Portfolio Insurance Strategies and their Applications

*Vladimír MLYNAROVIČ**

Abstract

At first the paper shortly characterizes basic classes of portfolio insurance strategies that provide the investor an ability to limit downside risk while allowing some participation in upside markets. Then some extensions of discrete Constant Part Portfolio Insurance (CPPI) methods that introduce risk budget, a stop loss rule, locking of the guaranteed value, the asset management fee and risk measures in the multiplier are presented and illustrated. Finally the paper presents a modification of CPPI method for pension funds with moving investment horizons. As the result user procedures in Excel environment that automatize the process of guaranteed strategies construction were developed.

Keywords: *portfolio insurance, CPPI, multipliers, lock in value, moving horizon*

JEL Classification: G11

1. Introduction

Portfolio insurance is designed to give to an investor the possibility to limit downside risk while allowing some participation in upside markets. Such approaches allow the investor to recover a given percentage of the initial investment at maturity, in particular in falling markets. There exists various portfolio insurance models, among them the Constant Proportion portfolio Insurance (CPPI), the Option Based Portfolio Insurance (OBPI) and Volatility Based Portfolio Insurance (VBPI). The goal of the paper is to suggest and illustrate some modifications and extensions in VBPI and CPPI approaches. The reminder of this paper is as follows. In section 2 basic portfolio insurance principles are shortly characterized, but in VBPI we suggest a modification that uses a goal programming approach. Section 3 describes some extensions of discrete CPPI

* Vladimír MLYNAROVIČ, Faculty of Social and Economic Sciences, Comenius University in Bratislava, Mlynské luhy 4, 82 105 Bratislava 2, Slovak Republic; e-mail: vladimir. mlynarovic @fses.uniba.sk

methods that introduce risk budget, a stop loss rule, locking of the guaranteed value, the asset management fee and risk measures in the multiplier. As a main result this section presents a modification of CPPI method for pension funds with moving investment horizons with the goal to guarantee the current average monthly level of a pension unit as the average monthly level of the pension unit in the stated number of months later. Illustrations of suggested approaches in excel applications are in section 4 and in conclusions we summarize basic results.

2. Portfolio Insurance Principles

The CPPI was introduced by Perold (1986) and Perold and Sharpe (1988) for fixed income instruments and Black and Jones (1987) for equity instruments. The method has been analyzed in Black and Rouhani (1989) and Black and Perold (1992). This method is based on a particular strategy to allocate assets dynamically over time. The investor starts by choosing a floor equal to the lowest acceptable value of the portfolio. Then he computes the cushion which is equal to the excess of the portfolio value over the floor. Finally, the amount invested to the risky asset, exposure, is determined by multiplying the cushion by a predetermined multiplier. The remaining funds are invested in the reserve asset, e.g. a liquid money market instrument. The method is based on the following mathematical background.

The portfolio manager is assumed to invest in two basic assets: a money market tool, denoted by B, and a portfolio of traded assets such as a composite index, denoted by S. The period of time considered is $[0, T]$. The strategy is self-financing. The values of the riskless asset B and risky asset S evolve according to

$$dB_t = B_t r\,dt, \quad dS_t = S_t\left[\mu\,dt + \sigma\,dW_t\right]$$

where

r – the deterministic interest rate,
W_t – a standard Brownian motion,
μ and σ – positive constants.

The CPPI method consists of managing a dynamic portfolio so that its value is above a floor P_t at any time t. The value of the floor gives the dynamic insured amount. It is assumed to evolve according to

$$dP_t = P_t r\,dt$$

Obviously, the initial floor P_0 is less than the initial portfolio value V_0. The difference $C_0 = V_0 - P_0$ is called the cushion. Its value at any time is given by

$$C_t = V_t - P_t$$

Denote by E_t the exposure, which is the total amount invested in the risky asset. The standard CPPI method consists of letting $E_t = mC_t$, where m is a constant called multiplier. The interesting case is when $m > 1$, that is, when the pay--off function is convex.

