

Wiredu contra Lewis on the Right Modal Logic

David B. Martens*

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Abstract: This paper is a critical study of an argument put forward by Kwasi Wiredu in his engagement with C. I. Lewis on choosing the right modal logic for logical necessity. Wiredu argues that Lewis “could have been more adventurous modally with perfect logicity” and could justifiably have accepted S4 rather than being “to the last cautious of any system stronger than S2” (Wiredu 1979). I address terse, incomplete, and provocatively incongruous notes on Wiredu’s paper by (Makinson 1980) and (Humberstone 2011), as well as a paper by (Cresswell 1965) that Humberstone cites, and I draw on recent work by (Lewitzka 2015; 2016). I conclude that Wiredu’s argument cannot be accepted as sound but a variant argument can be accepted as sound.

Keywords: C. I. Lewis; equivalence; identity; Kwasi Wiredu; modal logic; S4.

1 Introduction

Kwasi Wiredu argues against C. I. Lewis that the right modal logic for logical necessity is not S2 but “(at least) S4” (Wiredu 1979, 692; see Lewis

*University of the Witwatersrand

✉ Department of Philosophy, School of Social Sciences, University of the Witwatersrand, Johannesburg, Private Bag 3, WITS 2050, South Africa

✉ david.martens@wits.ac.za or david.martens@gmail.com



and Langford 1959). My questions are, what is Wiredu's argument exactly, and is it sound? I conclude that Wiredu's argument cannot be accepted as sound but a variant argument can be accepted as sound.

My questions are prompted by the provocative incongruity between, and incompleteness of, terse notes on Wiredu's paper by David Makinson and Lloyd Humberstone. Incongruously, while Makinson finds "a technical error" that he says "vitiates both parts of the paper" and "some confusion" in the second part (Makinson 1980), Humberstone credits Wiredu's paper with a worthwhile "observation" and "result" that Humberstone also credits to a paper by Max Cresswell (Humberstone 2011, 604, 1154; see Cresswell 1965, not cited by Wiredu). Moreover, Makinson's and Humberstone's notes are limited to only some formal aspects of Wiredu's paper, where the main concern really is pragmatic in a sense that borders on epistemic. (The broader philosophical contexts of Wiredu's and Lewis's formal investigations are sketched by Osha 2014 and Hunter 2016.) A satisfying treatment of Wiredu's paper thus should address the incongruity and the incompleteness of Makinson's and Humberstone's notes, and the relation of Wiredu's argument to Cresswell's.

As suggested by Makinson's comments quoted above, Wiredu frames his paper as providing two arguments for his main conclusion, one primary and the other "alternative" (Wiredu 1979, 692). The alternative argument is aimed at showing that "the addition of [a certain] definition of necessity [...] to a suitable system of propositional logic yields at least S4" (Wiredu 1979, 693). It seems to me that Humberstone is silent about Wiredu's primary argument and that his attention is focused on formal aspects of this alternative argument, which he takes to be in relevant respects essentially the same as Cresswell's argument, and which he thinks can be understood in such a way as to be sound and free from any insuperable technical error or confusion of the sort Makinson reports. I think in the end, though, Wiredu's paper is best understood as providing a single argument for his main conclusion, but with two formally distinct lines of support for a key premise in that argument.

My attention for the most part is focused on what Wiredu presents as his primary argument, where Wiredu's pivotal challenge to Lewis is this.

Lewis himself was to the last cautious of any [modal] system stronger than S2 [But] he could have been more adventurous modally with perfect logicality [For there is] a way of establishing the logical inevitability of S4 [using a certain substitution rule that

is] very plausible and intuitively acceptable [When either S1 or S2 is appropriately extended with that rule,] (at least) S4 becomes available [So,] by Lewis' own logical interpretation of the modalities he was committed to S4 . . . (Wiredu 1979, 689, 691, 692, 694).

Clearly, while Wiredu thinks S4 is the right modal logic—in the sense that the truth about logical necessity includes at least S4—that is not exactly what he intends to argue. Rather, his challenge is a circumstantial *ad hominem* to Lewis. Wiredu's argument in outline is that, because Lewis justifiably accepted that the truth about logical necessity includes at least S1 or S2, and Lewis could justifiably have accepted the substitution rule Wiredu has in mind, and S4 is deductively contained in the system that results when either S1 or S2 is appropriately extended with that substitution rule, therefore Lewis could justifiably have accepted that the truth about logical necessity includes at least S4. Fairly interpreting and assessing Wiredu's argument thus requires considering exactly what rule he has in mind and exactly why he thinks Lewis could justifiably have accepted that rule.

