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CHANGE DETECTION IN THE SLOPE PARAMETER OF A LINEAR REGRESSION MODEL

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ABSTRACT. In sequentially coming data following a linear regression model, we propose a test procedure based on weighted sum of L_1-residuals, to detect a change in the regression parameter, when training data with no change are available. The asymptotic properties are derived and checked in simulations.

1. Introduction

A sequential change-point problem can be found in many areas of real life, e.g., econometrics, finance, astrology, medicine etc. It is defined as follows. We observe data arriving one by one, and after each observation, we decide, whether a change occurs or not. We assume that training data with no change are available and that we should obey restrictions on probabilities of type I and type II errors.

Such formulated problem was first treated by Chu *et al.* [1]. The authors proposed two test procedures. One based on cumulative sums of recursive residuals, second based on fluctuation monitoring. Their results were generalized and new procedures were suggested in number of following papers. We recall Horváth *et al.* [3], where the authors worked with CUSUM procedures based on ordinary and recursive L_2-residuals.

Here we combine the ideas of Koubková [5] and Hušková and Koubková [4] and propose a testing procedure for detection of a change in linear regression model, based on weighted sum of L_1-residuals.

In Section 2 we introduce the model and the main theoretical results. Section 3 summarizes a simulation study and in Section 4, there we sketch the proofs of the theorems.

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2. Assumptions and main results

Assume sequentially coming data following a linear regression model

$$Y_i = \mathbf{X}_i^T \boldsymbol{\beta}_i + e_i, \qquad 1 \le i < \infty, \tag{1}$$

where Y_i are the observed data, $\mathbf{X}_i^T$ are p-dimensional vectors of regressors, which have the form $\mathbf{X}_i^T = (1, X_{i2}, \ldots, X_{ip})$, $\boldsymbol{\beta}_i \in \mathbb{R}^p$ are unknown vectors of regression parameters and e_i are random errors.

About the sequences $\{\mathbf{X}_i^T\}$ and $\{e_i\}$, $1 \le i < \infty$ we assume:

(i) $\{e_i,\, 1 \le i < \infty\}$ are independent identically distributed (i.i.d.) random variables with continuous distribution function F symmetric about zero, such that its second derivative in the neighborhood of zero is continuous and $F'(0) = f(0) > 0$,

(ii) there exist a positive definite matrix $\mathbf{C}$ and a constant $0 < \tau < 1/2$ such that
$$\left| \frac{1}{n} \sum_{i=1}^{n} \mathbf{X}_i \mathbf{X}_i^T - \mathbf{C} \right| = O(n^{-\tau}), \qquad \text{a.s.,}$$
where $|.|$ denotes the maximum norm of matrices,

(iii) $\{\mathbf{X}_i,\, 1 \le i < \infty\}$ create a strictly stationary sequence and there exists $\zeta > 0$ such that
$$\limsup_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} ||\mathbf{X}_i||^{2+\zeta} < \infty \quad \text{a.s.,}$$
where $||.||$ denotes the Euclidean norm, i.e., $||\mathbf{X}_i|| = (\mathbf{X}_i^T \mathbf{X}_i)^{1/2}$,

(iv) the sequences $\{e_i,\, 1 \le i < \infty\}$ and $\{\mathbf{X}_i^T,\, 1 \le i < \infty\}$ are independent.

The availability of training data of size m without any change, is represented by a so called *noncontamination assumption*

(v) $\boldsymbol{\beta}_1 = \ldots = \boldsymbol{\beta}_m$.

Denote the training data by $\boldsymbol{Y}^{(m)}$ and the regressors available at time i by $\mathbf{X}^{(i)} = (\mathbf{X}_1, \ldots, \mathbf{X}_i)$. At each time i we assume $\mathbf{Y}^{(m)}$ and $\mathbf{X}^{(i)}$ as fixed.

The given problem is treated via testing the null hypothesis of no change

$$H_0 : \boldsymbol{\beta}_i = \boldsymbol{\beta}_0, \quad 1 \le i < \infty$$

against the alternative that a change occurs H_A: there exists $k^* > 1$ such that

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_0,\ 1 \le i < m + k^*, \quad \boldsymbol{\beta}_i = \boldsymbol{\beta}_0 + \boldsymbol{\delta}_m,\ m + k^* \le i < \infty, \quad ||\boldsymbol{\delta}_m|| \ne 0.$$

The parameters $\boldsymbol{\beta}_0, \boldsymbol{\delta}_m$ and k^* are unknown.

