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# JÁN BORSÍK (1955 – 2019)

Roman Frič — Ľubica Holá

Professor Ján Borsík passed away in Košice, Slovakia, on Friday, September 6, 2019. The sad news reached the participants of the 33rd International Summer Conference on Real Functions held in Ustka, Poland, only three days later. True, Ján struggled with cancer over a longer period, however, was still an active member of the "real functions community". Exactly one year ago, he headed/managed the 32nd ISCRF in Stará Lesná and served as an editor of the present issue of Tatra Mountains Mathematical Publications. True, the treacherous illness severely limited his life and he had to step down from his teaching duties and editorial responsibilities in last months. Even though the participants of the 33rd ISCRF could not participate in his funeral, they expressed their deep sorrow, and during the conference commemorated Professor Ján Borsík, his personal and research qualities, friendship and services to the conference. It is only natural to devote the present issue of TMMP to his memory.

Ján Borsík was born on 11. 9. 1955 in Trstené, Slovakia. After completing his secondary education, in the period 1975–1980 he studied at the School of Natural Sciences, Comenius University in Bratislava, Slovakia. Already as a student, he participated in a scientific seminar devoted to real functions led by Professor Tibor Šalát and remained faithful to real functions all his life. After his graduation, Janko Borsík obtained his RNDr. title in 1981 and, under the supervision of Professor Šalát, the scientific degree CSc. (equivalent to PhD). Finally, based on his Habilitationshrift, in 1999 he was promoted to Docent in Mathematics. Since 1980 until his premature death, he was affiliated with the Mathematical Institute of Slovak Academy of Sciences, Extension in Košice, as a Senior Scientific Fellow. For a long time Professor Ján Borsík lectured at the Šafárik University and the Technical University in Košice and at the Prešov University in Prešov, where his colleagues and students appreciated his scientific and pedagogical masterships very much. The same can be said about several PhD students and members of the broader Slovak mathematical community.

Above all, from Professor Šalát and Professor Neubrunn he took over the responsibilities related to the ISCRF. Since 1994 he managed biennial conferences first in Liptovský Ján and then in Stará Lesná, on comfortable premises of Academia, the Congress Centre of Slovak Academy of Sciences. The conferences resulted in a cooperation with the Polish "real functions community" and since 2005, ISCRF is annually held alternatively on the Polish and Slovak side of the High Tatras. TMMP serves as the journal in which participants publish their contributions.

His scientific interests covered a wider range, however, his real interest and research concentrated on real analysis. In papers of Ján Borsík, mostly topological methods and measure theory methods are used. His scientific direction was influenced by his supervisor Professor Tibor Šalát.

In the beginning of his career (in collaboration with Jozef Doboš), Ján Borsík studied functions whose composition with every metric is a metric [1]. Results of the paper [1] were used in further joint papers with J. Doboš [2, 3, 6]. The paper [2], On a product of metric spaces, Math. Slovaca 31 (1981), 193–205, is by Google Scholar Citations the most cited paper of Ján Borsík, it has 35 records.

Ján Borsík also studied functions with closed graphs [28], functions that preserve cauchy sequences and cauchy nets [7, 34], functions that preserve convergence of infinite series [25], oscillations for quasicontinuity, almost continuity [33, 40], convergences of functions and generalized continuities.

Most of the research of Ján Borsík is devoted to the study of quasicontinuity and its weaker and stronger forms. The notion of quasicontinuity is a classical notion which was introduced by Kempisty in 1932 in [Kem32] for real functions of real variables and later generalized for topological spaces by many authors, see for eaxample [Neu88]. Perhaps the first mention of the condition of quasicontinuity can be found in the paper of R. Baire [Bai99] in the study of continuity points of separately continuous functions from $\mathbb{R}^2$ into $\mathbb{R}$.