I seek to understand and adjudicate Wiredu's challenge to Lewis as much as possible on its own terms. So I take Lewis's antecedent commitments, as Wiredu sees them and as much as possible, as given. First, like Wiredu, I do not dispute or attempt to fully articulate Lewis's pragmatism about choice of logical system. Lewis said that “the grounds of choice [among alternative logical systems] can only be pragmatic”, “such as simplicity or comprehensiveness or accord with our most frequent purposes in inference” (Lewis 1932, 507; 1934, 74). Though Lewis gave “little theoretical discussion of these ‘pragmatic considerations’” (Parry 1968, 153), I do not discourage any inclination readers may have to equate Lewis's ‘pragmatic grounds’ with ‘epistemic justification’. Granted, “unlike Wilfrid Sellars and many others in the latter half of the 20th century, Lewis never recognized such [pragmatic] factors as criteria of empirical [epistemic] justification” (Hunter 2016, §4). Nevertheless, Lewis did not dispute William Parry's account of Lewis's pragmatism about choice of logical system, as being concerned with a “question of *truth*” about our ordinary concept of logical implication (Parry 1968, 154) and as including intuitions about logical and modal concepts among ‘pragmatic grounds’ in Lewis's sense (Parry 1968, 126; Lewis 1968, 658; see also Cresswell 1967). (Obviously, equating Lewis's ‘pragmatic grounds’ with ‘epistemic justification’ would not

commit Lewis to reducing either necessity itself or the meaning of ‘necessary’ to epistemic terms.) Second, I do not dispute either Wiredu’s and Lewis’s shared (but not universally shared) intuitions about logical implication and about modality, or their shared (but problematic) acceptance of the view that “Strict implication is identical with deducibility”, that is, with logical implication (Wiredu 1973, 50; see Lewis and Langford 1959, ch. 8). Finally, I use ‘ S has justification to accept p ’ (and analogously for rejection of or suspension of judgment about p) to abbreviate ‘ S has some prima facie, pragmatic grounds (in Lewis’s sense) to accept p ’, where the latter is understood not to imply either that p is true, or that S is aware of having those grounds, or that S does not also have some contrary grounds, or that S does accept p .

In section 2, I reconstruct Wiredu’s primary argument and show that it cannot be accepted as sound. In section 3, I amend Wiredu’s argument and show that the initial variant argument I construct can be accepted as sound. In section 4, I further amend Wiredu’s argument and further address relations among Wiredu’s primary and alternative arguments, the final variant argument I construct, Makinson’s and Humberstone’s notes, and Cresswell’s argument.

2 Wiredu’s primary argument

In this section I reconstruct Wiredu’s primary argument and show that it cannot be accepted as sound, since two essential premises in the argument have indeterminate contents and Wiredu gives inadequate reasons for those premises.

Wiredu’s argument turns on the two special modal systems he constructs as extensions of S1 and S2, which I call W1 and W2, and on his special “strong rule of substitution” (Wiredu 1979, 691), which I call WR. Here is Wiredu’s primary argument as I see it.

Wiredu’s primary argument

- (P1) Lewis had justification to accept S2. (He was “cautious of any system stronger than S2”.)
- (P2) Lewis had justification to accept WR. (That rule is “very plausible and intuitively acceptable”.)

- (P3) S4 is deductively contained in W1 and in W2. (S4 “becomes available”.)
- (P4) Justification is relevantly transmitted by deduction. (Implicit.)
- (C5) So, Lewis had justification to accept S4. (He “could have been more adventurous modally [for] he was committed to S4”.)

The circumstantial *ad hominem* nature of Wiredu’s argument is clear from the way its premises and conclusion are centered on Lewis’s personal pragmatic (or, if readers prefer, epistemic) circumstances. The conclusion, for example, is not ‘the truth about logical necessity includes S4’ but ‘Lewis had justification to accept that the truth about logical necessity includes S4’. For charity to Wiredu, ‘had justification’ here means ‘had some prima facie grounds’, not ‘had overriding or all-in grounds’. Since Wiredu does not discuss Lewis’s objections to systems stronger than S2, my reading of the argument leaves it as a further case to be made that the grounds provided by WR to Lewis to accept S4 would have been sufficiently strong to trump those objections.

