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ANALYSIS OF VARIANCE OF ORE MINERALIZATION COMPONENTS OF THE ROZÁLIA VEIN AT THE HODRUŠA MINE IN CLASSIFICATION ACCORDING TO TWO FACTORS

(Figs. 5, Tabs. 6)

Abstract: In the contribution the influence of depth and horizontal factor on the components of ore mineralization and thickness of the Rozália vein at the Hodruša mine is examined. Derived is the mathematical model of the analysis of variance in classification according to two factors, which was used for solution of the task. The mathematical model takes into account the horizontal and vertical distribution of mining blocks, the average contents and thickness of which were the fundamentals of the analysis of variance. Besides them, the mathematical model makes possible to evaluate the influence of the effects of the main factors, as well as their interaction on the resulting sign and provides much more information than the simple model of the analysis of variance in classification according to one factor. The prerequisite of its aplication is, however, the requirement of full numbers for combinations of the levels of both factors, in which some of them must have the numebr of observations greater than one. In the analysis of variance of the components of ore mineralization it was proved at the significance level 10 %, that the depth factor takes part to a considerable extent in Pb, Zn contets and thickness of the vein. The factor of horizontal distribution in the ore dykes is important for Pb and Zn contents. The interaction has turned out in all cases as statistically insignificant.

Резюме: В статье исследуется влияние глубинного и горизонтального факторов на компоненты оруденения и мощность жилы Розалия на шахте Годруша. Выведена математическая модель дисперсионного анализа при классификации по двум факторам, использованная при решении задачи. Математическая модель учитывает горизонтальное и вертикальное распределение блоков добычи, средние содержания и мощности которых являлись основой для анализа изменения. Кроме того, математическая модель позволяет оценить влияние эффектов основных факторов и их взаимодействие на результативный знак и предоставляет гораздо больше информаций чем простая модель дисперсионного анализа при классификации по одному фактору. Но условием ее применения требование полных чисел для комбинаций уровней обоих факторов, причем некоторые из них должны иметь число наблюдений больше чем один. При дисперсионном анализе компонентов оруденения на уровне значительности 10 % было доказано, что глубинный фактор в значительной степени участвует в содержаниях Pb и Zn и мощности жилы. Фактор горизонтального распределения в рудных жилах важным для содержаний Pb и Zn. Взаимодействие показалось во всех случаях статистически неважным.

Brief geological characterization of the Rozália vein

The Rozália vein is developed at the Western margin of the Štiavnica part of the Štiavnica-Hodruša ore district. At the surface it is lying roughly at the

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boundary of rocks of the Hodruša-Vyhňe Island and the volcanics. The covering formations are mainly pyroclastic effusions of the 2nd and 3rd andesite phases of relatively small thicknesses, as a consequence of emergence and the following denudation of this deposit area.

The most important rock type of the deposit area is amphibole-biotite granodiorite, which occurs here as an elevation of a greater intrusive body in NE-SW direction. The granodiorite forms mainly the substratum of the Rozália vein. In the overburden its contact can be followed at the level of the 3rd and 8th horizon and toward depth the Rozália vein passes into the intrusive body proper with less important older sedimentary and metamorphic rocks.

The Rozália vein, apart from the southern margin, belongs to the so called copper zone (Koděra et al., 1978), which forms a distinct elevation here. Six supplying periods took part in the mineralization and they belong to veins mineralization of Štiavnica type. In this case the latest investigation (Koděra et al., 1978) showed that they belong mainly to the younger periods – 4. to 6. – which were affirmed.

The 4th younger ore-bearing period has a quartz-chalcopyrite development, which is typical for the copper zone and is the main bearer of mineralization. Besides the main ore minerals (chalcopyrite galena and brown sphalerite), bornite, chalcosine and unidentified Bi sulpho-salts were found in deeper parts in accessory amount.

The 5th period is formed by quartz and haematite with subordinate amount of ore minerals.

The 6th period is the youngest and formed by fine-grained siderite with small amount of barite, quartz and ore minerals, which fade out with depth. The main part of the vein is without ore minerals.