The OBPI was introduced by Leland and Rubinstein (1976) and analyzed by Martellini, Smisek and Goltz (2005).The method consists basically of purchasing, for simplicity, one share of the asset S and one share of European put option on S with maturity T and exercise price K. Thus, the portfolio value V^O is given at the terminal date by

$$V_T^O = S_T + \max\{K - S_T, 0\}$$

which is also

$$V_T^O = K + \max\{S_T - K, 0\}$$

due to Put/Call parity. This relation shows that the insured amount at maturity is the exercise price K. The value V_t^O of this portfolio at any time t in the period $[0, T]$ is

$$V_t^O = S_t + P(t, S_t, K) = Ke^{-r(T-t)} + C(t, S_t, K)$$

where $P(t, S_t, K)$ and $C(t, S_t, K)$ are the Black-Scholes values of the European put and call.

The volatility based portfolio insurance (VBPI), introduced in Temporis Presentation (2008), is a systematic investment process, where exposure to equity is adjusted routinely in order to maintain the volatility of the fund close to pre-determined target volatility levels. The guarantee mechanism is based on series of guaranteed bond and at maturity of the bond investor will receive the higher between:

- the final net asset value of the fund,
- the guaranteed net asset value determined on the issued of the bond

It means that each investment is (at least) guaranteed at the maturity. From a view point of a strategic allocation assets of the fund are invested in long-term assets and/or short-term assets (money market tools) according to a specific volatility based risk model. The exposure to the long term assets will be routinely adjusted depending on the prevailing historical volatility of long-term asset and pre-defined target volatility levels which become more conservative as the investment horizon approaches:

- at inception, assets are mainly invested in long term assets to boost returns,
- near to the investment horizon, assets are fully invested in short-term assets.

In a tactical asset allocation, asset mix within long-term assets can be actively managed to increase the exposure to the long-term assets and maximize the expected return for a given target volatility level.

Levels of target volatility on observation date t, σ_{tB}, $t = 0, 1,\ldots T$, are defined at inception, where

$$\sigma_{tB} \to 0, as\ t \to T, \quad \sigma_{TB} = 0$$

The fund manager determines the historical volatility σ_{tS} of the long-term asset and the exposure E_t to the risky (long-term) asset on observation date t, $t = 0, 1,\ldots T$, can be then determined, for example, by one of the following approaches:

a) by a simple approach (SM-approach) suggested in the Temporis presentation (2008), where

$$E_t = \frac{\sigma_{tB}}{\sigma_{tS}}, \quad E_t \le h \tag{2.1}$$

b) by a goal programming approach (GP-approach), where the following problems are solved

$$\min \quad p_t + n_t$$

subject to (2.2)

$$E_t^2 \sigma_{tS}^2 + (1 - E_t)^2 \sigma_{tM}^2 + 2E_t(1 - E_t)\sigma_{tSM} + n_t - p_t = \sigma_{tB}^2$$
$$p_t n_t = 0, \quad p_t \ge 0,\ n_t \ge 0, \quad 0 \le E_t \le h$$

where

p_t and n_t – deviation variables,

σ_{tM} – the historical volatility of the money market tool (riskless asset),

σ_{tSM} – the historical covariance between risky asset and riskless asset on observation date t.

In other words, investment in long-term (risky) asset us routinely adjusted such that the realized volatility of the fund remains close to the target volatility levels and the portion of the assets not invested in long-term assets is allocated to the short-term ("riskless") assets. Among the advantages of such approach there are:

- exposure to long-term assets may temporarily decrease when long-term assets´ realized volatility increases (when market conditions are hectic),
- the realized volatility usually converges back to its historical average (well known mean reverting pattern of volatility), which prevents the assets of the fund from being fully and permanently invested in short-term assets.

3. The Extensions of the CPPI Method

Bertrand and Pringet (2002; 2004) analyze and compare two standard portfolio insurance methods, CPPI and OBPI from a view point such criteria as pay--offs at maturity, stochastic or "quantile" dominance of their returns and examine dynamic hedging properties as well. They also compare the performance of two standard methods when the volatility of the stock index is stochastic and provide a quite general formula for the CPPI portfolio value. Boulier and Kanniganti (1995) examine expected performance and risk of various portfolio insurance strategies in a realistic case, where there are constraints on the maximum exposure to the market. The following analysis also concentrates on a realistic, discrete, CPPI method and develops two modification of the basic approach.

