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REPRESENTATION OF TECTONIC STRESS

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Abstract: A diagram for the presentation of the stress tensor is proposed. The diagram is an equal side triangle whose left side is labelled as [n $(\sigma_1 - \sigma_2)$] / σ_1, right side as [n $(\sigma_2 - \sigma_3)$] / σ_1 and base as R. The R value is computed as the stress ratio of Etchecopar et al. (1981); $R = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$. If $\sigma_1 = 1$ and $\sigma_3 = 0$, the remaining stress magnitude σ_2 is equal to R. Left and right side of the triangle are scaled to fit positive infinite interval to a finite interval <0, n>. Lines connecting the same values indicated by a left side are parallel to the right side and vice versa. The base is proportionally calibrated to indicate interval of R = <0, 1>. Lines connecting this calibration with the upper apex indicate stress ellipsoids with the same stress ratio, just progressively increasing the size of the greatest Mohr circle towards the base. Line having the R value 0.5 indicates the plane stress. Lines parallel to the left side show stress ellipsoids with the same $\sigma_2 - \sigma_3$ values, progressively changing the R value. Left side is a special line obeying in addition $\sigma_2 = \sigma_3$, thus indicating the axial compression increasing towards the base. Lines parallel to the right side show the stress ellipsoids with the same $\sigma_1 - \sigma_2$ values, progressively changing the R value. Right side is a special line obeying in addition $\sigma_1 = \sigma_2$, thus indicating the axial extension increasing towards the base. Upper apex indicates the hydrostatic stress state with all principal stress magnitudes equal.
Graph visualizes ellipsoidal shapes. The upper apex represents sphere and the line with R = 0.5 indicates ellipsoids with $\sigma_2 = (\sigma_1 + \sigma_3) / 2$. Left and right apexes represent an ideal cigar and pancake shape, respectively. Points of the diagram to the right from the plane stress line show stress ellipsoids with control constriction, points to the left from the plane stress control flattening. It is felt that this diagram ignoring the volume effect by a putting ellipsoids into a finite space provides a possibility either to compare the shapes of ellipsoids or to study continuous mutual changes of the stress axes.

Key words: stress diagram, stress ellipsoids.

Introduction

It is frequently necessary to represent paleostress states, in order to compare results of the performed structural analyses. They provide either orientations and magnitudes of the three principal stress axes or principal stress axis orientations and the ratio of their magnitudes given by various forms (e.g. Bott 1959; Angelier 1989). All these results have one thing in common: they describe the stress state which can be described as an ellipsoid. The shape of this ellipsoid is given by its three axes: principal stress axes.

There have been numerous graphical representation of stress states suggested, each of them with own advantages, which are determined by their purpose (e.g. Nadai 1963; Jaeger & Cook 1976; Lisle 1979; Simon-Gomez 1986; Oncken 1988; Fry 1992; Jamison 1992). Non of them is universal.

One of the frequently used graphical displays for a stress state is the Mohr diagram. The complete overview of various properties of this representation is given in Means (1990). This diagram is very useful for the study of specific stress state providing normal and shear stress magnitudes of this stress state acting upon the tested fault plane. However it is awkward either for describing stress state which followed the stress path changing by increments or for a comparison of a whole set of stress states.

Examples of other representations are J space, p-q space and σ space graphical formats discussed by Jamison (1992). Unlike the Mohr diagram, they allow a clear representation of a stress history. However, only a σ space format visualizes clearly the mutual relationship of the principal stress magnitudes. This three dimensional approach can be quite awkward if a large set of stress states needs to be compared. In such a case an apparent cluster shown in diagram can be caused by the projection regardless of the genesis of different data.

