A Performance and Risk Analysis on the Slovak Private Pension Funds Market

Veronika MIŤKOVÁ – Vladimír MLYNAROVIČ – Branislav TUŠ***

Abstract

This paper presents the results from two methodological approaches to the analysis of performance and risk of private pension funds in the Slovak Republic. In the first approach, the problem is formulated as a multiple criteria decision model, and Promethee methodology is used for outranking the pension funds. The second approach uses modern portfolio theory to analyze pension funds in a risk-return space, and presents results of the analysis of the efficiency on the private pension funds market in the Slovak Republic. Modern portfolio theory is used to construct efficient frontiers in selected risk-return spaces, using mean-CVaR and mean-standard deviation. The Black-Litterman approach is used to overcome a problem of sensitivity to small changes in inputs in mean-variance portfolio optimization.

Keywords: *pension funds, Promethee outranking, efficient frontiers, Black-Litterman portfolio optimization*

JEL Classification: C61, G11

1. Introduction

Since its reform, the pension system in the Slovak Republic consists of three pillars. First is the mandatory state pillar, second is the mandatory private pillar, and third is the private voluntary pillar. This reformed pension system is a topic of great interest among politicians, practitioners and academics. A description of the Slovak pension reform, calculations of the pension system balance, and

* Veronika MIŤKOVÁ – Vladimír MLYNAROVIČ, Faculty of Social and Economic Sciences, Comenius University in Bratislava, Odbojárov 10/A, 820 05 Bratislava 2, Slovak Republic; e-mail: veronika.mitkova@fses.uniba.sk; vladimir.mlynarovic@fses.uniba.sk

** Branislav TUŠ, Allianz Slovenská dochodková správcovská spoločnosť, Račianska 62, 831 02 Bratislava 3, Slovak Republic; e-mail: tusb@asdss.sk

expected level of pensions in the system, can be found in Goliaš (2003) and Melicherčík and Ungvarský (2004). Kiliánová, Melicherčík, and Ševčovič (2006) and Kiliánová and Pflug (2007) concentrate on the mandatory fully funded second pillar, and present dynamic accumulation models for determining the optimal switching strategy with different risk profiles among pension funds.

The subject of this analysis is the second pillar of the system. Our approach aims to analyze the market of the corresponding pension funds as a whole, to rank funds in the multiple criteria space, and to derive conclusions about investment efficiency strategies in the selected return-risk spaces. Initially the problem is formulated and solved as a multiple criteria decision one, and Promethee methodology is used for outranking the pension funds in a dynamic context. Applications of modern portfolio theory are then used to approximate efficient frontiers and allocate individual funds.

Within the second pillar, six companies (Aegon, Allianz, CSOB, VUB Generali, Winterthur and ING) run three types of pension funds – conservative, balanced and growth – according to the relatively restrictive rules stated in the legislature. The analysis in this report is based on the weekly data published from April 1, 2005, which reflects the performance of investment strategies. The following items are included for each fund:

- current value of the income unit (VIU) (starting value equal to 1)
- net asset value of the fund (NAV)
- asset management fee (as a percentage of the average monthly net asset)
- value of the fund.

Table 1a shows the current results of the pension funds to the specific datum, where the total investments are 29 790.52 mil. Slovak crowns and the structure of the investments are presented in Table 1b. The total number of clients in the second pillar is approximately 1.5 million.

T a b l e 1a

Results of Pension Funds (January 26, 2007)

	Conservative Fund			**Balanced Fund**			**Growth Fund**		
Company	VIU	NAV (in mil. Sk)	Charge (in %)	VIU	NAV (in mil. Sk)	Charge (in %)	VIU	NAV (in mil. Sk)	Charge (in %)
AEGON	1.0677	111.94	0.000	1.0903	745.76	0.069	1.0900	2 108.43	0.069
Allianz	1.0719	444.56	0.070	1.0877	2 860.59	0.070	1.0950	5 723.74	0.070
CSOB	1.0626	55.52	0.000	1.0946	522.72	0.070	1.0978	1 119.10	0.070
ING	1.0657	100.15	0.075	1.0642	963.66	0.075	1.0660	2 208.66	0.075
VÚB Generali	1.0664	221.57	0.000	1.0806	1 674.75	0.075	1.0863	2 584.24	0.075
Winterthur	1.0683	276.28	0.075	1.0844	2 209.46	0.075	1.0914	5 859.41	0.075
Sum		1 210.01			8 976.94			19 603.57	

T a b l e 1b
The Structure of Investments (January 26, 2007) (in %)

Company	Conservative fund	Balanced fund	Growth fund
AEGON	3.77	25.14	71.08
Allianz	4.92	31.68	63.39
CSOB	3.27	30.80	65.93
ING	3.06	29.45	67.49
VUB Generali	4.95	37.38	57.68
Winterthur	3.31	26.48	70.21

2. Pension Funds Outranking

Pension funds outranking can be written as a multiple criteria decision making problem:

$$\text{'max'} \quad \left\{ y = \left(y_1, y_2, \ldots, y_k \right) \quad \middle| \quad y \in Y \right\}$$

Elements of the set Y are assumed pension funds and each is evaluated on the base of k selected criteria. Without a loss of universality, it can be assumed that 'the more the better' can be applied to each criterion. The goal is to rank the funds in the form of preference structure (*P, S, I)*, or (*P, S, I, R*) where P means a strict preference, S means a weak preference, I denotes indifference and R denotes incomparability.