3.1. CPPI Method with the *Stop Loss Rule*

The following modification of discrete CPPI method takes into account fees for the asset management, which are derived from the starting value of the portfolio at each observation moment, and introduce a lock in of the guaranteed value, floor, when current portfolio value arises, and the stop loss rule for the zero level of the exposure. For the portfolio value at the moment *t* we have

$$V_t = V_{t-1}(1 + E_{t-1}s_t + (1 - E_t)r_t) - V_0 f \frac{d_t - d_{t-1}}{365}, \quad V_0 = 100 \tag{3.1}$$

where

$$s_t = \frac{S_t}{S_{t-1}} - 1, \quad r_t = \frac{B_t}{B_{t-1}} - 1$$

and

- f – the yearly fee for the asset management, e.g. 2%,
- S_0 – the official closing price of the risky asset (reference index) on the start date,
- S_t – the official closing price of the risky asset (reference index) on observation date *t*,
- B_t – the theoretical price on observation date *t* of a synthetic zero coupon bond with nominal 1 paid on the terminal date,
- r_t – the price return of the riskless asset for observation date *t,*
- d_t – the number of calendar day between the start date and observation date *t.*

The actual proportion of the net asset value which is allocated to the risky asset, the exposure, on observation date *t* equals:

$$E_t = m_t C_t = m_t (V_t - P_t), \quad 0 \le E_t \le 1 \tag{3.2}$$

where

$$m_t = \frac{1}{lV_t}$$

while *l is* the *stop loss*, the percentage loss that the risky asset is allowed to have in 1 day before the exposure in the risky asset drops to 0%, e.g. 15%, and

$$P_t = V_0 B_t \left(k_t + (1+f)^{\frac{d_T - d_t}{365}} - 1\right) \tag{3.3}$$

where

$$k_t = \max\left\{k_{t-1}, \frac{V_t}{V_0}\right\}, \quad k_0 = 1$$

is the so called *lock in* value on observation date that locks in the net allocation value, which is guaranteed at maturity. It holds

- if $E_t = 0$ on any observation date, then all subsequent values for exposure will be set to 0%,
- if there is percentage loss higher then the stop loss, the guarantee at maturity would still be

$$k_T = V_0 \max\left\{1, k_{t-1}\right\}$$

3.2. CPPI with the Risk Budget and the Multiplier Modification

The next modification of the discrete CPPI method, similarly as the previous one, takes into account fees for the asset management, but the value of the fee on each observation date t is derived from the net asset value on observation date $t-1$. It also uses lock in system for the guaranteed floor, but introduce its discrete shifts in both directions (up, down) in dependence from the current net asset value with the restriction that its minimum must not drop under the defined minimal level. The minimal level of the floor is directed through the risk budget or, in other words, through the feasible level of the guaranteed value drop below the starting investment. Observe that an advantage of such kind of the lock in system, which enables limited shifts in the both directions, is that in the case of exposure falls create potential for exploitation of future possible growth on the stock market.

The new element of this CPPI method modification consists in introducing the risk in the multiplier construction. The method looks for such multiplier value that exploit the current value of the reduced cushion as much as possible. The reduction size is derived from the risk for period from the observation date to terminal date, or for defined minimal period. As the risk measure the historical CVaR is used.

For the portfolio value on the observation date t we now have

$$V_t = V_{t-1}(1 + E_{t-1}s_t + (1 - E_t)r_t - f\frac{d_t - d_{t-1}}{365}), \quad V_0 = 100 \tag{3.4}$$

where

$$s_t = \frac{S_t}{S_{t-1}} - 1, \quad r_t = \frac{B_t}{B_{t=1}} - 1, \quad r_t^{pa} = (1 + r_t)^{\frac{365}{d_t - d_{t-1}}}$$

and

B_t – the theoretical price of the riskless asset on observation date t,
r_t^{pa} – annualized riskless asset return on observation date t.