According to Horváth *et al.* [3], we denote a suitable test statistic by $Q(m,k)$ and an appropriate boundary function by $g(m,k)$. We reject the null hypothesis at a so called *stopping time*

$$\tau(m) = \inf\left\{k \geq 1 : \frac{|Q(m,k)|}{g(m,k)} \geq c(\alpha)\right\}$$

with the understanding that $\inf\{\emptyset\} = \infty$, and such that the critical values $c(\alpha)$ satisfy the later assumptions (2) and (3).

Typically we have to compromise two requests

(a) if a change occurs, we should detect it as soon as possible,
(b) if no change occurs, we should continue the observation as long as possible.

We express these requests via restrictions on probabilities on type I and type II errors. We bound the probability of false alarm with $\alpha \in (0,1]$

$$\lim_{m\to\infty} P\big(\tau(m) < \infty | H_0\big) \leq \alpha, \tag{2}$$

and we require the power of the testing procedure to tend to 1

$$\lim_{m\to\infty} P\big(\tau(m) < \infty | H_A\big) = 1. \tag{3}$$

The task now is to determine all the particular terms necessary to state the decision rule, i.e., $Q(m,k) = Q(Y_1,\ldots,Y_{m+k})$, $g(m,k)$ and $c(\alpha)$.

In Koubková [5], there is proposed a CUSUM type test statistic based on L_1-residuals, which is insensitive to changes with $\boldsymbol{\delta}_m^T \mathbf{c}_1 = 0$, where $\mathbf{c}_1$ is the first column of the matrix $\mathbf{C}$. Motivated by the idea of Hušková and Koubková [4] we propose a test statistic based on weighted L_1-residuals, which is able to detect all changes in $\boldsymbol{\beta}$, i.e.,

$$\widetilde{Q}(m,k) = \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widetilde{e}_i\right)^T \mathbf{C}_m^{-1} \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widetilde{e}_i\right), \quad k = 1,2,\ldots \tag{4}$$

Here $\mathbf{C}_m = \sum_{j=1}^{m} \mathbf{X}_j \mathbf{X}_j^T$ and $\widetilde{e}_i$ are the L_1-residuals: $\widetilde{e}_i = \operatorname{sign}(Y_i - \mathbf{X}_i^T \widetilde{\boldsymbol{\beta}}_m)$ with $\widetilde{\boldsymbol{\beta}}_m = \arg\inf_{\mathbf{b}} \sum_{i=1}^{m} |Y_i - \mathbf{X}_i^T \mathbf{b}|$. Note that $\operatorname{Var} \widetilde{e}_i = 1$.

We adopt the boundary function from Horváth *et al.* [3] and Hušková and Koubková [4], i.e., we use

$$g^2(m,k,\gamma) = \left(\sqrt{m}\left(1 + \frac{k}{m}\right)\left(\frac{k}{m+k}\right)^{\gamma}\right)^2, \qquad \gamma \in \big[0, \min\{\tau, 1/2\}\big)$$

This function was shown to be appropriate for CUSUM's of residuals. The tuning constant γ modifies the ability of the testing procedure to detect better early (γ close to 1/2) or late (γ close to 0) changes.

In the following two theorems the limit behaviour of $\widetilde{Q}(m,k)$ under the null as well as under the alternative hypotheses is described.

Theorem 1. *Let the model* (1) *and the assumptions (i)–(v) be satisfied. Let* $\gamma \in \big[0, \min\{\tau, 1/2\}\big)$ *and* $k = O(m^q)$, *where* $q < 1 + \zeta/2$. *Then, under the null hypothesis* H_0

$$\lim_{m\to\infty} P\left(\sup_{1\le k<m^{1+\zeta/2}} \frac{\widetilde{Q}(m,k)}{g^2(m,k,\gamma)} \le c\right) = P\left(\sup_{0\le t\le 1} \frac{\sum_{i=1}^{p} W_i^2(t)}{t^{2\gamma}} \le c\right),$$

holds for all $c > 0$ *and where* $\big\{W_i(t), t \in < 0,1)\big\}$, $i = 1,\dots,p$ *are independent Wiener processes.*