Premises P1 and P4 can be granted for present purposes. P1 reflects Wiredu’s decision to take Lewis’s antecedent commitments as given. P4 ensures that the argument’s conclusion in this case validly follows from the first three premises jointly, but does not imply that justification in general is deductively closed.

Premises P2 and P3, on the other hand, cannot simply be granted. On the one hand, since S4 is not deductively contained in S2, Wiredu’s conclusion C5 does not follow from P1 and P4 alone, and P2 and P3 are essential to his argument. On the other hand, it is not clear what P2 and P3 amount to, or why they should be thought to be true.

P2’s content is indeterminate since WR is indeterminate. Adopting Lewis’s notation of = for strict equivalence (mutual strict implication) (Wiredu 1979, 689), Wiredu states his “strong rule of substitution” as follows.

- (WR) “If $(p = q)$, then $[A(p) = A(q/p)]$ (Here... $A(q/p)$ is like $A(p)$ except for containing q in place of p in all or some of the occurrences of p in A)” (Wiredu 1979, 691).

The difficulty is how to understand this. Wiredu says he intends WR as a “strengthening of the rule for the substitution of strict equivalents”, that is, “the rule that is standardly used in the formulation of S1 and of the Lewis systems, generally” (Wiredu 1979, 692, 691). But he does not say exactly

what strengthening measure he has in mind from among indefinitely many conceivable sorts. In another paper he acknowledges the convention of suppressing turnstiles in statements of inference rules (Wiredu 1973, 39). But if WR were read in light of that convention, then WR would turn out just to be the standard rule for substitution of proved strict equivalents (Eq'), already available in S1 and the Lewis systems generally, which Wiredu insists is distinct from his rule.

(Eq') If $\vdash (\gamma = \delta)$, then $\vdash (\alpha = \beta)$, provided that α differs from β only in having γ in some of the places where β has δ . (Hughes and Cresswell 1996 [hereafter NIML], 199–200; Hughes and Cresswell 1968 [hereafter IML], 246–247; Wiredu 1979, 690).

Wiredu invokes WR in his proofs only once, to license a single step in his ostensible derivation of “the characteristic thesis of S4” (4.1) in W1 and W2 (Wiredu 1979, 690).

(4.1) $\Box p \rightarrow \Box \Box p$

Wiredu evidently intends some sort of weakening or other of the restrictions on Eq', sufficient for WR to license that one proof-step. More than that is not easy to say.

WR's indeterminateness also infects the reasons Wiredu offers for P2. On the one hand, Wiredu's assertion that WR is “very plausible and intuitively acceptable” is empty if he actually does not have any determinate rule in mind. On the other hand, though the “very informal” subsidiary argument Wiredu offers for P2 (Wiredu 1979, 691–692) is too terse and opaque for me to confidently reconstruct or remark upon, that subsidiary argument does seem to involve three elements available in S4 but not available to Wiredu on pain of circularity of justification. First, proved material equivalents seem to be taken as everywhere substitutable, though that rule (Eq) is not available in S1, S2, or S3 (IML, 228).

(Eq) If $\vdash (\gamma \equiv \delta)$, then $\vdash (\alpha \equiv \beta)$, provided that α differs from β only in having γ in some of the places where β has δ . (NIML, 32; IML, 35)

Second, necessity seems to be taken as everywhere inferable from provability, though that rule (N) is not available in S1, S2, or S3 (IML, 235).

(N) If $\vdash \alpha$, then $\vdash \Box\alpha$. (NIML, 361; IML, 346)

Third, strict equivalence seems to be taken as substitutably equivalent to necessary material equivalence, though that thesis (2.1) is not available in S1 (Feys 1965, 45, 62, 72).

(2.1) $(p = q) = \Box(p \equiv q)$

If the subsidiary argument does involve elements available in S4 but not in S1 or S2, then, regardless of however WR might be rendered determinate, it is difficult to see how the subsidiary argument can render WR “very plausible and intuitively acceptable” in a way or to a degree that 4.1 was not already plausible and acceptable to Lewis.

P3’s content is also infected by WR’s indeterminateness, since W1 and W2 are defined in terms of WR along with specific definitions of strict equivalence (D=) and possibility (D \diamond) (Wiredu 1979, 689).