Another stress ratio representation was made by Lisle (1979), who suggests a two-dimensional diagram whose x and y axes are labelled as the $\sigma_1 - \sigma_2$ and $\sigma_2 - \sigma_3$ differences, respectively. The stress ratio is represented by a tangens of the line created by equation $y = f(x)$. The length of this line beginning in the origin is infinite as well as the length of the x and y axes. This display is capable to record stress paths and visualizes a mutual relationship of the three principal stress magnitudes. However, the tensor with all three magnitudes multiplied by a positive number is represented by a point lying further from the origin than the point indicating the original tensor, despite of the fact that it has the same proportion of all three magnitudes; i.e. the same shape factor.

If an attention is focused to the description of the ellipsoidal shape, any finite interval between the maximum and minimum principal stress magnitudes can be fixed. Medium stress magnitude will then directly indicate mutual changes of the three principal stress magnitudes and stress ratio. This stress ratio in a graphic display becomes a shape factor of the stress ellipsoid given by its three principal stress axes. Using the ratio of Etchecopar et al. (1981) $R = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$ ranging from 0 to 1, ellipsoids of any scale can be compared by their R shape factors. Such an approach allows to compare the stress ellipsoids regardless of their volume, removing thus the scale effect

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and focusing to the genetic point of view. This paper presents a graph fit to the finite space by constraining the $\sigma_1 - \sigma_2$, $\sigma_1 - \sigma_2$ and R values to the finite graphic intervals. It involves the simple linear scaling of mentioned differences and R ratio along the sides of the equal-angle triangle. A diagram allows to study continuous mutual changes of the principal stresses obeying the Coulomb-Mohr criteria, visualizes the shape of the stress ellipsoid and provides stress paths.

Graph design

The co-ordinate system is formed by an equal-angle triangle (Fig. 1a). Left side is labelled as $[n\ (\sigma_1 - \sigma_2)]\ /\ \sigma_1$, right size as $[n\ (\sigma_2 - \sigma_3)]\ /\ \sigma_1$. In the case when stress magnitudes are not available, values $\sigma_1 = 1$, $\sigma_3 = 0$ and $\sigma_2 = R$ are used. Any stress ratio other than one sensu Etchecopar et al. (1981), given by stress inversion techniques, can be recomputed. For example the Lisle's (1979) ratio expressed by means of the Etchecopar et al. (1981) ratio is: $L = R\ /\ (1 - R)$. In the case when only these reduced tensors are available their points on the graph will occupy only the base of the triangle.

Co-ordinate axes are inclined to each other at the angle of 60°, thus a point **A** with co-ordinates $[n\ (\sigma_1 - \sigma_2)]\ /\ \sigma_1 = 1$ and $[n\ (\sigma_2 - \sigma_3)]\ /\ \sigma_1 = 4$ is to be found by the intersection of lines parallel to these co-ordinate axes (Fig. 1a). Related R ratio is then found as the intersection of the line going from the upper apex of the triangle through the point **A** and the scaled base.

Ellipsoidal shapes are clearly visualized by their position in a graph, comparable with the **K** parameter of Flinn (1964). Position in upper apex indicates sphere, position in the lower right apex indicates an ideal pancake and position in the lower left apex indicates an ideal cigar. R = 0.5 line from uper apex to base divides the field representing an affinity of ellipsoidal shapes to the pancake from the field representing an affinity of ellipsoidal shapes to the cigar.

Graph properties

Various properties of this graphical display can be demonstrated if the points on a graph are compared with the Mohr's stress representation.

First of the special lines of a graph is the line connecting the upper apex with the R = 0.5 value on the base. Points of this line indicate plane stress, i.e. stress ellipsoids with $\sigma_1 - \sigma_2 = \sigma_2 - \sigma_3$. As the point moves progressively from the apex towards the base, the size of the biggest Mohr circle becomes larger (Fig. 2a). The graph is designed in such a way that each such a shift of the Mohr stress circles along lines radiating from the upper apex to the base progressively approaches larger $\sigma_1 - \sigma_3$ value. Further shift away of the upper apex would require the negative σ_3 stress magnitude, that is not real for the stresses in the nature, but occurring rather in metallurgy etc. as pointed by Mandl (1988). This 0.5 line divides the triangle into the two sectors; right one with stress ellipsoids controlling constriction and left one with the stress ellipsoids controlling flattening.

