commutative, it is not associative.*

Indeed. According to Proposition 7, the associativity implies $p \circ q = p$, for $p, q \in X$, $p \neq q$. If X is also commutative, then $p = p \circ q = q \circ p = q$. But this would be in contradiction with $p \neq q$. □

PROPOSITION 9. *Let* $(U(R), \leq, 0, 1, ')$ *be the logic of all idempotents of the ring* R. *Then* $U(R)$ *is a left Jordan groupoid if the operations* $\circ$ *and* $'$ *are defined as follows:*

$$p \circ q = p - 2pq - 2qp + 4qpq \text{ for } p, q \in U(R) \text{ and } p' = 1 - p,\ p \in U(R).$$

Moreover, we have $p \circ q_1 = p \circ q_2$ *iff* $q_1 q_2 = q_2 q_1$ *and* $p, q_1, q_2 \in U(R)$.

Proof.

a) If $p \circ q_1 = p \circ q_2$, then $(1 - 2q_1)p(1 - 2q_1) = (1 - 2q_2)p(1 - 2q_2)$. From this it follows that $(1 - 2q_1 - 2q_2 + 4q_1q_2)p = (1 - 2q_1 - 2q_2 + 4q_2q_1)p$. Suppose that $p = 1$, then $q_1q_2 = q_2q_1$.

b) If for $p, q_1, q_2 \in U(R)$ is $q_1q_2 = q_2q_1$, then one can show that $p \circ q_1 = p \circ q_2$. □

COROLLARY. *Let* $U(R)$ *be the logic of all elements of a ring* R, *and let* B *be a Boolean subalgebra of* $U(R)$. *Then for each* $p \in U(R)$ *and* $q_1, q_2 \in B$, *it follows that* $p \circ q_1 = p \circ q_2$.

Proof. If $q_1, q_2 \in B$, then $q_1 \leftrightarrow q_2$ and this implies that $q_1q_2 = q_2q_1$. Now from Example 1 it follows that $p \circ q_1 = p \circ q_2$.

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