(D=) $p = q =_{df} [(p \rightarrow q) \& (q \rightarrow p)]$

(D \diamond) $\diamond p =_{df} \sim [(p = (r \& \sim r))]$

(W1) W1 is S1 with D= and extended with WR as an additional primitive inference rule and D \diamond as an additional definition, and with the extended system closed also under S1’s primitive inference rules.

(W2) W2 is S2 with D= and extended with WR as an additional primitive inference rule, and with the extended system closed also under S2’s primitive inference rules.

(W2’s definition omits D \diamond because S2 has equivalent thesis 2.2. See Wiredu 1979, 692; Lewis and Langford 1959, 506.)

(2.2) $\sim \diamond p = [(p = (r \& \sim r))]$

So, W1 and W2 are indeterminate because WR is indeterminate. (D= and D \diamond are also problematic for W1 and W2, since Lewis’s bases for S1 and S2 seem to have both strict equivalence and possibility as “primitive or undefined

ideas”. See Lewis and Langford 1959, 123; but compare IML, 217. No ‘idea’ can be at once both undefined and defined in any basis for a system. See Parry 1968, 129, note 51. But Wiredu does not specify non-Lewis bases for S1 and S2 in W1 and W2.)

Besides the indeterminateness of P3’s content, Wiredu gives faulty proofs for his claim that extending either S1 to W1 or S2 to W2 makes 4.1 provable and thus yields at least S4. Wiredu repeatedly but mistakenly says that, except for WR and $D\Diamond$ (or 2.2), his proofs use “only principles available in S1” (Wiredu 1979, 691; see also 690). The “technical error” reported by Makinson is that Wiredu twice relies on the rule of conditional proof for strict implication, which is not a valid rule in S1, S2, or S3. If that rule were valid in any of those systems, then replacement of \supset with \rightarrow as the main operator in a thesis would always yield another thesis. But each of those systems has some “theses in which the main operator is \supset but which cease to be theses if the \supset is replaced by \rightarrow ” (IML, 228). Another error, not reported by Makinson, is that Wiredu also relies on a “simple lemma” (L0) that he mistakenly claims is valid in S1 (hence in S2 and S3) (Wiredu 1979, 690).

$$(L0) \quad \sim (p = p) = \perp$$

L0 fails, for example, left-to-right in the S3 (hence S2 and S1) model where $W = \{w_1, w_2\}$, $N = \{w_1\}$, $Q = \{w_2\}$, $R = \{\langle w_1, w_1 \rangle, \langle w_1, w_2 \rangle\}$, and $V(p, w_1) = V(p, w_2) = 1$. (Here $=$ is identity. Kripke models for non-normal systems distinguish between normal and non-normal worlds. See IML, 274–276; NIML, 201–202. Here N comprises the normal worlds in W , while Q comprises the non-normal worlds in W .) All these faults in Wiredu’s proofs are independent of WR’s indeterminateness and independent of WR’s role in the proofs. So, Wiredu’s proofs fail on account of these faults regardless of however WR (hence also W1 and W2) might be rendered determinate and regardless of whether the single proof-step that Wiredu intends WR to license is a valid proof-step or not. Wiredu’s faulty proofs thus do not provide any reason to accept that S4 is deductively contained in W1 and in W2.

In sum, Wiredu’s primary argument cannot be accepted as sound, since P2 and P3, while essential to the argument, have indeterminate contents and Wiredu gives inadequate reasons for those premises.

3 Initial variant argument

In this section I amend Wiredu's primary argument in a number of specific ways that jointly change the argument's view of Lewis's relevant circumstances (pragmatic or epistemic, as readers prefer). The resulting initial variant of Wiredu's argument can be accepted as sound.

First, I replace the weak non-normal systems S1 and S2 in P1, P2, and P3 with the only slightly stronger normal system T (not mentioned by Wiredu) that contains both S1 and S2 and like them is contained in S4. This amendment reflects Cresswell's suggestion that, rather than S1 or S2, "the system most closely satisfying Lewis's intuitions would be T" (Cresswell 1967, 204). The suggestion in other words is that, Lewis had grounds to accept T even though he did not accept S1 or S2. Now, T lacks 4.1 but has 2.1, 2.2, Eq, Eq', N, L0, conditional proof for strict implication, and 1.1.

$$(1.1) \quad (p = q) = [(p \supset q) \& (q \supset p)]$$

So, T is weaker than S4 but shares crucial features of S2 with Wiredu's W1 and W2, and also has non-S2 principles whose intuitive attractiveness for Wiredu seems likely responsible for his failure to see the formal faults in his own proofs and arguments.