The Rozália vein is in the area of a horst structure lying on a normal fault with 110 m amplitude, on a separately uplifted block with a distinct elevational effect of the granodiorite intrusion. It is situated in the middle of a 11 km wide zone system of parallel veins of Carpathian direction. The general trend of the vein is NNE-SSW 205°, with variations from 170° to 240°, dipping at 45°. In greater depth it steepens to 55°–60° to the East. The ore mineralization at the vein is concentrated into ore dykes reaching to great depth, with ends not attested so far. A common mark of all ore dykes is their continuation in SSW-SW direction in the longitudinal section of the vein – Fig. 1 and forming a common veil.

Mathematical model of analysis of variance in sorting according to two factors

The Rozália vein is made accessible by mining operations to the level of the 14th horizon. At present a quick solution to the problem of its continuation to greater depth is necessary. Therefore we wanted to test the influence of the depth factor and factor of ore dykes on the components of ore mineralization on the contents of Cu, Pb, Zn and the thickness of vein. For the basis of this research we employed the following data: the average contents of the mentioned components and average thicknessed of individual mining blocks horizontally distributed in ore dykes I.—V. and vertically from the Upper Rozália adit to the XIVth horizon — Fig. 1. The list of blocks with avkerage analyses, included in the analysis ofvariance is in Tab. 1.

We used variance analysis which by splitting the total variance of the examined statistical sing into components belonging to the individual factors, their interaction with the resulting statistical, sign, which on our case was the content of Cu, Pb, Zn and the thickness of the vein.

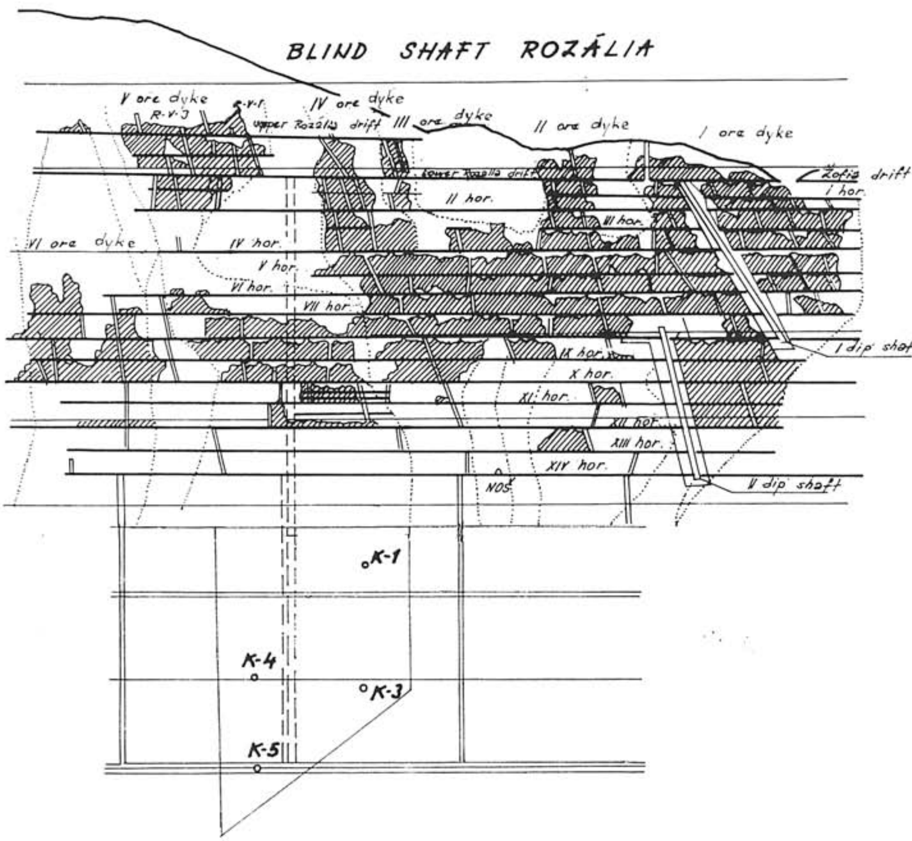

Fig. 1. Longitudinal section of the Rozália vein (according to Ing. Gavura, GP Nová Baňa).