The sector of constriction is bounded by the upper apex indicating the hydrostatic state of stress with all three stress magnitudes equal. R representations of ellipsoids of the various volumes fall into this point when their three magnitudes are equal. This relative recomputation of the shape factor regardless of the volume is valid for each point of the graph. Right

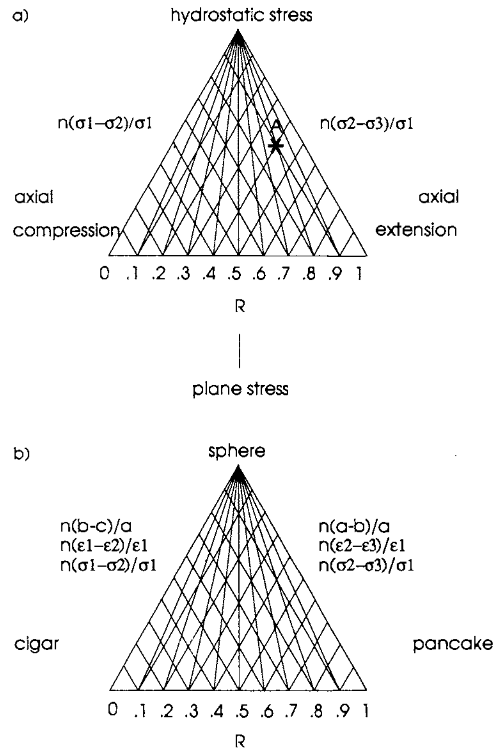

Fig. 1. Stress diagram. Explanation in text.

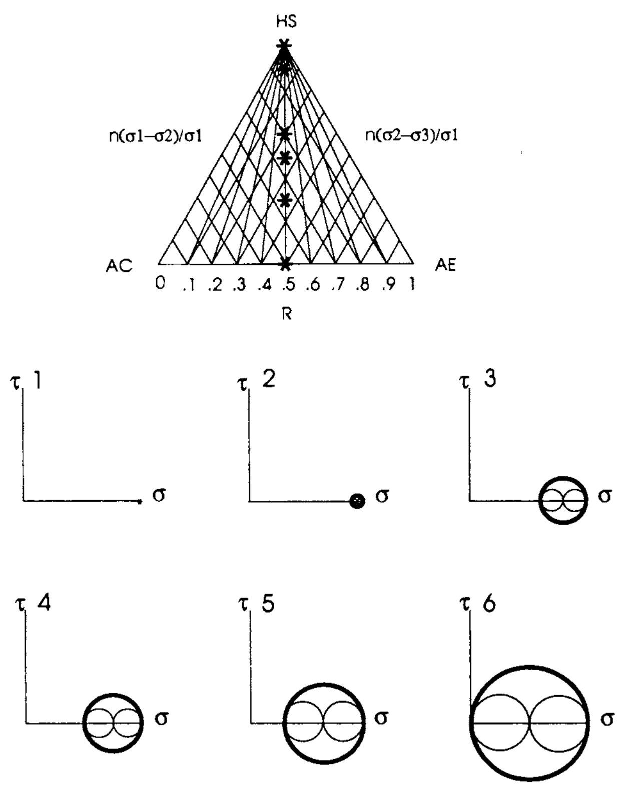

Fig. 2a. Plane stress states coming from the hydrostatic stress towards the base ($\sigma 1\ /\ \sigma 2\ /\ \sigma 3$): **1** - 10 / 10 / 10, **2** - 10 / 9.5 / 9, **3** - 10 / 8 / 6, **4** - 10 / 7.5 / 5, **5** - 10 / 6.5 / 3, **6** - 10 / 5 / 0 represented by triangular and Mohr representation AC and AE are axial compression and extension, respectively, τ and σ are shear and normal stress, respectively, star represents point.