In paper [10] J. Borsík, J. Doboš, On decompositions of quasicontinuity, the authors introduced the notion of almost quasicontinuity and proved that every function from a topological space into a metric space is quasicontinuous if and only if it is almost quasicontinuous and cliquish. T. Šalát and A. Neubrunnová in [NŠ92] proved that the uniform convergence preserves almost quasicontinuity and the transfinite convergence does not preserve it.

It was well-known that the uniform convergence preserves quasicontinuity. J. Doboš in [Dob81] showed that the quasi-uniform convergence does not preserve it. However, by the paper [Ble52] of Bledsoe, even the pointwise limit of a sequence of quasicontinuous mappings from a Baire space into a metric space is cliquish. Ján Borsík in his paper [14], Quasiuniform limits of quasicontinuous functions, proved that every cliquish function $f : \mathbb{R} \to \mathbb{R}$ is a quasi-uniform limit of a sequence of quasicontinuous functions.

The paper [20], On F-continuity of real functions, is a joint paper with Professor Tibor Šalát. The authors proved that from $F$-continuity of a function from $\mathbb{R}$ to $\mathbb{R}$ at a point its linearity follows. The concept of $F$-continuity of real numbers is based on the well-known notion of $F$-convergence from [Lor48].

In 1985, Z. Grande [Gra85] showed that there is a cliquish function of a real variable which cannot be written as the finite product of quasicontinuous functions and asked for characterization of such functions. This characterization was given by T. Natkaniec [Nat90] in 1990 for real functions of a real variable. In [31], Ján Borsík showed that every cliquish function $f$ defined on a pseudometrizable space for which the preimages of the positive and negative half-axes are simply open sets is the product of two quasicontinuous functions. The results improve his previous ones from [26].

In paper [38], Ján Borsík solved a problem of Z. Grande concerning strong quasicontinuity points from the paper [Gra96]. He characterized the pair $\big(C(f), S(f)\big)$, where $C(f)$ is the set of all continuity points and $S(f)$ is the set of all strong quasi-continuity points of a function $f : \mathbb{R} \to \mathbb{R}$.

Another interesting result concerning points of continuity, quasicontinuity and cliquishness was proved in [43]: For a real function $f$ defined on a topological space denote $C(f)$ the set of all continuity points of $f$, $Q(f)$ the set of all quasicontinuity points of $f$ and $A(f)$ the set of all cliquishness points of $f$. Let X be a Baire metric space without isolated points. Let $C, Q$ and $A$ be subsets of $X$. Then, $C = C(f)$, $Q = Q(f)$ and $A = A(f)$ for some function $f : X \to \mathbb{R}$ if and only if $C \subset Q \subset A$ , $C$ is a $G_\delta$-set, $A$ is a closed set, and $A \setminus C$ is of first category.

In [49] (with J. Holos), Ján Borsík introduced the notion of porouscontinuous function on the base of porous set and investigated relations between porouscontinuous, continuous and quasicontinuous functions. Porouscontinuous functions have $\sigma$-porous sets of discontinuity points [49].

Works [45, 48, 50, 51] are chapters in 4 monographs: Real functions, density topology and related topics (eds. M. Filipczak), Lodz University Press, 2011, 91–100; Traditional and present-day topics in real analysis, Lodz University Press 2013; Monograph on the occasion of 100th birthday anniversary of Zygmunt Zahorski, Wydawnictwo Politechniki Slaskiej, Gliwice 2015; Modern Real Analysis, Lodz University Press, Łódź, 2015.

In his last paper [52], Points of uniform convergence and quasicontinuity, which appeared in European Journal of Mathematics in 2019, sets of points of uniform convergence for sequences of quasicontinuous functions and for convergent sequences of functions are characterized. It is proved that a subset of a metric space is the set of points of uniform convergence for some convergent sequence of functions if and only if it is a $G_\delta$-set containing all isolated points.

On the other hand, an arbitrary $G_\delta$-set is equal to the set of points of uniform convergence of some sequence of quasicontinuous functions.

R. I. P.

## LIST OF PUBLICATIONS OF JÁN BORSÍK


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