There are several classes of method for solving such problems. In the application, the method PROMETHEE II was used (Brans, Mareschall, and Vincke, 1986; Mlynarovič, 1998). This method is based on a construction of generalised criteria, and indices of multiple criteria preferences. Intensity of one fund preference over a second is a function of the difference in performances, according to individual criteria, and takes a value from 0 to 1. If y and z are two funds from the set Y which are to be compared from the viewpoint of criterion i, then

$$d_i = y_i - z_i$$

and preference function value is

$$F_i(y_i, z_i) = P(d_i) = 1 - e^{-\frac{d_i^2}{2\sigma^2}}, \; d_i \geq 0$$

where σ represents standard deviation, and measures the contribution of criterion i to the total preference of y over z. Note that this Gaussian preference function is not the only possible function: *usual criterion, quasi-criterion, criterion with linear preference level criterion* or *criterion with linear preference and indifference area* can also be used.

Assuming that for each criterion i a preference function F_i was defined, and w_i expresses relative importance of criterion i, then for all couples of pension funds y and z, a following index of multiple criteria preferences is defined.

$$\pi(y,z) = \frac{\sum_{i=1}^{k} w_i F_i(y,z)}{\sum_{i=1}^{k} w_i}$$

The index measures a client's preference intensity for fund y over fund z in such a way that all criteria are taken into account simultaneously. From these calculations, a matrix of indices can be developed. For each fund y, the mean of preference intensities over all other funds is defined in the form of outgoing flow, where n is the number of assumed funds:

$$\Phi^{+}(y) = \frac{\sum_{z \in Y} \pi(y,z)}{n-1}$$

In turn, the mean of preference intensities of all other funds over fund y is defined as incoming flow:

$$\Phi^{-}(y) = \frac{\sum_{z \in Y} \pi(z,y)}{n-1}$$

The net flow is then defined as

$$\Phi(y) = \Phi^{+}(y) - \Phi^{-}(y)$$

and PROMETHEE II outranking relationships are defined as:

Fund y outranks fund z iff $\Phi(y) > \Phi(z)$

Fund y is indifferent to fund z iff $\Phi(y) = \Phi(z)$.

The criteria used in the methodology application for pension funds outranking are presented in Table 2. The relative importance weights were stated as a result of consultation with pension funds portfolio managers.

An application of the described methodology for outranking of conservative, balanced and growth pension funds on the base of weekly data provides three types of results:

• funds outranking on the base of the current week data

• average results for the last 52 weeks

• results that present long-term tendencies of funds performance developments.

T a b l e 2

Criteria for Pension Funds Outranking

Criterion	Type	Weights
The current value of the income unit – average for the last four weeks	max	0.10
The weekly return in % – average for the last four weeks	max	0.15
The net asset value of the fund	max	0.15
The relative weekly change in the nest asset value of the fund	max	0.05
The charge for asset management in % of average monthly net asset value of the fund	min	0.05
The historical weekly Value at Risk (95% confidence level)	min	0.05
The historical weekly Conditional Value at Risk (95% confidence level)	min	0.05
The lower semi standard deviation of returns for the last 26 weeks	min	0.10
The difference between the short run (the last 8 weeks) and long run (the last 26 weeks) average weekly returns	max	0.10
The difference between the return of the fund and the return of the market competition – average for the last four weeks	max	0.20

T a b l e 3

Funds Outranking in Current Week (January 26, 2007)

	Net Flows of the Funds		
Company	growth	balanced	conservative
Aegon	–0.56725	–0.52725	–0.01942
Allianz	0.20148	0.15773	0.28032
CSOB	0.29953	0.32261	0.20775
ING	–0.17621	–0.19941	–0.34757
VUB	–0.01210	–0.01428	–0.28469
Winterthur	0.25455	0.26059	0.16361

The current weekly results as of January 26, 2007 are presented in Table 3. Table 4 illustrates the corresponding funds outranking based on the last 52 weeks. It includes the average net flows, and the standard deviations of these values, for the last 52 weeks. The values provide a risk measure, by determining volatilities of results for the period. Combining the two results leads to the construction of an efficient funds boundary, which consists of funds where a better average result can be achieved only with a higher risk. Such constructions create starting points for modern portfolio theory applications, in decisions concerning assumed investment opportunities space.

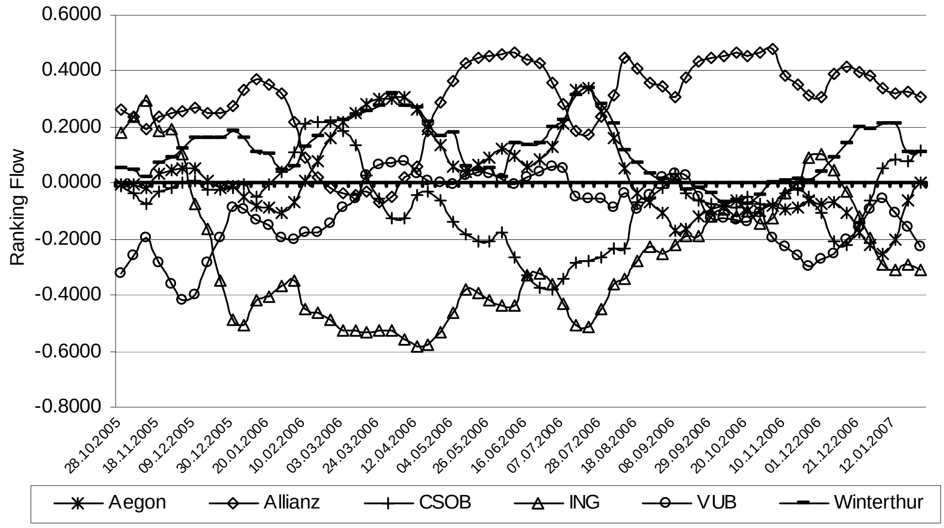
T a b l e 4

Funds Outranking Based on the 52-Week Average (January 26, 2007)