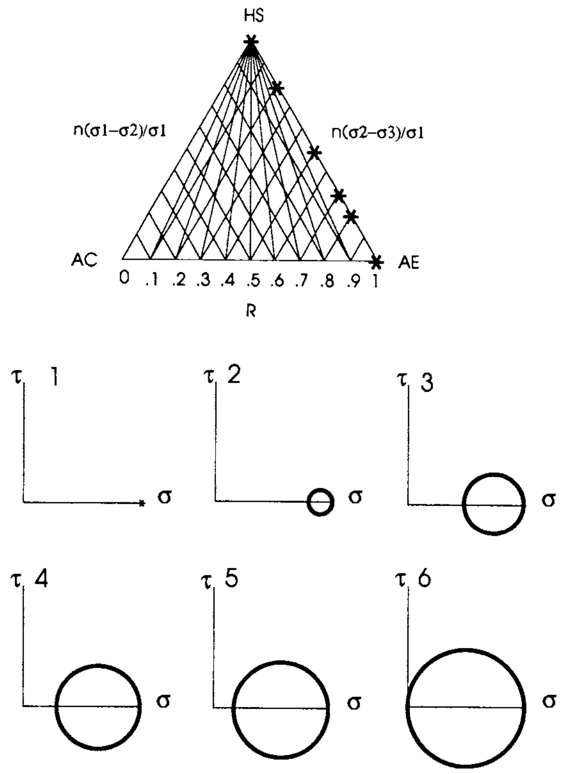

Fig. 2b. Stress states coming from the hydrostatic state towards the base along the right co-ordinate ($\sigma 1 / \sigma 2 / \sigma 3$): **1** - 10 / 10 / 10, **2** - 10 / 10 / 8, **3** - 10 / 10 / 5, **4** - 10 / 10 / 3, **5** - 10 / 10 / 2, **6** - 10 / 10 / 0 represented by triangular and Mohr representation symbols as in Fig. 2a.

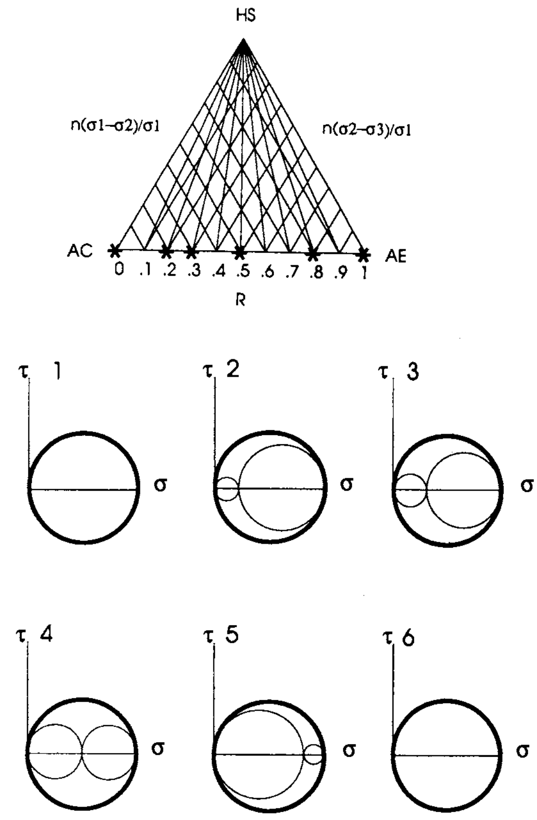

Fig. 2c. Stress states coming from the left to right along the base ($\sigma 1 / \sigma 2 / \sigma 3$): **1** - 10 / 0 / 0, **2** - 10 / 2 / 0, **3** - 10 / 3 / 0, **4** - 10 / 5 / 0, **5** - 10 / 8 / 0, **6** - 10 / 10 / 0 represented by triangular and Mohr representation symbols as in Fig. 2a.

margin of the constrictional sector is the $[n(\sigma_2 - \sigma_3)] / \sigma_1$ line, line of the axial extension, indicating the change of the Mohr circle representation, when $\sigma_1 = \sigma_2$ and σ_3 progressively approaches values from 100 % of σ_1 to 0 % of σ_1 as the representation moves away from the hydrostatic stress state (Fig. 2b). Lower margin of the constrictional sector is the R line indicating how the shape factor of the stress ellipsoid changes from the plane stress towards the pure axial extension through the interval <0.5, 1> (cases 4-6 in Fig. 2c).