As we intended to consider the influence of two factors — depth and the presence of ore chimneys — on the components of ore mineralization and on the vein thickness, we chose a two-factor mathematical model with several repetitions. We set out from the fact, that we have N observations of y_{ijk} ($i = 1, 2 \ldots, p$); ($j = 1, 2, \ldots, q$); ($k = 1, 2, \ldots, n_{ij}$) which represent the values of the resulting sign — the Cu, Pb, Zn contents and the thickness of mining blocks, which we obtained for the individual combinations of the levels

Table 1
List of blocks with average analyses

Horizon	Number of block	Ore dyke	Thickness	Average content		
				Cu	Pb	Zn
1	2	3	4	5	6	7
VI	R II – 6	II	0.94	0.98		
	R II – 6A	II	1.09	1.06		
	R II – 6B	II	1.51	1.07		
	R III – 6	III	1.84	0.78		
	R III – 6B	III	0.80	1.06		
	R IV – 6	IV	1.18	1.11	0.47	0.46
	R IV – 6B	IV	2.26	0.51	0.39	0.32
	R V – 34	V	1.09	0.59	2.64	4.84
	R V – 37	V	0.85	0.51	1.18	0.26
VII	R II – 7	II	1.07	1.27		
	R II – 7A	II	0.71	0.44	0.10	0.31
	R II – 8	II	1.11	1.13		
	R III – 7	III	1.35	0.68		
	R III – 7B	III	1.38	0.71		
	R IV – 7	IV	1.23	1.27	0.32	0.45
	R IV – 7B	IV	0.82	0.50	0.05	0.10
	R V – 35	V	1.09	0.50	1.06	1.94
	R V – 23C	V	1.30	0.67	0.24	0.40
	R V – 31C	V	0.86	0.80	0.49	0.86
	R V – 38	V	0.94	0.58	1.07	0.34
VIII	R II – 7B	II	0.54	0.69	0.17	0.19
	R II – 9	II	1.13	1.36		
	R III – 8	III	0.87	0.83		
	R IV – 8	IV	1.21	1.31	0.13	0.15
	R V – 21	V	0.80	0.46	0.07	0.12
	R V – 22	V	1.39	1.49	0.79	1.83
	R V – 23	V	1.50	2.92	1.36	2.85
	R V – 31	V	0.95	1.58	0.21	0.39
	R V – 53	V	1.39	0.64	0.17	0.30
IX	R II – 10	II	1.15	0.89	0.05	0.06
	R III – 9	III	0.74	0.70		
	R IV – 9	IV	1.07	2.12	0.06	0.06
	R V – 24A	V	0.80	0.46	0.07	0.12
	R V – 25	V	1.03	0.94	0.74	1.85
	R V – 26	V	1.41	1.29	0.87	1.90
	R V – 27	V	1.69	3.52	1.50	2.69
	R V – 32	V	0.95	1.58	0.21	0.39
	R V – 54	V	1.11	0.81	0.51	0.42
X	R II – 10B	II	1.27	0.69	0.09	0.20
	R III – 10	III	0.34	0.81	0.09	0.03
	R IV – 10	IV	1.28	0.51	0.08	0.10
	R V – 24B	V	1.77	0.52	0.26	0.16
	R V – 28	V	1.39	1.37	0.54	1.18
	R V – 29	V	1.53	1.92	0.45	0.47
	R V – 30	V	1.12	2.11	0.28	0.51
	R V – 51	V	0.67	0.38	0.02	0.05