	Average Net Flows of the Funds and their Standard Deviations					
	growth		balanced		conservative	
Company	average	st. deviation	average	st. deviation	average	st. deviation
Aegon	–0.04667	0.28822	–0.05702	0.282409	0.040145	0.184276
Allianz	0.25383	0.144607	0.240728	0.151984	0.295262	0.188559
CSOB	0.038563	0.181973	0.031283	0.214102	–0.07928	0.181777
ING	–0.42418	0.2511	–0.37254	0.241249	–0.31008	0.206857
VUB	–0.01438	0.115514	0.028995	0.13032	–0.07771	0.133281
Winterthur	0.220219	0.110465	0.155942	0.126674	0.122585	0.143669

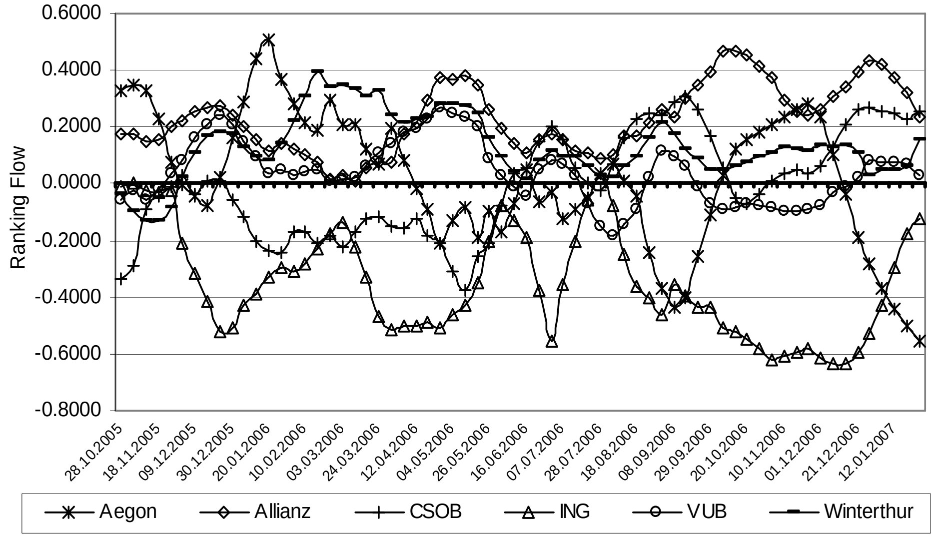
F i g u r e 1

Average Results for the Last Four Geeks – Conservative Funds



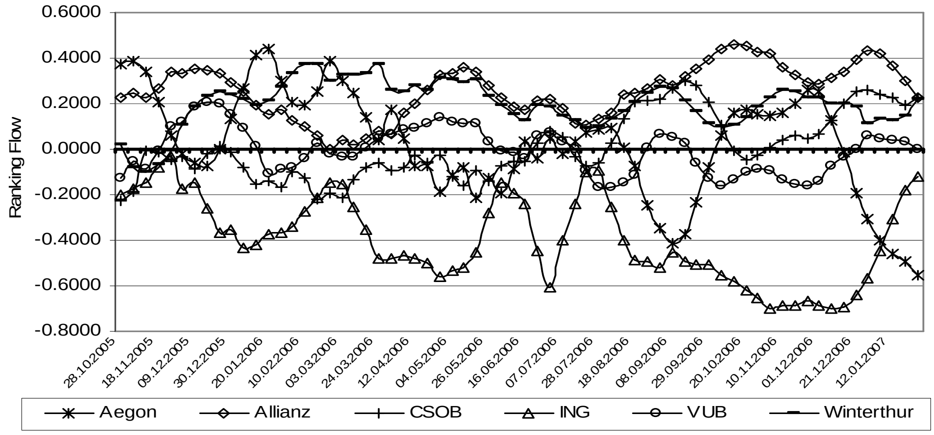
F i g u r e 2

Average Results for the Last Four Weeks – Balanced Funds



A long-term view of funds performance can be described in average sequences of four weeks, for the whole period of evaluation beginning April 1, 2005. Such results are illustrated for conservative, balanced and growth pension funds in Figures 1 – 3.

F i g u r e 3
Average Results for the Last Four Weeks – Growth Funds



The multiple criteria approach makes it possible to rank funds, or describe them as relatively *good* or *bad*, as illustrated in Tables 5 – 7. The current rankings of companies, and the rankings based on average results for the last 52 weeks, are presented for each type of fund. As stated previously, the funds are ranked on the base of net, or ranking, flows. A positive value of the flow means that the corresponding fund is relatively good, and a negative value means that the fund is relatively bad. The last two columns of Tables 5 – 7 show the total number of positive and negative values of the flows in the 68 rankings.

T a b l e 5
Conservative Funds on January 26, 2007

Company	Current Ranking	Last 52 Weeks Average Ranking	Number of Ranking Flows Values	
			positives	negatives
Aegon	4	3	32	36
Allianz	1	1	61	7
CSOB	2	5	24	44
ING	6	6	11	57
VUB	5	4	17	51
Winterthur	3	2	55	13

T a b l e 6
Balanced Funds on January 26, 2007

Company	Current	Last 52 Weeks Average Ranking	Number of Ranking Flows Values	
			positives	negatives
Aegon	6	5	37	31
Allianz	3	1	66	2
CSOB	1	3	32	36
ING	5	6	8	60
VUB	4	4	39	29
Winterthur	2	2	56	12

T a b l e 7

Growth Funds on January 26, 2007

Company	Current	Last 52 Weeks Average Ranking	Number of Ranking Flows Values	
			positives	negatives
Aegon	6	6	37	31
Allianz	3	1	64	4
CSOB	1	3	34	34
ING	5	5	7	60
VUB	4	4	32	36
Winterthur	2	2	62	6

3. Efficiency Analysis of Pension Funds Market in Risk-Return Spaces

Based on the published weekly data, the risk-return characteristics of the funds investment strategies can be computed using an average four-week logarithmic return, standard deviation, historical VaR, historical Conditional VaR, lower semi-standard deviation (SSD), lower semi-absolute deviation (SAD), and kurtosis and skewness of returns distribution. Tables 8 – 10 present such characteristics for conservative, balanced and growth pension funds for the period of September 1, 2005 to January 21, 2006. This limited period was used because in the first stages, pension funds dramatically rebalanced their portfolios which resulted in high movement of pension units; whereas recent developments show that the portfolios are much more stable. In Tables 8 – 10, MC denotes 'Market Competition'; 'simple' refers to the simple average of returns; and 'weighted' refers to weighted average of returns, where weights are devoted from the capitalization level that is measured by the relative level of the fund net asset value.