The sector of flattening is bounded by the hydrostatic stress state. Left margin of the sector of flattening is the $[n(\sigma_1 - \sigma_2)] / \sigma_1$ line, line of the axial compression, indicating the change of the Mohr circle representation, when $\sigma_2 = \sigma_3$ and this value progressively approaches values from 100 % of σ_1 to 0 % of σ_1 as the representation moves away from the hydrostatic stress state (Fig. 2d). Lower margin of the sector of flattening is the R line indicating how the shape factor of the stress ellipsoid changes from the plane stress towards the pure axial compression through the interval <0.5, 0> (cases 1-4 in Fig. 2c).

Other special lines are lines parallel to the co-ordinate axes.

Lines parallel to the $[n(\sigma_2 - \sigma_3)] / \sigma_1$ line show the changes in the Mohr circle representation when the $\sigma_1 - \sigma_2$ value remains the same and σ_3 value progressively approaches values from 100 % of σ_1 to 0 % of σ_1 as the representation moves towards the R line (Fig. 2e). Position of these parallel lines towards the axial compression apex indicate the change of the Mohr stress representation when the σ_2 value progressively approaches less % of σ_1.

Lines parallel to the $[n(\sigma_1 - \sigma_2)] / \sigma_1$ line show the changes in the Mohr stress circle representation when the $\sigma_2 - \sigma_3$ value remains the same and this value progressively approaches values from 100 % of σ_1 to 0 % of σ_1 as the representation moves towards the R line (Fig. 2f). Position of these parallel lines towards the axial extension apex indicate the change of the Mohr stress representation when the σ_2 value progressively approaches more % of σ_1.

Lines parallel to the R line represent the Mohr stress circle representation when the value $\sigma_1 - \sigma_3$ remains the same and the only σ_2 changes progressively from 0 % to 100 % of σ_1 as the representation moves from the axial compression to the axial extension (Fig. 2g). The position of these parallel lines towards the hydrostatic state indicates a progressive shrinking of the greatest Mohr circle ($\sigma_1 - \sigma_3$).

Explanation

If Etchecopar et al. (1981) ratio $R = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$ is taken, R varies in interval <0, 1>. All possible ellipsoids can be described by keeping σ_1 and σ_3 values constant and by σ_2 moving between them. Knowing this, the constant values can be fixed arbitrarily as $\sigma_1 = 1$ and $\sigma_3 = 0$. In order to represent shapes of ellipsoids the values $\sigma_2 - \sigma_3$ have to be put on y axis and values $\sigma_1 - \sigma_2$ on x axis – right and left side of the triangle, respectively. If $\sigma_1 = 1$, $\sigma_2 = R$ and $\sigma_3 = 0$, then $\sigma_2 - \sigma_3$ value becomes R and $\sigma_1 - \sigma_2$ value becomes $1 - R$.

The lines radiating from the upper apex to the base lines drawn by a function $y = f(x)$ on a two dimensional graph. Substituting values $y = R$ and $x = 1 - R$ the function becomes: $R = f(1 - R)$.