Continuation of Tab. 1

1	2	3	4	5	6	7
XI	R II – 15	II	5.39	0.68		0.02
	R II – 16	II	4.37	1.71	0.02	0.03
	R III – 11	III	0.88	0.94	0.04	0.04
	R IV – 11	IV	0.45	2.30	0.07	0.18
	R V – 40	V	6.02	0.85	0.15	0.17
	R V – 41	V	8.16	1.05	0.13	0.21
	R V – 42	V	1.98	1.76	0.12	0.19
	R V – 42N	V	0.84	2.20	0.87	1.32
	R V – 52	V	1.35	0.76	0.09	0.08
	R V – 52N	V	0.56	0.66	0.06	0.05
XII	R II – 17	II	4.37	1.75	0.02	0.03
	R II – 11	II	3.25	1.42	0.01	0.03
	R III – 12	III	1.02	0.75	0.03	0.04
	R IV – 12	IV	0.78	0.87	0.11	0.06
	R V – 43	V	6.23	1.10	0.09	0.10
	R V – 44	V	7.25	1.01	0.02	0.06
	R V – 45	V	9.28	0.94	0.04	0.06
	R V – 56	V	1.25	0.77	0.04	0.10
XIII	R II – 12	II	0.80	1.68		
	R III – 13	III	0.56	0.33	0.02	0.05
	R IV – 13	IV	8.24	0.61	0.04	0.02
	R V – 46	V	8.69	0.91	0.01	0.03
	R V – 47	V	9.85	0.67	0.04	0.03
	R V – 55	V	7.28	1.00	0.03	0.04
	R V – 57N	V	1.23	1.05	0.03	0.03
	R V – 57	V	1.21	0.77	0.05	0.08
XIV	R II – 13	II	0.83	0.98	0.02	0.05
	R IV – 14	IV	7.37	0.69	0.01	0.03
	R V – 49	V	7.02	0.84	0.02	0.02
	R V – 50	V	8.56	0.54	0.11	0.02
	R V – 58	V	5.20	0.75	0.04	0.03
	R V – 59N	V	2.51	1.09	0.05	0.03
	R V – 60	V	1.07	0.91	0.03	0.02

of two factors. The first factor – depth- was traced at p levels and the second factor – the presence of ore dykes at q levels where for each combination of factor levels we have k observations (of mining blocks). Then at the i-th level of the first factor and j-th level of the second factor each observation can be expressed by the following mathematical model:

$$y_{ijk} = \mu + \tau_i + \gamma_j + \lambda_{ij} + \varepsilon_{ijk} \qquad (1)$$

Thus we assume that each observation is composed of the total average μ, effect τ_i at the i-th level of the first factor, efect γ_j of the j-th level of the second factor, of systematic deviation λ_{ij} of each observation with combination of the i-th level of the first factor and j-th level of the second factor, from the sum of the first three members of the model (1) and of the random error ε_{ijk}. The λ_{ij} are the effects, which belong to certain combinations of the levels of both factors in case, when the effects of both factors interact.

For the sums of the effects we assume these conditions:

$$\sum_{i}^{p} \tau_i = \sum_{j}^{q} \gamma_j = \sum_{i}^{p} \lambda_{ij} = \sum_{j}^{q} \lambda_{ij} = 0 \tag{2}$$

The mentioned conditions are the hypotheses, the testing of which is the main aim of the analysis of variance.

In this model we thus suppose the possibility of interaction of both factors, i. e., that the difference between the actual effects of two levels of the first factor is different at all the levels of the second factor, i. e. both factors are inter dependent in their influence on the resulting sign.

The parameters μ, τ_i, γ_j, λ_{ij} of the mathematical model (1) we estimate by the least square method. When we call the estimates of the mentioned parametres m, t_i, g_j, l_{ij}, then we can express the basic condition of the least square method as follows:

$$S = \sum_{i}^{p} \sum_{j}^{q} \sum_{k}^{n_{ij}} (m + t_i + g_j + l_{ij} - y_{ijk})^2 = \min \tag{3}$$

what means, that the sum of squares of deviations of the theoretical, mathematical model (1) from the empiric values of the resulting sign must be equal to minimum. When the mentioned condition has to be fulfilled, then the partial derivatives of the S sum for the unknown m, t_i, g_j, and l_{ij} must be equal to zero. We calculate now these partial derivatives, so:

$$\frac{\delta S}{\delta m} = 2 \sum_{i}^{p} \sum_{j}^{q} \sum_{k}^{n_{ij}} (m + t_i + g_j + l_{ij} - y_{ijk}) = 0$$

or

$$\sum_{i}^{p} \sum_{j}^{q} n_{ij} \,.\, m + \sum_{i}^{p} n_{ij} \,.\, t_i + \sum_{j}^{q} n_{ij} \,.\, g_j + \sum_{i}^{p} \sum_{j}^{q} n_{ij} \,.\, l_{ij} = \sum_{i}^{p} \sum_{j}^{q} \sum_{k}^{n_{ij}} y_{ijk}$$