Second, I replace Wiredu's indeterminate rule WR in P2 and P3 with a determinate rule that strengthens Eq' in a precise way to permit substitution, not only of proved strict equivalents (whose substitution Eq' permits), but also of strict equivalents that are logically implied by (deducible from) assumptions or premises that are not themselves provable. In connection with Wiredu's paper, Humberstone uses a rule he calls ($\Box \leftrightarrow E$), but which for present purposes I rewrite as MSR.

(MSR) If $\vdash (\chi \supset \Box(\gamma \equiv \delta))$, then $\vdash (\chi \supset (\alpha \supset \beta))$, provided that α differs from β only in having γ in some of the places where β has δ . (See Humberstone 2011, 604; compare Cresswell 1965, 192, Axiom Schema I and Def =. I follow Wiredu in using \supset for material implication, \equiv for material equivalence, and no symbol for propositional identity, while Humberstone uses \rightarrow for material implication, \leftrightarrow for material equivalence, and \equiv for propositional identity.)

But, at least in his primary argument, Wiredu frames WR in terms of strict equivalence (mutual strict implication), not in terms of necessary material equivalence. In another context and without mentioning Wiredu, Steffen Lewitzka strengthens Eq' to a rule he calls SR, which for present purposes I rewrite as follows.

- (SR) If $\vdash (\chi \supset (\gamma = \delta))$, then $\vdash (\chi \supset (\alpha = \beta))$, provided that α differs from β only in having γ in some of the places where β has δ . (See Lewitzka 2016, 1771. Lewitzka uses \rightarrow for material implication, \leftrightarrow for material equivalence, and \equiv for both strict equivalence and propositional identity.)

But Wiredu follows Lewis in identifying logical implication (deducibility) with strict implication, not material implication (even if proved). For sake of least divergence from Wiredu's text, I prefer to amend his primary argument by replacing WR, not with MSR or SR, but with the rule I call \Box SR.

- (\Box SR) If $\vdash (\chi \supset (\gamma = \delta))$, then $\vdash (\chi \supset (\alpha = \beta))$, provided that α differs from β only in having γ in some of the places where β has δ . (See Martens 2019. Though Lewitzka 2016, 1774, describes an equivalent axiom schema, which he calls \Box SP, he does not describe \Box SR.)

\Box SR permits substitution, not only of proved strict equivalents, but, more generally, of strict equivalents that are formally deducible by Wiredu's lights. Replacing WR with \Box SR in Wiredu's proof of 4.1 renders the amended proof valid in T. The amended proof thus establishes that S4 is deductively contained in T+ \Box SR. (S4 has \Box SR, but T lacks \Box SR. See Martens 2019.)

Third, I add to P1 the assumption that Lewis had some prima facie grounds to accept the problematic view that strict equivalence is propositional identity, or, in other words, that to say ' p and q are strictly equivalent' is to say ' p and q are the same proposition'. (For favorable treatments of the view, see Stalnaker 1984 and Cresswell 1965; for criticisms, see Prior 1963 and Cresswell 1988.) I think Lewis's exact position on this view is not easy to pin down and there is no space here for full discussion of the interpretive issues. What is important for present purposes is that Lewis arguably took the view

seriously and often wrote in way that invites attribution of the view to him. For example, he repeatedly said of various statements of strict equivalence that they assert “identity” of propositions (Lewis and Langford 1959, 128, 129). As for Wiredu, though he does not mention propositional identity, I am tempted to conjecture that his engagement with Lewis has encouraged him to tacitly and unreflectively see Lewis as identifying strict equivalence with propositional identity, and to tacitly and unreflectively make that identification himself. (See IML, 347, on “the chance of confusion” when = is used for strict equivalence.)

Fourth, I further amend P2 to reflect the fact that, since \Box SR is an unqualified form or analogue of Leibniz’s Law (‘identicals are indiscernible’) for propositional identity if strict equivalence is propositional identity, the intuitive plausibility of that Law provides anyone who has grounds to accept the latter view with grounds also to accept \Box SR. (Prior 1963, §1, discusses propositional identity in relation to Leibniz’s Law and substitution of equivalent propositions. Cresswell 1965, 195, constrains propositional identity with an unqualified form or analogue of Leibniz’s Law that he calls “the identity schema”. Humberstone 2011, 604, 603, calls $(\Box \leftrightarrow E)$ “the analogous ‘strict equivalence’ version of $(\equiv E)$ ”, where the latter is an unqualified form or analogue of Leibniz’s Law for propositional identity. Lewitzka 2016, 1771, associates his strengthened substitution rule SR with “a general ontological law known as *the Indiscernibility of Identicals*”.) However, since T lacks \Box SR despite having Eq, N, and 2.1, the “very informal” subsidiary argument Wiredu offers to justify WR cannot be rehabilitated to provide additional grounds for \Box SR beyond the latter’s intuitive plausibility as an unqualified form or analogue of Leibniz’s Law.