The actual proportion of the net asset value which is allocated to the risky asset, the exposure, on observation date t equals:

$$E_t = m_t C_t = m_t(V_t - P_t), \quad d \le E_t \le h \tag{3.5}$$

where

$$P_t = k_t(1 + r_t^{pa})^{-\frac{d_T - d_t}{365}}$$

$$k_t = \begin{cases} \max\{\max\{k_{t-1}, V_0\}(1+q), uV_t\}, & V_t > k_{t-1}(1+q) \\ \max\{k_{t-1}(1-q), V_0(1-b_r), cV_t\}, & V_t < k_{t-1} \\ k_{t-1}, & else \end{cases}$$

while

$$k_0 = V_0(1 - b_r)$$

and

d – the lower bound on the exposure,
h – the upper bound on the exposure,
q – the bound for lock in of the guaranteed value, e.g. 2%,
u – the lock in parameter in „up“ direction with possible values 0 or 1,
c – the lock in parameter in „down“ direction with possible values 0 or 1,
b_r – the risk budget, e.g. 3%.

The multiplier value m_t is computed according to the following iteration procedure:

1. $m_t^i = 1, \quad i = 1$
2. $\omega = \max\{m_t^i(V_t - P_t), 0\} \times CVaR_t\sqrt{\min\{d_T - d_t, o\}}$
3. $Q = V_t - \omega$
4. $(Q - P_t)\begin{cases} > 0, & m_t^{i+1} = m_t^i + 1, i = i + 1, go to\ step\ 2 \\ \le 0, & m_t = m_t^i, the\ end \end{cases}$

where

$CVaR_t$ – the historical conditional VaR on observation date t,
o – the minimal length of the period for the risk accounting.

3.3. CPPI Modification for Moving Horizon

The legislative changes approved by Slovak Parliament in 2009 established the commitment for the companies that run pension funds in the second pillar of the Slovak pension system to guarantee average level of the pension unit in the current month by not less average level of the pension unit in six (= n) months later. It led to an introduction of the moving horizon into the method presented in the previous part. The average level of the pension unit thus defines its guaranteed value for six months later. The system was started in July 2009 and the first six guaranteed values were gradually established. The guaranteed values are moving in time on the month base and create moving investment horizons. At each time t the corresponding floor P_t for CPPI is selected as the highest discounted guaranteed value from all known guaranteed values. The strategy works on daily base and the guaranteed value being derived from the data of current months is gradually updated on the base of known values of pension unit value.

Let us assume that we have month time intervals with the starting points d_i, $-n \le i \le n$, the current day t is from the interval $[d_0, d_1)$ and v_j is the value of a pension fund unit at time (day) j, $j \in [d_{-n}, t]$. Then

$$V_i = \frac{\sum_{j=d_{i-6}}^{d_{i-(n-1)}-1} v_j}{d_{i-(n-1)} - d_{i-n}}, i = 0,1,\dots,n-1 \tag{3.6}$$

and

$$V_n = \frac{\sum_{j=d_0}^{t} v_j}{t - d_0 + 1} \tag{3.7}$$

are the guaranteed values of pension unit for time intervals starting at points d_i, $I = 0, 1, \dots, n$, that are known at the current day t. In the suggested CPPI modification all these values are taken into account in the definition of the floor value P_t, where