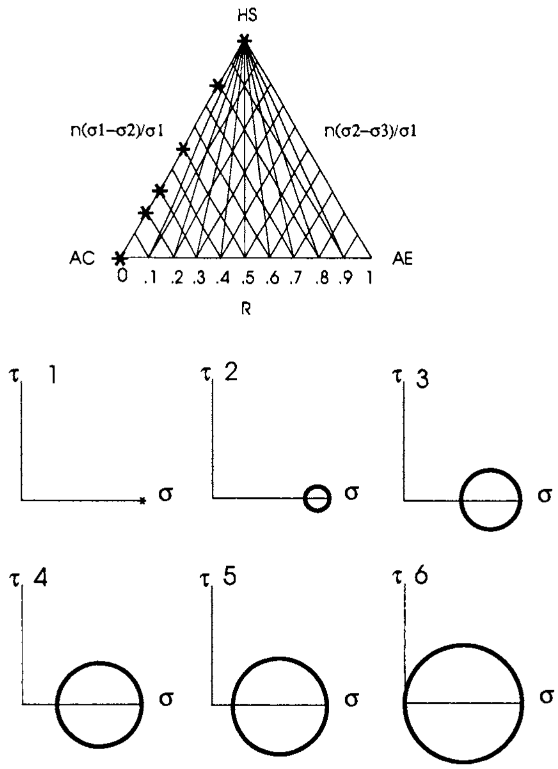

Fig. 2d. Stress states coming from hydrostatic state towards the base along the left co-ordinate $(\sigma 1 / \sigma 2 / \sigma 3)$: **1** - 10 / 10 / 10, **2** - 10 / 8 / 8, **3** - 10 / 5 / 5, **4** - 10 / 3 / 3, **5** - 10 / 2 / 2, **6** - 10 / 0 / 0 represented by triangular and Mohr representation symbols as in Fig. 2a.

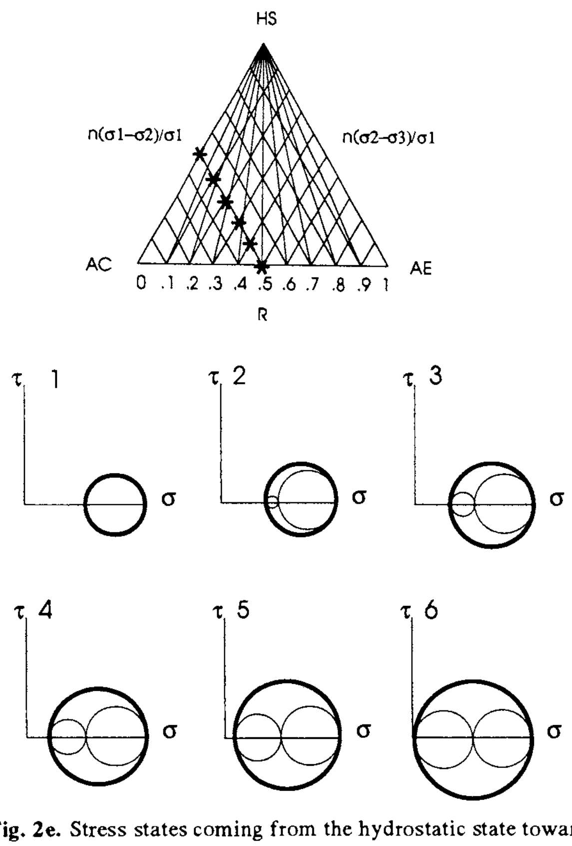

Fig. 2e. Stress states coming from the hydrostatic state towards the base along the line parallel to the right co-ordinate $(\sigma 1 / \sigma 2 / \sigma 3)$: **1** - 10 / 5 / 5, **2** - 10 / 5 / 4, **3** - 10 / 5 / 3, **4** - 10 / 5 / 2, **5** - 10 / 5 / 1, **6** - 10 / 5 / 0 represented by triangular and Mohr representation symbols as in Fig. 2a.