T a b l e 8

Conservative Funds Risk-Return Characteristics Based on 4-Week Logarithmic Returns

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Return, p.a.	3.15%	3.45%	3.24%	3.82%	3.05%	3.20%	3.32%	3.27%
St. dev., p.a	0.28%	0.32%	0.25%	0.92%	0.34%	0.22%	0.25%	0.26%
Average return	0.24%	0.26%	0.25%	0.29%	0.23%	0.24%	0.25%	0.25%
Standard deviation	0.08%	0.09%	0.07%	0.26%	0.10%	0.06%	0.07%	0.07%
Kurtosis	26.22	–1.32	–0.86	10.97	1.31	–1.09	–1.55	–1.36
Skewness	4.20	0.34	0.70	3.30	–1.11	0.58	0.20	0.37
VaR (0.95)	–0.18%	–0.15%	–0.18%	–0.11%	–0.05%	–0.18%	–0.17%	–0.16%
CVaR (0.95)	–0.18%	–0.13%	–0.16%	–0.09%	0.02%	–0.18%	–0.15%	–0.15%
Lower SSD	0.04%	0.06%	0.04%	0.10%	0.08%	0.04%	0.05%	0.05%
Lower SAD	0.02%	0.04%	0.03%	0.07%	0.04%	0.03%	0.03%	0.03%

T a b l e 9

Balanced Funds Risk-Return Characteristics Based on 4-Week Logarithmic Returns

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Return, p.a.	4.66%	4.41%	4.71%	3.34%	3.98%	4.19%	4.21%	4.17%
St. dev., p.a	1.24%	1.08%	1.17%	1.12%	0.98%	1.02%	1.00%	1.00%
Average return	0.35%	0.33%	0.35%	0.25%	0.30%	0.32%	0.32%	0.31%
Standard deviation	0.34%	0.30%	0.33%	0.31%	0.27%	0.28%	0.28%	0.28%
Kurtosis	0.80	1.27	0.64	1.39	1.87	1.69	1.73	1.80
Skewness	–0.09	–0.61	–0.57	–0.74	–1.01	–0.99	–1.01	–0.97
VaR (0.95)	0.32%	0.31%	0.24%	0.18%	0.25%	0.29%	0.27%	0.30%
CVaR (0.95)	0.46%	0.42%	0.42%	0.57%	0.42%	0.42%	0.43%	0.43%
Lower SSD	0.24%	0.23%	0.25%	0.24%	0.22%	0.23%	0.22%	0.22%
Lower SAD	0.13%	0.11%	0.12%	0.12%	0.10%	0.10%	0.10%	0.10%

Such characteristics allow us to compare pension funds in the selected risk--return space, and derive conclusions about their efficiency on the Slovak pension fund market. It must be noted that this market is very young, and these companies built up their investment strategies very conservatively. For example, their investments into equities are still well under the planned levels, and they are below 20 per cent in growth funds. It is stated in the legislature that no fund can be worse than the market competition (simple) more than 5 per cent.

Such approaches are also reflected in the corresponding correlation coefficients. Tables 11 – 13 present the coefficients for conservative, balanced and growth funds.

T a b l e 10

Growth Funds Risk-Return Characteristics Based on 4-Week Logarithmic Returns

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Return, p.a.	4.65%	4.87%	4.98%	3.28%	4.36%	4.64%	4.48%	4.54%
St. dev., p.a	1.34%	1.29%	1.34%	1.39%	1.19%	1.25%	1.20%	1.22%
Average return	0.35%	0.37%	0.37%	0.25%	0.33%	0.35%	0.34%	0.34%
Standard deviation	0.37%	0.36%	0.37%	0.39%	0.33%	0.35%	0.33%	0.34%
Kurtosis	1.17	1.39	1.13	2.49	1.82	1.80	2.23	2.09
Skewness	–0.51	–0.73	–0.84	–1.09	–1.07	–1.09	–1.19	–1.13
VaR (0.95)	0.39%	0.42%	0.37%	0.36%	0.34%	0.39%	0.40%	0.41%
CVaR (0.95)	0.61%	0.55%	0.56%	0.85%	0.55%	0.55%	0.60%	0.58%
Lower SSD	0.27%	0.28%	0.29%	0.31%	0.26%	0.28%	0.27%	0.27%
Lower SAD	0.14%	0.13%	0.13%	0.14%	0.12%	0.12%	0.12%	0.12%

It is very simple to compare couples of funds or investment strategies, and derive conclusions about dominance relations. However, in our opinion it is more interesting to determine the position of the fund in the risk-return space, owing to the efficient frontier of the market.