Theorem 2. *Let the model* (1) *and the assumptions (i)–(v) be satisfied. Let* $\gamma \in \big[0, \min\{\tau, 1/2\}\big)$ *and* $k = O(m^q)$, *where* $q < 1 + \zeta/2$. *Let* $\boldsymbol{\delta}_m$ *is a* p*-dimensional nonzero vector such that*

$$\lim_{m\to\infty} \boldsymbol{\delta}_m^T \boldsymbol{\delta}_m \to 0, \qquad \textit{and} \qquad \lim_{m\to\infty} m\boldsymbol{\delta}_m^T \boldsymbol{\delta}_m \to \infty$$

Then, under the alternative hypothesis H_A

$$\sup_{1\le k<m^{1+\zeta/2}} \frac{\widetilde{Q}(m,k)}{g^2(m,k,\gamma)} \xrightarrow{P} \infty, \qquad \textit{as} \quad m \to \infty.$$

Theorem 1 describes the limit distribution of the test statistic under the null hypothesis and provides the possibility to determine the critical values. Since the exact form of this distribution is known only for $\gamma = 0$, the critical values have to be obtained via simulations (for $p = 2$ they are published in Hušková and Koubková [4]). The range of the tuning constant $\gamma \in [0, \min\{\tau, 1/2\})$ ensures the almost sure finiteness of the distribution.

3. Simulations

To illustrate the behaviour of the proposed testing procedure, we conduct a simulation study, in which we compare this procedure with three others:

$$Q_1(m,k)\mathbf{:} = \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widetilde{e}_i\right)^T \mathbf{C}_m^{-1} \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widetilde{e}_i\right) \dots \text{ here},$$

$$Q_2(m,k)\mathbf{:} = \sum_{i=m+1}^{m+k} \widetilde{e}_i \dots \text{ Koubková [5]},$$

$Q_3(m,k)$: $= \ \widehat{\sigma}_m^{-2} \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widehat{e}_i \right)^T \mathbf{C}_m^{-1} \left(\sum_{i=m+1}^{m+k} \mathbf{X}_i \widehat{e}_i \right) \ldots$ Hušková and Koubková [4],

$Q_4(m,k)$: $= \widehat{\sigma}_m^{-2} \sum_{i=m+1}^{m+k} \widehat{e}_i \ldots$ Horváth *et al.* [3],

where $\widehat{e}_i = Y_i - \mathbf{X}_i^T \widehat{\boldsymbol{\beta}}_m, \widehat{\boldsymbol{\beta}}_m = \left(\sum_{j=1}^{m} \mathbf{X}_j \mathbf{X}_j^T \right)^{-1} \sum_{j=1}^{m} \mathbf{X}_j Y_j$ and $\widehat{\sigma}_m^2 = (m-p)^{-1} \sum_{j=1}^{m} (Y_j - \mathbf{X}_j^T \widehat{\boldsymbol{\beta}}_m)$.

To compare different situations we consider several values of key parameters:

- length of the training period $m = 50, 100, 500$,
- time of change $k^* = 50, 100, 500$,
- tuning constant $\gamma = 0, 0.25, 0.49$,
- error distribution Laplace, $t_4, N(0,1)$
- distribution of X_i: AR(1), i.i.d. $N(0,1), U[-\sqrt{3}, \sqrt{3}]$
- the size and shape of the alternative - namely the null hypothesis, change in the intercept and change in the slope, such that $\boldsymbol{\delta}_m^T \mathbf{c}_1 \neq 0$ and $\boldsymbol{\delta}_m^T \mathbf{c}_1 = 0$.

The simulation procedure proceeds as follows. We generate a sequence of 7000 observations Y_i following the linear regression model (1) under various hypotheses, evaluate the test statistics $Q_j(m,k)$, $j = 1, \ldots, 4$, and determine the corresponding stopping times $\tau_1(m), \ldots, \tau_4(m)$. Selected results of 3000 replications (obtained for $\mathbf{X}_i \sim$ i.i.d. $N(0,1)$, since there was almost no difference between all the distributions considered) are summarized in Table 1 and Figure 1. Here we compare only normal and Laplace error distribution, since the conclusions for the t-distributed errors are not so obvious.

In Table 1, there are the extremes and quartiles of the observed stopping times. The results provide comparison of the four testing procedures with respect to three kinds of alternatives (change in intercept, change only in slope such that $\boldsymbol{\delta}_m^T \mathbf{c}_1 \neq 0$ and change only in slope such that $\boldsymbol{\delta}_m^T \mathbf{c}_1 = 0$) and two error distributions. We consider moderate change ($k \doteq m$) with $\gamma = 0.25$.