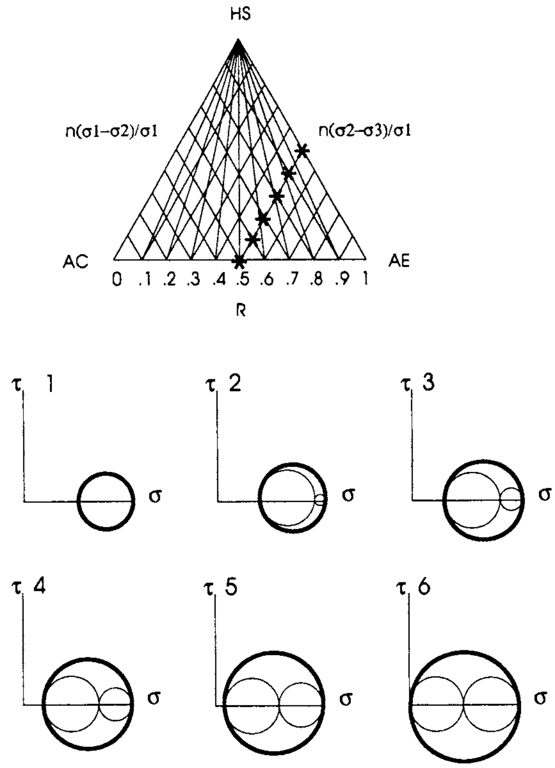

Fig. 2f. Stress states coming from the hydrostatic state towards the base along the line parallel to the left co-ordinate $(\sigma 1 / \sigma 2 / \sigma 3)$: **1** - 10 / 10 / 5, **2** - 10 / 9 / 4, **3** - 10 / 8 / 3, **4** - 10 / 7 / 2, **5** - 10 / 6 / 1, **6** - 10 / 5 / 0 represented by triangular and Mohr representation symbols as in Fig. 2a.

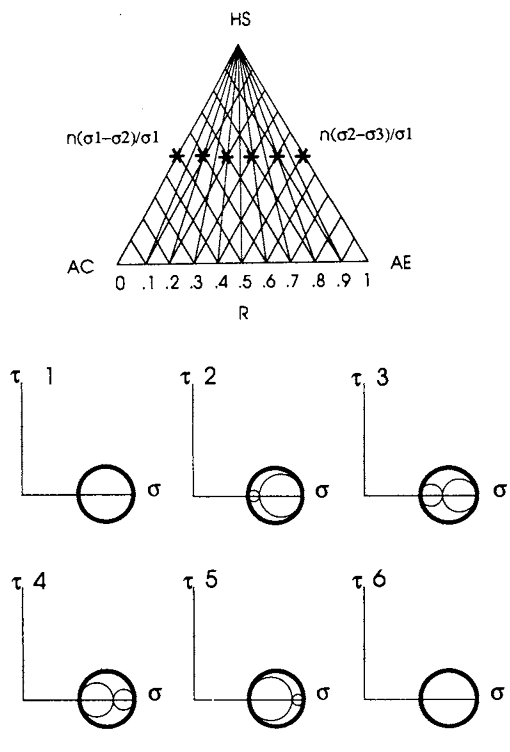

Fig. 2g. Stress states coming from the left to right along the diagonal $(\sigma 1 / \sigma 2 / \sigma 3)$: **1** - 10 / 5 / 5, **2** - 10 / 6 / 5, **3** - 10 / 7 / 5, **4** - 10 / 8 / 5, **5** - 10 / 9 / 5, **6** - 10 / 10 / 5 represented by triangular and Mohr representation symbols as in Fig. 2a.

The function $f = y/x$ is known as a tangens of the angle formed by a and b, shortest and medium, sides of the right triangle. Thus the side $a = R$ and it is projected on the base by a line drawn from the upper apex to the base. This is the way how the point given by $(1 - R, R)$ co-ordinates is directly projected on the base of the triangle as R ratio.

The correctness of the projection is then given by a proper scaling of all sides of the triangle. The base is calibrated to fit a finite $<0, 1>$ interval to a finite $<0, n>$ interval. The right side, y axis, is scaled to take the σ_1 value as the 100 % of its graphical length, equal n in this case. The R value is then scaled as a % of the total graphical length: $[n(\sigma_2 - \sigma_3)] / \sigma_1$. The same is valid for the left side, x axis: $[n(\sigma_1 - \sigma_2)] / \sigma_1$.

Based on this scaling, the ellipsoids provide proportional axes regardless of their volume and the base represents various stress ratio intervals projected within the finite interval $<0, 1>$.

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