T a b l e 11

The Correlation Coefficients – Conservative Funds

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Aegon	1.000	0.470	0.480	–0.003	0.541	0.496	0.557	0.572
Allianz	0.470	1.000	0.924	0.118	0.752	0.918	0.827	0.974
CSOB	0.480	0.924	1.000	0.093	0.743	0.930	0.810	0.941
ING	–0.003	0.118	0.093	1.000	–0.325	0.066	0.583	0.112
VUB	0.541	0.752	0.743	–0.325	1.000	0.760	0.522	0.835
Winterthur	0.496	0.918	0.930	0.066	0.760	1.000	0.798	0.948
MC-simple	0.557	0.827	0.810	0.583	0.522	0.798	1.000	0.863
MC-weighted	0.572	0.974	0.941	0.112	0.835	0.948	0.863	1.000

T a b l e 12

The Correlation Coefficients – Balanced Funds

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Aegon	1.000	0.739	0.624	0.598	0.701	0.727	0.815	0.766
Allianz	0.739	1.000	0.832	0.725	0.970	0.966	0.958	0.984
CSOB	0.624	0.832	1.000	0.744	0.850	0.830	0.898	0.870
ING	0.598	0.725	0.744	1.000	0.770	0.760	0.846	0.808
VUB	0.701	0.970	0.850	0.770	1.000	0.968	0.962	0.983
Winterthur	0.727	0.966	0.830	0.760	0.968	1.000	0.961	0.985
MC-simple	0.815	0.958	0.898	0.846	0.962	0.961	1.000	0.989
MC-weighted	0.766	0.984	0.870	0.808	0.983	0.985	0.989	1.000

To determine this, we need to know the corresponding efficient frontier. This is very simple, because we are now in the space of a generalized Markowitz portfolio selection problem, which can be written as:

T a b l e 13

The Correlation Coefficients – Balanced Funds

	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-simple	MC-weighted
Aegon	1.000	0.775	0.625	0.570	0.630	0.652	0.790	0.708
Allianz	0.775	1.000	0.894	0.770	0.968	0.966	0.963	0.980
CSOB	0.713	0.894	1.000	0.800	0.897	0.896	0.934	0.918
ING	0.688	0.770	0.800	1.000	0.819	0.810	0.881	0.852
VUB	0.741	0.968	0.897	0.819	1.000	0.969	0.967	0.980
Winterthur	0.779	0.966	0.896	0.810	0.969	1.000	0.971	0.987
MC-simple	0.844	0.963	0.934	0.881	0.967	0.971	1.000	0.993
MC-weighted	0.809	0.980	0.918	0.852	0.980	0.987	0.993	1.000

$$eff\left\{ \mathbf{E}^T\mathbf{w}; \Omega(\mathbf{w})\right\}$$

subject to

$$\mathbf{e}^T\mathbf{w} = 1$$

$$\mathbf{w}^l \leq \mathbf{w} \leq \mathbf{w}^u$$

where

Ω(w) – the risk measure scalar function
w – the vector of portfolio weights
E – the vector of expected returns
$\mathbf{w}^l$ – the vector of lower bounds on portfolio weights
$\mathbf{w}^u$ – the vector of upper bounds on portfolio weights
e – the vector which elements equal 1.

Beginning with Zeleny (1982) and Konno, Waki, and Yuuki (2002), we can write:

$$\Omega(\mathbf{w}) = \begin{cases} \Omega(\mathbf{w}, c, \alpha, \lambda) = \Omega(c, \alpha, \lambda) = \left[\sum_{r_k \le \lambda} \left| \mathbf{r}_k^T \mathbf{w} - c \right|^\alpha p_k \right]^{\frac{1}{\alpha}}, \quad \alpha > 0 \\ CVaR_\beta(\mathbf{w}, \beta) = \dfrac{1}{1-\beta} \mathrm{E}\left[-(\mathbf{r}^T \mathbf{w}) \middle| -(\mathbf{r}^T \mathbf{w}) \ge VaR_\beta(\mathbf{w}) \right] \end{cases}$$

where p_k is the probability of *k*th level $\mathbf{r}_k^T\mathbf{w}$ of portfolio return, and *c* is a given reference level of wealth from which deviations are measured. For example, *c* could represent expected return of the asset, zero, the initial wealth level, the mode, or the median. Parameter α is the power to which deviations are raised, and thus α reflects the relative importance of large and small deviations. Parameter λ specifies the deviations to be included in the risk measure if $\alpha > 0$. Possible choices for parameter λ include ∞, *c*, a desired target level return, and several others. Conditional value at risk (*CVaR*) is an alternative measure of risk that maintains advantages of *VaR*, yet is free from computational disadvantages of *VaR*, where β, $0 < \beta < 1$, is the confidence level.

It must also be noted that the *VBA procedures* (Jackson and Staunton, 2001; Mlynarovič, 2004) present an effective execution of the solution process in the Excel environment, which provides an approximation of the efficient frontier. From a practical application view, there are two main problems: • how to select the particular risk measure function • how to estimate the expected return of assets.

For $\alpha = 2$, $\lambda = \infty$ and $c = \mathbf{E}^T$w, we have the model of portfolio selection in the mean-variance space that is broadly used in fund management. It is used to allocate assets for the purpose of setting fundamental fund management policy; for managing individual assets that form the portfolio; and for risk management and performance measurement.

The model is further being used for specifying the proportion of funds allocated to passive (index) management, and for different types of active management. Its utility is determined by the following:

• If the rate of return has a normal distribution of probability, which was usually considered presumption fulfilled for common stock, then the model is consistent with 'expected utility maximization' principle.

• Quadratic programming problems, representing technical execution of the model, are solvable considering the existing knowledge of mathematical programming methodology.

Nevertheless, in recent years radical changes have been observed in the investment environment. Now there are different financial instruments with asymmetric distribution of yield, such as options and bonds. Recent statistical studies have also shown that normal distribution of return is not recorded with all common stock. As a result of these factors, one can never rely on a standard model of portfolio selection.