The comparison of the tuning constant for various times of change are shown in Figure 1. The figure shows how the power functions of the statistics $Q_1(m,k)$ (bold lines) and $Q_2(m,k)$ (normal lines) increase in time, after change: $\boldsymbol{\delta}_m = (0,1)^T$, $\boldsymbol{\delta}_m^T \mathbf{c}_1 \neq 0$ occurs.

Based on the results, we conclude that the statistics based on L_2-residuals work nicely for normally distributed data, whereas the statistics based on L_1-residuals we recommend for Laplace-like distributed data. Changes in the intercept are detected faster with statistics based on simple residuals and changes only in the slope with those based on weighted residuals. Changes with the property $\boldsymbol{\delta}_m^T \mathbf{c}_1 = 0$ are detected just with $Q_1(m,k)$ and $Q_3(m,k)$. The early changes

TABLE 1. Summaries of stopping times obtained by statistics $Q_j(m,k)$, $j = 1, 2, 3, 4$.

		Laplace errors					$N(0,1)$ errors				
change	statistic	min	$1^{st}Q$	med	$3^{th}Q$	max	min	$1^{st}Q$	med	$3^{th}Q$	max
$\boldsymbol{\delta} = (1,0)$	$Q_1(m,k)$	17	180	215	258	648	18	172	203	241	588
	$Q_2(m,k)$	24	172	201	238	536	24	164	191	225	678
	$Q_3(m,k)$	4	199	244	307.3	2340	40	177	203	239	605
	$Q_4(m,k)$	26	181	219	268	1084	59	164	185	214	465
$\boldsymbol{\delta} = (0,1)$	$Q_1(m,k)$	23	163	183	205	311	24	158	175	194	279
$\mathbf{c}_1^T\boldsymbol{\delta} \neq 0$	$Q_2(m,k)$	21	154	172	193	315	27	149	164	186	288
	$Q_3(m,k)$	24	132	144	157	244	50	127	134	143	188
	$Q_4(m,k)$	64	132	143	156	244	105	126	134	142	197
$\boldsymbol{\delta} = (0,1)$	$Q_1(m,k)$	25	177	246	358	924	14	170	222	316	3331
$\mathbf{c}_1^T\boldsymbol{\delta} = 0$	$Q_2(m,k)$	21	7000	7000	7000	7000	24	7000	7000	7000	7000
	$Q_3(m,k)$	20	188	242	325.3	7000	72	173	204	244	931
	$Q_4(m,k)$	15	7000	7000	7000	7000	104	7000	7000	7000	7000

are detected faster with $\gamma = 0.49$, for moderate changes $\gamma = 0.25$ is suitable and for late changes $\gamma = 0$ works best.

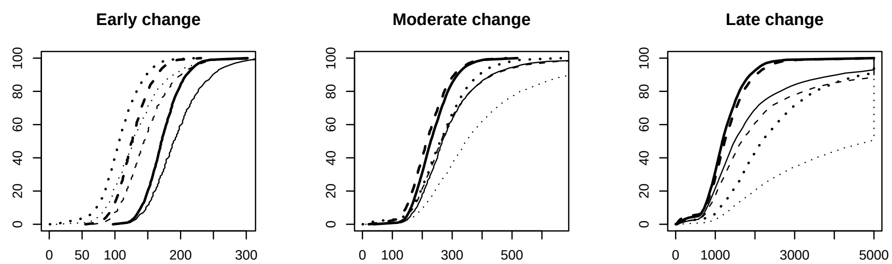

FIGURE 1. Power functions of test statistics $Q_1(m,k)$ and $Q_2(m,k)$, with solid line for $\gamma = 0$, dash line for $\gamma = 0.25$ and dotted line for $\gamma = 0.49$.

4. Proofs of Theorems 1 and 2

In the proofs we use the results of Horváth *et al.* [3] (in the underlain calculations), Koubková [5] and mainly Hušková and Koubková [4].