Fifth, I amend the argument’s form to make it conditional, since P1 as thus far amended raises interpretive issues for which there is no space to discuss fully here. For example, though Lewis “does not mention T” (Cresswell 1967, 204, note 18), the fact that he did not accept T but stood fast at S2 “to the last” (as Wiredu says), suggests that either he thought he did not have justification to accept T (contrary to Cresswell’s suggestion), or he thought he had stronger justification not to accept T. Moreover, though above I cited some places where Lewis seems to have accepted that strict equivalence is propositional identity, other places can be cited where he seems to have rejected that view (for example, Lewis and Langford 1959, 285, note 2). To set aside interpretive issues raised by P1 but without amending that

argument-element's content, I change its function from asserted premise to assumption for conditional proof and accordingly change its designation to A1. I discharge the dependence on A1 of now-intermediate conclusion C5 by conditional proof to a new final conclusion C6.

Here now is my initial variant of Wiredu's primary argument.

Initial variant argument

- (A1) Lewis had justification to accept T and he had justification to accept that strict equivalence is propositional identity. (Assumption for conditional proof.)
- (P2) If Lewis had justification to accept that strict equivalence is propositional identity, then he had justification to accept \Box SR. (\Box SR is an unqualified form or analogue of Leibniz's Law for propositional identity if strict equivalence is propositional identity.)
- (P3) S4 is deductively contained in T+ \Box SR. (By amendment of Wiredu's 'primary argument' proof.)
- (P4) Justification is relevantly transmitted by deduction. (Granted to Wiredu.)
- (C5) So, Lewis had justification to accept S4. (From A1, P2, and P3, by P4.)
- (C6) So, if Lewis had justification to accept T and he had justification to accept that strict equivalence is propositional identity, then he had justification to accept S4. (From A1 and C5, by conditional proof, but retaining C5's dependence on P2, P3, and P4.)

The variant argument retains the circumstantial *ad hominem* nature of Wiredu's argument, for the premises and conclusions remain centered on Lewis's personal pragmatic (or epistemic) circumstances. Also as before, 'had justification' here still means 'had some prima facie grounds', not 'had overriding or all-in grounds'. Since the variant argument still does not address Lewis's objections to systems stronger than S2, a further case still remains to be made that the grounds provided, by \Box SR and by the view that strict equivalence is propositional identity, to Lewis to accept S4 would have had sufficient collective strength to trump those objections. The variant argument thus leaves open the possibility that Lewis could justifiably have resisted accepting S4, for example, by justifiably rejecting the view that strict equivalence is propositional identity or by justifiably accepting Leibniz's Law for propo-

sitional identity only in a qualified form. (Systems for propositional identity with necessity weaker than S4 are investigated, for example, by Ishii 1998; 2000; Martens 2004; and Lewitzka 2015; 2016.)

Since the initial variant argument is valid and its asserted premises (P2, P3, P4) all can reasonably be accepted as true, the initial variant argument can reasonably be accepted as sound.

4 Final variant argument

In this section I further amend Wiredu's argument and further address relations among Wiredu's primary and alternative arguments, the final variant argument I construct, Makinson's and Humberstone's notes, and Cresswell's argument.

In section 2 above I reconstructed Wiredu's primary argument, confirmed Makinson's report of "a technical error" in it, and showed that it cannot be accepted as sound. In section 3, reluctant to accept Makinson's assessment that its problematic character "vitiates" Wiredu's primary argument, I noted Cresswell's suggestion that Lewis's explicit expressions might not always accurately reflect his motivating intuitions, applied that suggestion with similar charity to Wiredu's expression of his primary argument, and amended the argument to yield an initial variant argument that can be accepted as sound. I now show how Wiredu's primary and alternative arguments can be understood as providing two formally distinct lines of support for a key premise in a single overall argument for Wiredu's main conclusion.