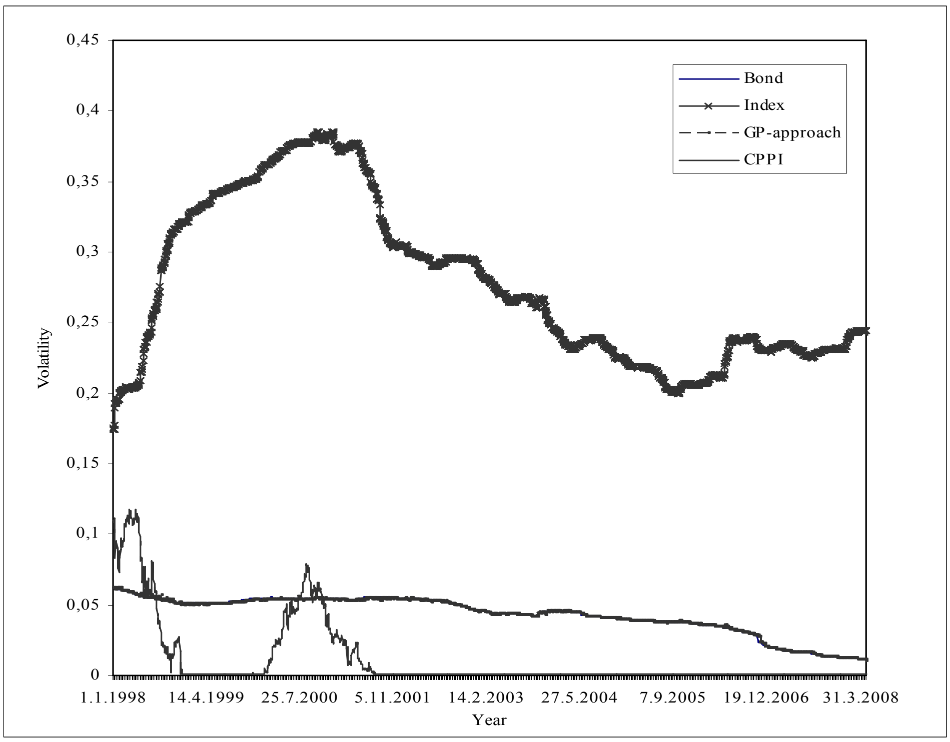
$$P_t = \max\left\{ V_i (1 + r_t^{pa})^{-\frac{d_{i+1}-t}{365}}, i = 0,1,\dots n-1; V_n (1 + r_t^{pa})^{-\frac{(d_n - t)+(t-d_0)}{365}} \right\} \tag{3.8}$$

and CPPI strategy building continues as it was described in the previous part.

4. Applications Illustrations

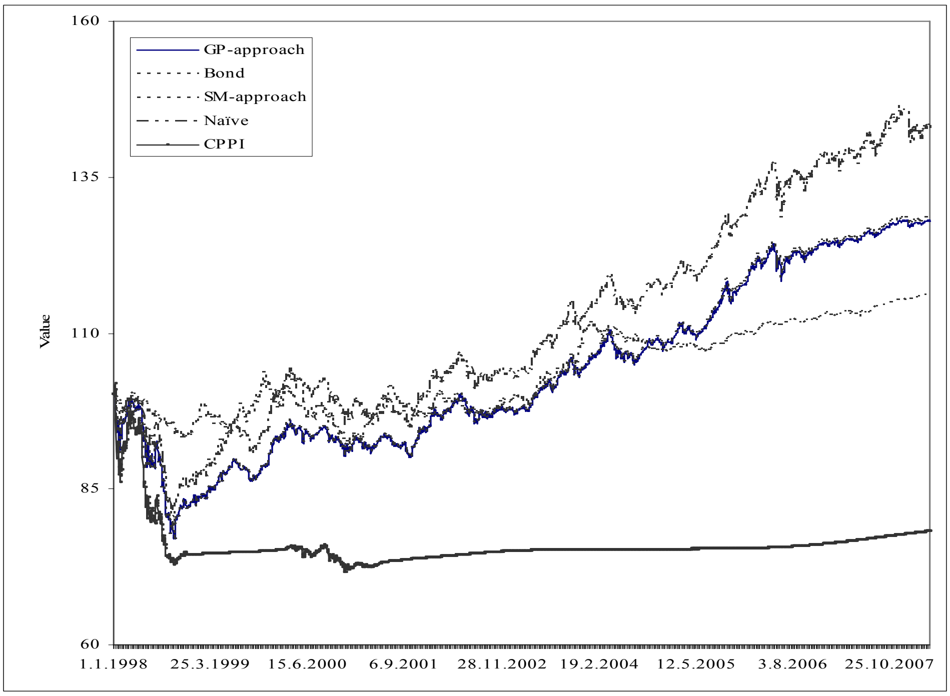
The first developed decision support system developed in excel environment realizes VBPI approaches described in the section 2. In the first illustration we have used the daily data for MSELEMEE commodity index for the period from January 3, 1995 to May 7, 2008 together with daily data for bond a and money market tool to compare SM-approach and GP-approach VBPI strategies with the standard CPPI strategy with multiplier value equal 2 and a naïve portfolio with 20% exposure. The results are presented in Figure 1, Figure 2 and Figure 3. In Figure 1 we can see that GP-approach replicates the volatility of the bond. Results in Figure 2 show the development of corresponding "portfolio" values that start from value 100. SM-approach and GP-approach provide better results than bond. The best result has the naïve portfolio, but its high volatility is in general non acceptable for pension funds. This illustration shows a case where a basic CPPI approach fails. The high volatility of this portfolio in the beginning part of the period caused that the portfolio was forced on that money market as one can se from the Figure 3.