Proof of Theorem 1. The proof consists of two parts. First we show that the test statistic can be expressed as a sum of i.i.d. random variables, and then, by an invariance principle, we represent this sum by a sum of squared independent Wiener processes. As in Hušková and Koubková [4], we start with the key relation of the proof:

$$\sup_{1\le k<\infty} \frac{1}{g(m,k,\gamma)} \Bigg| m^{1/2}\mathbf{C}_m^{-1/2} \sum_{i=m+1}^{m+k} \mathbf{X}_i \operatorname{sign}\big(Y_i - \mathbf{X}_i^T \widetilde{\boldsymbol{\beta}}_m\big) - m^{1/2}\mathbf{C}_m^{-1/2} \sum_{i=m+1}^{m+k} \mathbf{X}_i \operatorname{sign}(e_i) + \frac{k}{m}\mathbf{C}_m^{-1/2} \sum_{j=1}^{m} \mathbf{X}_j \operatorname{sign}(e_j) \Bigg| = o_p(1). \tag{5}$$

To show this, we define random variables $Z_i = \operatorname{sign}(Y_i - \mathbf{X}_i^T\widetilde{\boldsymbol{\beta}}_m) - \operatorname{sign}(e_i)$, $i = m+1, m+2, \dots$, similarly to K o u b k o v á [5]. Note that given the historical data $\mathbf{Y}^{(m)} = (Y_1, \dots, Y_m)^T$ and the available regressors $\mathbf{X}^{(i)} = (\mathbf{X}_1, \dots, \mathbf{X}_i)^T$, the random variables Z_i are conditionally independent and for their first two conditional moments we get

$$\mathsf{E}\left(\sum_{i=m+1}^{m+k} \mathbf{X}_i Z_i | \mathbf{Y}^{(m)}, \mathbf{X}^{(i)} \right) = -\frac{k}{m} \sum_{j=1}^{m} \mathbf{X}_j \operatorname{sign}(e_j)(1+o(1)), \quad \text{a.s.}$$

$$\mathsf{Var}\left(\mathbf{C}_m^{-1/2} \sum_{i=m+1}^{m+k} \mathbf{X}_i Z_i | \mathbf{Y}^{(m)}, \mathbf{X}^{(i)} \right) = O\left((m+k)^{1+1/(2+\delta)} m^{-3/2} \ln m \right), \quad \text{a.s.,}$$

uniformly in $k = O(m^q)$, where $q < 1+\zeta/2$ and $\mathbf{C}_m = \sum_{i=1}^{m} \mathbf{X}_i \mathbf{X}_i^T$. Now we follow the steps in K o u b k o v á [5].

The interior of the absolute value in (5) can be written as

$$\left| m^{1/2}\mathbf{C}_m^{-1/2} \sum_{i=m+1}^{m+k} \mathbf{X}_i Z_i - \mathsf{E}\left(m^{1/2}\mathbf{C}_m^{-1/2} \sum_{i=m+1}^{m+k} \mathbf{X}_i Z_i | \mathbf{Y}^{(m)}, \mathbf{X}^{(i)} \right) \right| \big(1+o(1)\big).$$

The Kolmogorov-Hájek-Rényi-Chow inequality (see, e.g., S e n [6]) gives

$$P\left(\sup_{1\le k< m^{1+\zeta/2}} \frac{m^{1/2}\mathbf{C}_m^{-1/2} \left| \sum_{i=m+1}^{m+k} \Big(\mathbf{X}_i Z_i - \mathsf{E}\left(\mathbf{X}_i Z_i | \mathbf{Y}^{(m)}, \mathbf{X}^{(i)}\right) \Big) \right|}{g(m,k,\gamma)} > B | \mathbf{Y}^{(m)}, \mathbf{X}^{(i)} \right) = O\Big(B^{-2} m^{-1/2+1/(2+\zeta)} \ln m \Big).$$

asymptotically as $m \to \infty$, for arbitrary $B > 0$. So the relation (5) is proved.