The alternative argument is aimed at showing that "the addition of [a certain] definition of necessity [...] to a suitable system of propositional logic yields at least S4" (Wiredu 1979, 693). On Makinson's report, the same technical error that "vitiates" the primary argument also "vitiates" the alternative argument. Makinson's reason for saying this clearly is Wiredu's statement in justification of a crucial step in the alternative argument that "The proof of this is essentially the same as the one already given for [4.1 in the primary argument]" (Wiredu 1979, 693), that is, the one with a technical error. But Makinson clearly has been hasty, for Wiredu immediately completes his sentence with "except that here the conclusion is a material implication instead of a strict implication". Since the conclusion here is by conditional proof for

material implication, not for strict implication, one instance of the technical error, at least, has not carried over from the primary argument to the alternative argument. On the other hand, Makinson also reports “some confusion” in the alternative argument and it is not immediately clear how that charge might be avoided. Wiredu does not say anything to suggest that the proof carried over from the primary argument to the alternative argument is not still intended to rely on L0 and WR. But he does say that in the alternative argument = is no longer strict equivalence (mutual strict implication). He says, “In the [alternative argument] we understand $p = q$ as an abbreviation for ‘ $p \equiv q$ is logically true (i.e., tautological)’” (Wiredu 1979, 693). On the one hand, it is not immediately clear how nested occurrences of =, as in L0, should now be understood. On the other hand, the shift in WR’s content between the primary argument and the secondary argument aggravates suspicion that WR is indeterminate.

Faced with my ambivalent assessment of Makinson’s report on Wiredu’s alternative argument, I defer to Humberstone. It seems to me that Humberstone’s attention is focused on formal aspects of the alternative argument, which he takes to be in relevant respects essentially the same as Cresswell’s 1965 argument, and which he thinks can be understood in such a way as to be sound and free from any insuperable technical error or confusion of the sort Makinson reports. I take Humberstone’s report to be that, with suitable amendments in either case, Wiredu’s alternative argument and Cresswell’s argument both establish that S4 results when the smallest normal system K (not mentioned by Wiredu) is extended with MSR.

Here now is my final variant of Wiredu’s overall argument, which incorporates his primary and alternative arguments at P3a and P3b below.

Final variant argument

- (A1) Lewis had justification to accept T and he had justification to accept that strict equivalence is propositional identity. (Assumption for conditional proof.)

- (P2) If Lewis had justification to accept T and he had justification to accept that strict equivalence is propositional identity, then he had justification to accept K and he had justification to accept $\Box SR$ and he had justification to accept MSR . (K is deductively contained in T , and $\Box SR$ is an unqualified form or analogue of Leibniz's Law for propositional identity if strict equivalence is propositional identity, as is MSR , given that T has 2.1 and Eq'.)
- (P3a) $S4$ is deductively contained in $T + \Box SR$. (By amendment of Wiredu's 'primary argument' proof.)
- (P3b) $S4$ is deductively contained in $K + MSR$. (By amendment of Wiredu's 'alternative argument' proof or that of Cresswell 1965. See Humberstone 2011, 604.)
- (P4) Justification is relevantly transmitted by deduction. (Granted to Wiredu.)
- (C5) So, Lewis had justification to accept $S4$. (By P4, from A1, P2, and either P3a or P3b.)
- (C6) So, if Lewis had justification to accept T and he had justification to accept that strict equivalence is propositional identity, then he had justification to accept $S4$. (From A1 and C5, by conditional proof, but retaining C5's dependence on P2, P4, and either P3a or P3b.)

The final variant argument retains the circumstantial *ad hominem* nature of Wiredu's argument and 'had justification' here still means 'had some prima facie grounds', not 'had overriding or all-in grounds'. Since the final variant argument is valid and its asserted premises (P2, P3a, P3b, P4) all can reasonably be accepted as true, the final variant argument can reasonably be accepted as sound.

The final variant argument is not exactly what Wiredu says but it is what he means, I think, in the sense that it accurately expresses the interesting philosophical intuitions motivating him. The argument also serves as a model of the Lewisian pragmatic approach to the question of choice of modal logic. On that approach, the question is never settled by formal methods but only by explicit recourse to specific pragmatic factors rooted in specific circumstances. There is some irony in Wiredu's teasing of Lewis for being "cautious" rather than "adventurous modally", when Wiredu himself is so philosophically careful and circumspect in his respectful treatment of Lewis's pragmatic circumstances.

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