3.1. Mean-Conditional Value at Risk Space

In the past there were several risk measures proposed, different from the variance, including s*emi-standard deviation,* s*emi-absolute deviation* and below target *risk*. There are also models explicitly examining *skewness* of return distribution. A relatively new measure of lower partial *risk* also comprises *Value at Risk*, which is widely used for market risk measurement. This risk measure is very popular in the conservative environment, because the probability of huge loss-larger than $VaR_{0.99}$ – is very low, provided that the portfolio's returns have normal distribution. However, considering the existing methodologies of non-linear programming, it is impossible to find a portfolio with the lowest *VaR*. For this reason, the *CVaR* (*conditional value at risk* or *expected loss*) becomes more attractive as a risk measure, because of its theoretical and computing features.

Pictures 4 – 6 present analysis in the mean – CvaR space for conservative, balanced and growth funds for confidence level 0.95

Figure 4

Risk-Return Profile for Conservative Funds

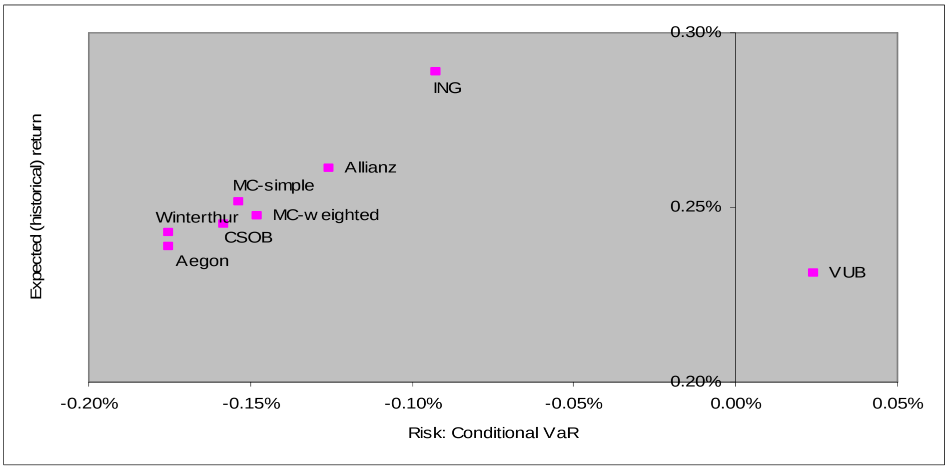


Figure 5

Risk-Return Profile for Balanced Funds

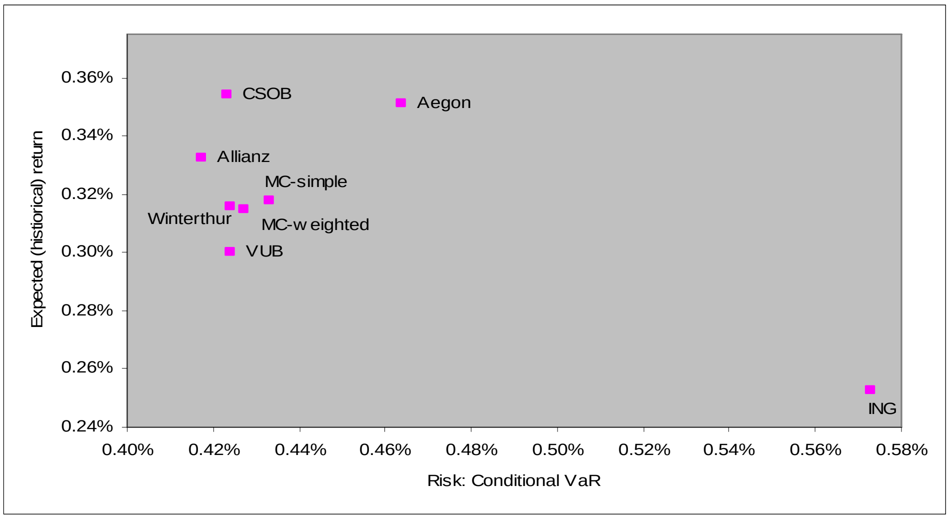
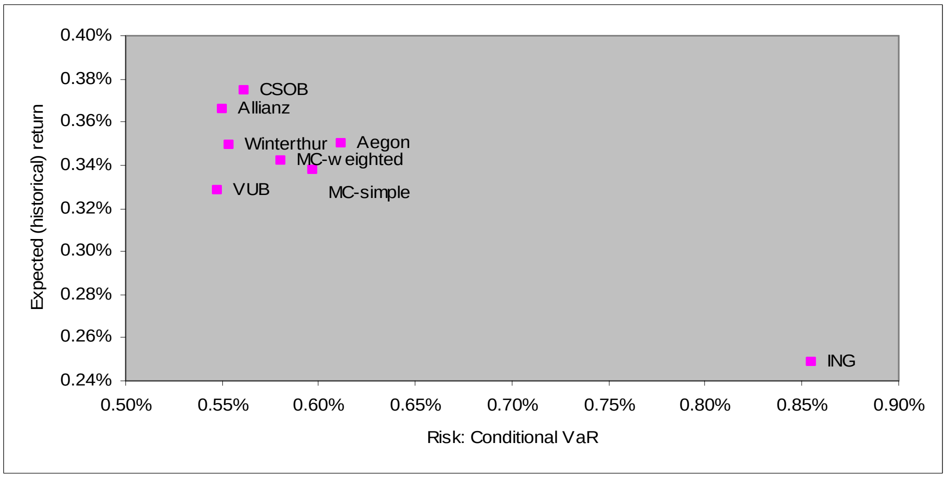


Figure 6

Risk-Return Profile for Growth Funds



3.2. Black-Litterman Portfolio Optimization

The second limitation of the mean-variance approach is that the recommended asset allocations are highly sensitive to small changes in inputs, and therefore to estimation errors. In its impact on the results of a mean-variance approach to asset allocation, estimation error in expected returns has been estimated as roughly 10 times as important as estimation error in variances, and 20 times as important as estimation error in covariance.