By Lemma 3.1.7 in C s ö r g ő a n d H o r v á t h [2] we have that for each m, there exist two independent p-dimensional Wiener processes $\big\{\mathbf{W}_{1,m}(t),\ t \in$

$[0,\infty)\big\}$ and $\big\{\mathbf{W}_{2,m}(t), t \in [0,\infty)\big\}$ with independent components such that

$$\sup_{1\le k<\infty} \frac{1}{g(m,k,\gamma)} \left|\left|\left| m^{1/2}\mathbf{C}_m^{-1/2}\left(\sum_{i=m+1}^{m+k}\mathbf{X}_i\,\mathrm{sign}(e_i) - \frac{k}{m}\sum_{j=1}^{m}\mathbf{X}_j\,\mathrm{sign}(e_j)\right)\right|\right| - \left|\left|\mathbf{W}_{1,m}(k) - \frac{k}{m}\mathbf{W}_{2,m}(m)\right|\right|\right| = o(1),$$

as $m \to \infty$. Now it remains to prove that

$$\sup_{1\le k<\infty} \frac{||\mathbf{W}_{1,m}(k) - \frac{k}{m}\mathbf{W}_{2,m}(m)||^2}{g^2(m,k,\gamma)} \xrightarrow{D} \sup_{0<t\le 1} \frac{\sum\limits_{i=1}^{p} W_i^2(t)}{t^{2\gamma}},$$

where $\big\{W_i(t),\, t \in (0,\infty)\big\}$, $i = 1,\dots,p$ are independent Wiener processes. This is done in Hušková and Koubková [4]. $\square$

Proof of Theorem 2. Until the time of change, the behaviour of the cumulative sum of weighted residuals follows Theorem 1. Hence it is enough to show that there exists a sequence k_m, such that $k_m - k_m^* \to \infty$ as $m \to \infty$, and

$$Q_3^{(2)}(m,k_m) = m\left|\left|\left(\sum_{i=m+k_m^*}^{m+k_m}\mathbf{X}_i\mathrm{sign}\Big(e_i - \mathbf{X}_i^T\big(\widetilde{\boldsymbol\beta}_m - \boldsymbol\beta_0 - \boldsymbol\delta_m\big)\Big)\right)^T \mathbf{C}_m^{-1/2}\right|\right|^2$$

tends to infinity faster than $g^2(m,k_m,\gamma)$. We can, e.g., choose $k_m = m^\rho + k_m^*$.

As in Koubková [5], we approximate the statistic $Q_3^{(2)}(m,k_m)$ by its conditional expectation given $\mathbf{Y}^{(m)}$ and $\mathbf{X}^{(i)}$. For each $\varepsilon_1,\varepsilon_2 > 0$ we define a set $B_m(\varepsilon_1,\varepsilon_2)$, such that

$$B_m(\varepsilon_1,\varepsilon_2) = \left\{\max_{1\le i\le m^\rho}\left|\mathbf{X}_i^T\big(\widetilde{\boldsymbol\beta}_m - \boldsymbol\beta_0\big)\right| < \varepsilon_1 m^{-q/2}, \max_{1\le i\le m^\rho}\left|\mathbf{X}_i^T\boldsymbol\delta_m\right| < \varepsilon_2\right\}.$$

It can be shown that $P\big(B_m^c(\varepsilon_1,\varepsilon_2)\big) \to 0$, as $m \to \infty$, for each $\varepsilon_1,\varepsilon_2 > 0$. We choose $\varepsilon_1 m^{-q/2} + \varepsilon_2 < D$, where $D > 0$ encloses the neighborhood of zero, where we can apply Taylor expansion of F.

On the set $B_m(\varepsilon_1,\varepsilon_2)$ we obtain, by the assumptions (i) and (iii), as $m \to \infty$

$$\mathsf{E}\left(Q_3^{(2)}(m,k_m)|\mathbf{Y}^{(m)},\mathbf{X}^{(i)}\right) = (k_m - k_m^*)^2\boldsymbol\delta_m^T\mathbf{C}^T\boldsymbol\delta_m + O\Big((k_m - k_m^*)^2 m^{-\tau}||\boldsymbol\delta_m||\Big).$$

Note that $\mathsf{Var}\big(Q_3^{(2)}(m,k_m)|\mathbf{Y}^{(m)},\mathbf{X}^{(i)}\big) = G_1(k_m - k^*)^2$, where $G_1 > 0$ is a constant. Since $\widetilde{e}_i$ are conditionally i.i.d., we can apply the Chebyshev inequality, which enables the desired representation. Therefore, we have, as $m \to \infty$

$$\frac{\left|Q_3^{(2)}(m, k_m)\right|}{g^2(m, k_m, \gamma)} \geq G_2 m \boldsymbol{\delta}_m^T \mathbf{C} \boldsymbol{\delta}_m \big(1 + o(1)\big)$$

with some positive constant G_2, which finishes the proof. □

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