F i g u r e 1
Volatilities of the Strategies



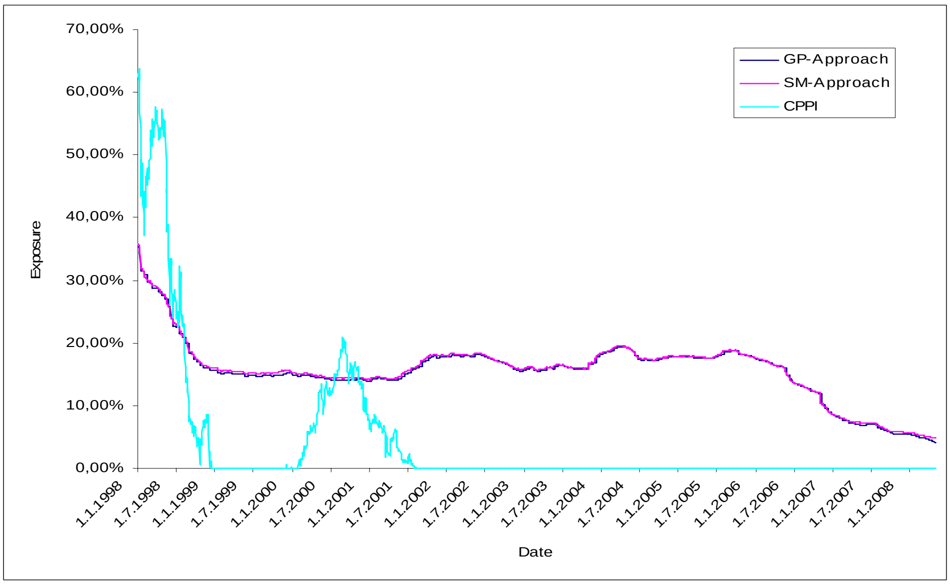
Source: Our computations.

Figure 2
Portfolio Values



Source: Our computations.

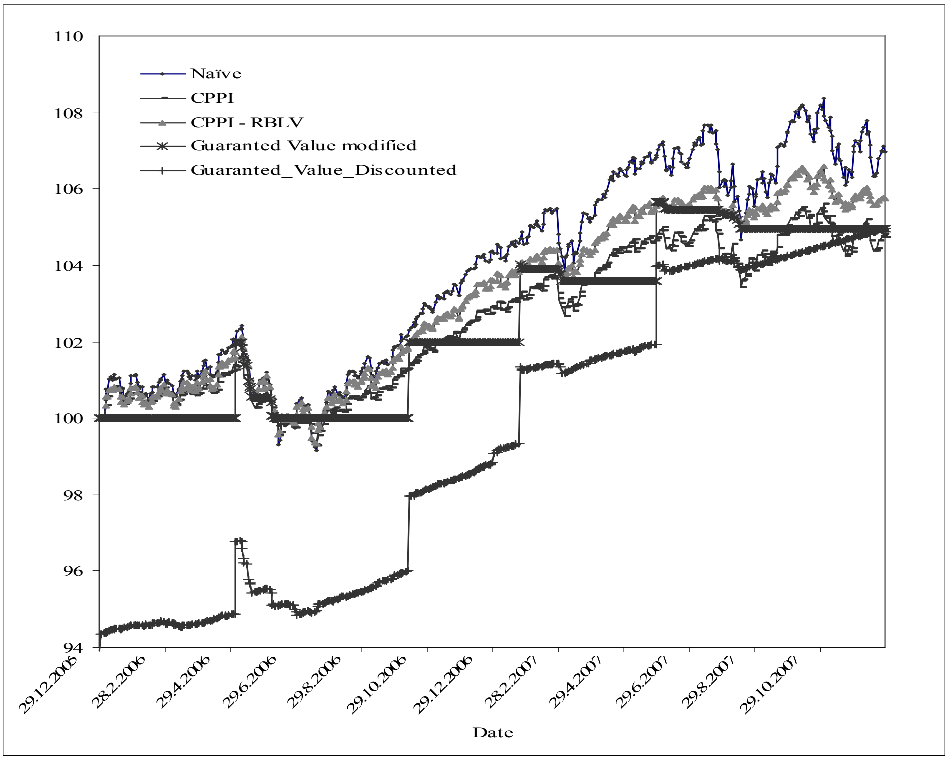
Figure 3
The Exposures of the Strategies



