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REMARK TO A PROBLEM POSED BY S. PULMANNOVÁ

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PROBLEM. [1] *Let A, B be self-adjoint operators affiliated with a von Neumann algebra $\mathcal{A}$ such that $A+B$ is densely defined. What can be said about self-adjoint extensions of $A+B$?*

In the case when $\mathcal{A}$ is finite, the solution is as follows. Since $\mathcal{A}$ is finite, any densely defined operator which is affiliated with the algebra and closed is Segal measurable [2]. Hence the closure of the sum $(A+B)^-$, the so-called strong sum $\dot{+}$ of A and B, is a self-adjoint extension of $A+B$ (indeed, $(A\dot{+}B)^* = A^*\dot{+}B^*$). Thus, $A+B$ is essentially self-adjoint and cannot have any other self-adjoint extension (if C is another extension of $A+B$, then $C \supset (A+B)^- = A\dot{+}B$, so that $C = C^* \supset (A\dot{+}B)^* = A\dot{+}B$, which implies $C = A\dot{+}B$).

In the case when $\mathcal{A}$ is arbitrary, one can show, as above, that the sum of two locally measurable (see [3]) self-adjoint operators has a unique self-adjoint extension, obviously, affiliated with the algebra.

REFERENCES

[1] *Problem Session*, Proceedings of the Second Winter School on Measure Theory, Liptovský Ján, January 7–12, 1990, Czechoslovakia, eds. A. Dvurečenskij and S. Pulmannová, Slovak Academy of Sciences, Bratislava, 1990, 219–221.

[2] SEGAL, A.: *A noncommutative extension of abstract integration*, Ann. of Math. **57** (1953), 401–457.

[3] YEADON, F. J.: *Convergence of measurable operators*, Proc. Camb. Phil. Soc. **74** (1973), 257–268.

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