Thus the most important inputs in mean-variance optimization are the expected returns. Fisher Black and Robert Litterman (Black and Litterman, 1992) developed a quantitative approach to dealing with the problem of estimation error. The goal of this model is to create stable, mean-variance efficient portfolios, which overcome the problem of input sensitivity. The Black-Litterman model uses 'equilibrium' returns as a neutral starting point. Equilibrium returns are calculated using either CAPM or the reverse optimization method, in which the vector of implied expected equilibrium returns **P** is extracted from known information, where

$$\mathbf{P} = \delta \mathbf{C}\mathbf{w}$$

and **w** in this case is the vector of market capitalization weights, **C** is the covariance matrix, $n \times n$, where n is the number of assets, and δ is risk-aversion coefficient, which represents the market average risk tolerance. In general, the Black-Litter-man approach consists of the following steps:

1. Define equilibrium market weights and covariance matrix for all asset classes.
2. Calculate the expected return implied from the market equilibrium portfolio.
3. Express market views and confidence for each view.
4. Calculate the view adjusted market equilibrium returns.
5. Run mean-variance optimization.

In our application we use this approach without market views expressions to describe the efficient frontier of the Slovak pension funds market in the following way. Vector $\mathbf{w}_c$ describes the capitalization on the market of the funds, and E_c is the corresponding return of the weighted market competition for the current period. The risk adjusted return can be written in the form

$$E_c - \delta \mathbf{w}_c^T \mathbf{C} \mathbf{w}_c$$

and we assume that this return is for the weighted market competition zero. Therefore we have

$$\delta_c = \frac{E_c}{\mathbf{w}_c^T \mathbf{C} \mathbf{w}_c}$$

and the vector

$$\mathbf{P}_c = \delta_c \mathbf{C} \mathbf{w}_c$$

is used as the vector of expected returns in mean-variance optimization.

Tables 14 – 16 present the corresponding long-run equilibrium returns for the pension funds, the market weights for the last 4-week period to January 26, 2007, and the risk adjusted returns of the pension funds. Figures 7 – 9 illustrate the corresponding efficient frontiers, together with the positions of individual pension funds.

Table 14

Long-Run Equilibrium Log Four Weeks Return (in %)

Conservative Funds	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-weighted	MC-simple
Market weight	9.25	36.74	4.59	8.28	18.31	22.83		
Equilibrium return	0.24	0.45	0.34	0.39	0.37	0.30	0.37	0.35
Risk adjusted return	–0.22	–0.15	–0.03	–4.50	–0.30	0.02	0.00	–0.02

Figure 7

The Efficient Frontier of the Slovak Conservative Pension Funds

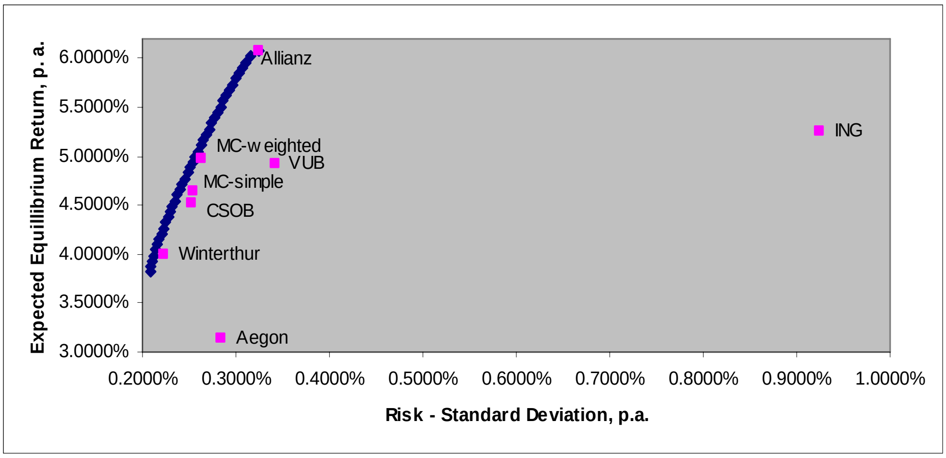


Table 15

Long-Run Equilibrium Log Four Weeks Return (in %)

Balanced Funds	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-weighted	MC-simple
Market weight	8.31	31.87	5.82	10.73	18.66	24.61		
Equilibrium return	0.48	0.52	0.51	0.45	0.48	0.50	0.50	0.49
Risk adjusted return	–0.28	–0.05	–0.18	–0.17	0.00	–0.02	0.00	0.00

Table 16

Long-Run Equilibrium Log Four Weeks Return (in %)

Growth Funds	Aegon	Allianz	CSOB	ING	VUB	Winterthur	MC-weighted	MC-simple
Market weight	10.76	29.20	5.71	11.27	13.18	29.89		
Equilibrium return	0.48	0.55	0.53	0.52	0.50	0.53	0.53	0.52
Risk adjusted return	–0.16	–0.05	–0.11	–0.18	0.00	–0.02	0.00	0.00