Source: Our computations.

F i g u r e 4

CPPI Portfolio Values



Source: Our computations.

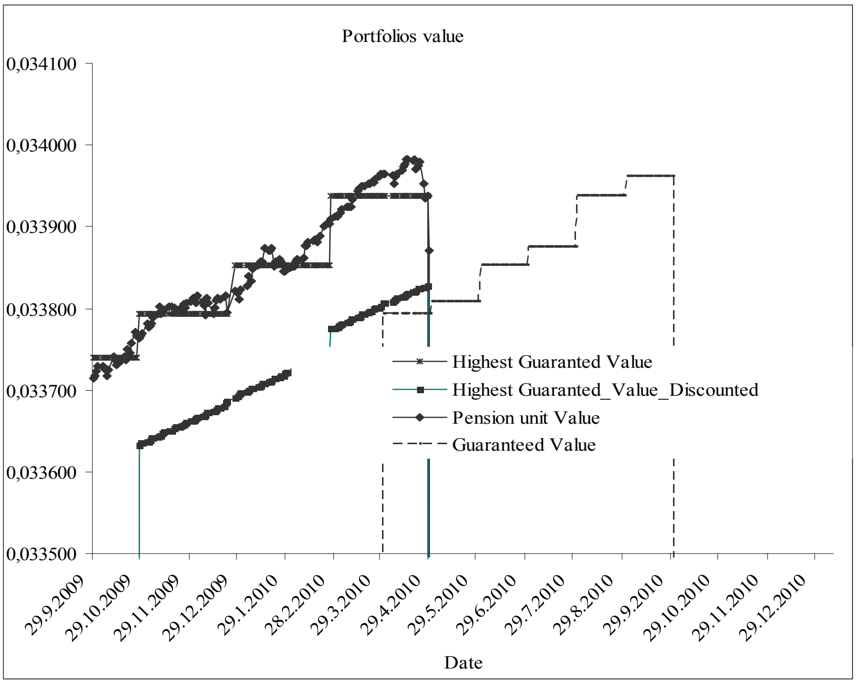
The second developed decision support system developed in excel environment realizes CPPI approaches described in the section 3.2. In the second illustration we have used the daily data for MXWO Index (but portfolio manager has a possibility to choose among a class of risky assets from Bloomberg) for period from January 2, 2006 to December 31, 2007 to compare the standard CPPI strategy with multiplier value equals 3 with the CPPI-RBLV strategy presented above in the part 3.2. The results are presented in Picture 4. In the picture together with the portfolio value for the CPPI strategies there are values of the naive portfolios value, with the exposure 25%, and evolution of guaranteed value and its discounted value as well. In this illustration 3% yearly fee is assumed.

The decision support system developed in excel environment for CPPI with moving horizons combines modifications of the CPPI method presented in the section 3.2 and 3.3. It uses daily data form Bloomberg of selected risky assets and money market tool according to the decision of portfolio (fund) manager. It is relatively fully automatized in the sense that for $d_0 \le t \le d_1 - 1$ there is only one another input, the daily value v_t of the pension unit for the selected fund.