Figure 8

The Efficient Frontier of the Slovak Balanced Pension Funds

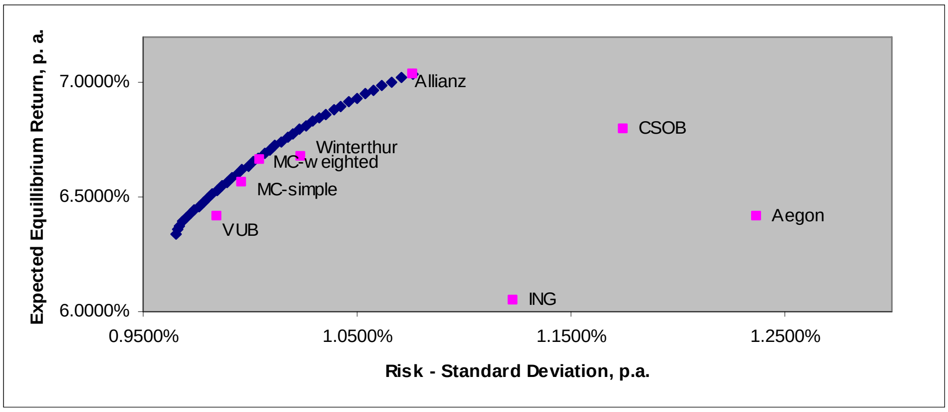


Figure 9

The Efficient Frontier of the Slovak Growth Pension Funds

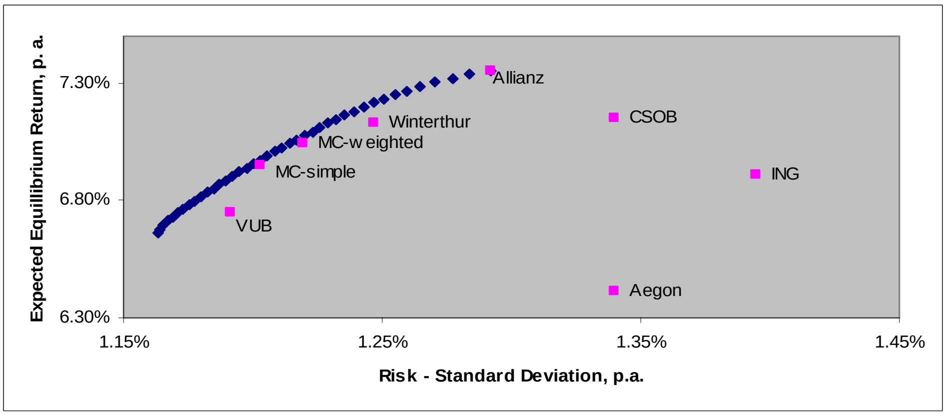
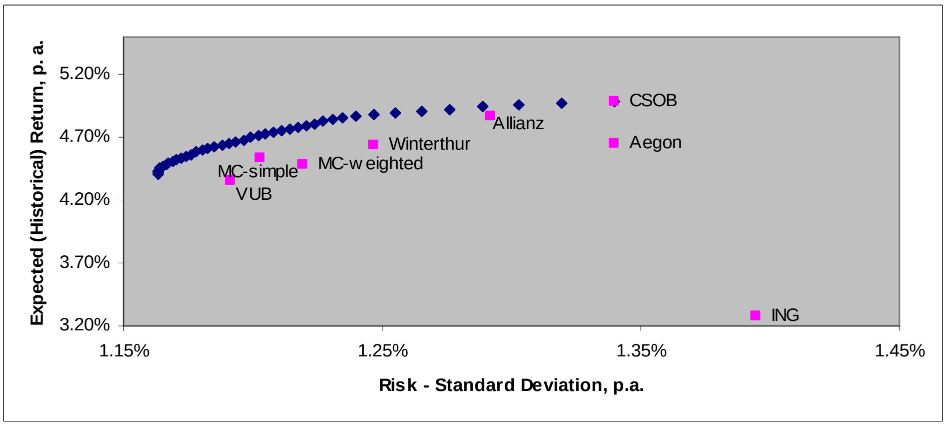


Figure 10

The Efficient Frontier of the Slovak Growth Pension Funds – Historical Returns



It could be interesting to compare the above results-where long-run equilibrium returns were used in the mean-variance optimization-with results where, as is usual, a historical average returns is used in the mean-variance optimization. To illustrate, this comparison was performed for growth pension funds, using average returns from Table 7. The corresponding efficient frontier, together with the positions of individual funds, can be seen in Figure 10. The comparison with Figure 9 confirms that mean-variance optimization is very sensitive to input data.

Mean-variance optimization and the corresponding efficiency analysis provide conclusions that are illustrated in Figures 7 – 10. Table 17 presents additional results, where the funds are ranked based on two return-risk ratios.

Table 17
Funds Ranking Based on Return-risk Ratios

	Conservative Funds		**Balanced Funds**		**Growth Funds**	
Company	average return/risk ratio	equilibrium return/risk ratio	average return/risk ratio	equilibrium return/risk ratio	average return/risk ratio	equilibrium return/risk ratio
Aegon	3	5	5	6	5	6
Allianz	4	1	3	3	1	1
CSOB	2	2	4	4	4	4
ING	6	6	6	5	6	5
VUB	5	4	2	2	3	2
Winterthur	1	3	1	1	2	3

Conclusions

This paper presents a number of approaches for measuring performance and risk in the Slovak market of private pension funds. We have found that in spite of the different approaches, the results are rather similar.

Initially, we ranked pension funds in time, based on a suggested set of characteristics that measure returns and their volatility, lower partial risks, fees, short and longer returns and returns from the viewpoint of market competition. For the three types of funds, it is clear that Allianz and Winterthur are among the relatively good ones, and ING and VUB (possibly) are among the relatively bad ones. The highest volatility of the results is characteristic for CSOB and Aegon funds; however, CSOB funds improve their relative ranking in time.

From the computed return and risk characteristics, it is our opinion that mainly growth funds do not exploit the potential of investment possibilities owing to low levels, in comparison with legislature possibilities and equity investments. Differences between balanced and growth funds are too small. A high positive correlation coefficient points out that all companies, at least at this time, build up their investment strategies in very similar and conservative ways. In spite of this similarity of results, from a view point of absolute levels, the application of modern portfolio theory and resulted approximation of efficient frontiers have shown differences in the context of efficiency. The presented results show that mainly ING, Aegon and CSOB (with the exception of the conservative fund) could achieve their return with lower risks.

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