These values are provided by the Slovak National Bank on the daily base. For the $t = d_1$ the value V_0 loses the importance and corresponding VBA procedure moves the strategy one month forward. The user has the opportunity to choose among the companies that runs the pension funds in the Slovak Republic. There are six companies and each of them runs three funds: conservative, balanced and growth one. User has also the opportunity to define the parameters that control the CPPI modification described in the part 3.2 of the paper. Finally the system compares the resulted strategy with the standard CPPI method with the fixed multiplier and with the naïve strategy with the fixed exposure. In the third illustration the graphical output of the decision support system that realizes the CPPI modification for moving horizon in excel environment is presented in Figure 5 for one of the pension fund in Slovak Republic for data to April 30, 2010.

F i g u r e 5

CPPI Strategy for Moving Horizon



Source: Our computations.

Conclusions

Guaranteed funds have had great success in a theory and practice of financial investment. These products are complex, but their behavior is not always well understood at the first glance, and questions about their expected returns and risk profiles are in many cases opened. Presented applications confirm the great flexibility of the approaches to specific market conditions and their usefulness in practical management of investment strategies. Described results and their illustrations show that exist a space for such modifications of basic principles that in our opinion have a potential to provide interesting improvements. In volatility based approach one can, for example, maximize portfolio returns for feasible interval of deviations from the bond volatility. Instead of a one risky asset we can also use a class of risky assets where the portfolio of risky asset is implicitly constructed inside the portfolio insurance strategy building. Including a principle that gives a possibility to lock and modify the floor in CPPI could limit situations as presented in the first illustration, when the strategy relatively soon ends on the money market and fails. As a one of the important message of this paper we would like to stress that practical applications such results as the one in the section 3.3 assumes used friendly decision support system. In our opinion excel environment with its optimization solvers and possibilities to communicate with such data sources as Bloomberg provide excellent space for such developments.

References

BERTRAND, P. – PRIGENT, J.-L. (2002): Portfolio Insurance Strategies: OBPI versus CPPI. [GREQAM Seminar February.] Montpelier: Université Montpelier.

BERTRAND, P. – PRIGENT, J.-L. (2004): Portfolio Insurance Strategies: A Comparison of Standard Methods when the Volatility of the Stocks is Stochastic. [GREQAM Seminar November.] Montpelier: Université Montpelier.

BLACK, F. – JONES, R. (1987): Simplifying Portfolio Insurance. The Journal of Portfolio Management, *14*, No. 1, pp. 48 – 51.

BLACK, F. – ROUHANI, R. (1989): Constant Portfolio Insurance and the Synthetic Put Option: A Comparison. In: FABOZZI, F. J. (ed.): Institutional Investor Focus on Investment Management. Cambridge, MA: Ballinger, pp. 695 – 708.

BLACK, F. – PEROLD, A. R. (1992): Theory of Constant Proportion Portfolio Insurance. The Journal of Economics, Dynamics and Control, *16*, No.3 – 4, pp. 403 – 426.

BOULIER, J.-F – KANNIGANTI, A. (1995): Expected Performance and Risks of Various Portfolio Insurance Strategies. [Proceedings of the 5th AFIR International Colloquium, pp. 1093 – 1122.] Brussels: AFIR International Colloquium.

LELAND, H. E. – RUBINSTEIN, M. (1976): The Evolution of Portfolio Insurance. In: LUSKIN, D. L. (ed.): Portfolio Insurance: A Guide to Dynamic Hedging. New York: Wiley.

MARTELLINI, L. – SIMSEK, K. – GOLTZ, F. (2005): Structured Forms of Investment Strategies in Institutional Investor's Portfolios. An Edhec Risk and Asset Management Research Centre Publication. Lile – Nice: EDHEC Bussines School.

PEROLD, A. (1986): Constant Portfolio Insurance. [Unpublished Manuscript.] Harward: Harvard Business School.

PEROLD, A. – SHARPE, W. (1988): Dynamic Strategies for Asset Allocation. Financial Analyst Journal, January – February, pp. 16 – 27.

Temporis Presentation (2008): Global Equities and Derivative Solutions. Paris: Société Générale, Corporate and Investment Banking.