As far as at the individual combinations of the levels of both factors we assume different number of repetitions of n_{ij}, for the estimates of parametres of the model (1) according to (2) the following must be true:

$$\sum_{i}^{p} n_{ij} \,.\, t_i = 0; \qquad \sum_{j}^{q} n_{ij} \,.\, g_j = 0 \tag{4}$$

$$\sum_{i}^{p} n_{ij} \, . \, l_{ij} = \sum_{j}^{q} n_{ij} \, . \, l_{ij} = 0; \qquad \text{and} \sum_{i}^{p} \sum_{j}^{q} n_{ij} = N$$

then

$$m = \frac{\sum_{i}^{p} \sum_{j}^{q} \sum_{k}^{n_{ij}} y_{ijk}}{N} = y_{...} \tag{5}$$

what is the total average of all observations and we shall designate it $y_{...}$

$$\frac{\delta S}{\delta t_i} = 2 \sum_{j}^{q} \sum_{k}^{n_{ij}} (m + t_i + g_j + l_{ij} - y_{ijk}) = 0$$

or

$$\sum_{j}^{q} n_{ij} \, . \, m + \sum_{j}^{q} n_{ij} \, . \, t_i + \sum_{j}^{q} n_{ij} \, . \, g_j + \sum_{j}^{q} n_{ij} \, . \, l_{ij} = \sum_{j}^{q} \sum_{k}^{n_{ij}} y_{ijk}$$

As far as $\sum_{j}^{q} n_{ij}$ are the row frequencies, we designate them $n_{i..}$ and regarding to (4), we can write then:

$$t_i = \frac{\sum_{j}^{q} \sum_{k}^{n_{ij}} y_{ijk}}{n_{i..}} - m$$

The first summand in the last equation on the right side is the average value of the resulting sign at the i-th level of the first factor and we shall designate it $y_{i..}$.

Then, regarding to (5) we can write

$$t_i = y_{i..} - y_{...} \tag{6}$$

what means that the best estimate of the effect τ_i of the first factor at the i-th level is the difference between the row average and the total mean value of the resulting sign.

$$\frac{\delta S}{\delta g_j} = 2 \sum_{i}^{p} \sum_{k}^{n_{ij}} (m + t_i + g_j + l_{ij} - y_{ijk}) = 0$$

or

$$\sum_{i}^{p} n_{ij} \,.\, m + \sum_{i}^{p} n_{ij} \,.\, t_i + \sum_{i}^{p} n_{ij} \,.\, g_j + \sum_{i}^{p} n_{ij} \,.\, l_{ij} = \sum_{i}^{p} \sum_{k}^{n_{ij}} y_{ijk}$$

As far as $\sum_{i}^{p} n_{ij}$ are the column frequencies, we designate them as $n_{.j.}$ and regarding to (4), we can write that:

$$g_j = \frac{\sum_{i}^{p} \sum_{k}^{n_{ij}} y_{ijk}}{n_{.j.}} - m$$

The first summand on the right side of the last equation represents the average value of the resulting sign at j-th level of the second factor and we shall designate it as $y_{.j.}$. Then regarding to (5) we can write:

$$g_j = y_{.j.} - y_{...} \tag{7}$$

This means that the best estimate of the effect y_j of the second factor at the j-th level is the difference between the column average and the total mean value of the resulting sign.

$$\frac{\delta S}{\delta l_{ij}} = 2 \sum_{k}^{n_{ij}} (m + t_i + g_j + l_{ij} - y_{ijk}) = 0$$

or

$$n_{ij} \,.\, m + n_{ij} \,.\, t_i + n_{ij} \,.\, g_j + n_{ij} \,.\, l_{ij} = \sum_{k}^{n_{ij}} y_{ijk}$$

where n_{ij} are the frequencies at the i-th level of the first factor and at the j-th level of the second factor, or the number of data at the given combination of the levels of both factors and we shall designate it $n_{ij.}$.
Then

$$l_{ij} = \frac{\sum_{k}^{n_{ij}} y_{ijk}}{n_{ij.}} - t_i - g_j - m$$

The first summand on the right side represents the average value of the resulting sign at the i-th level of the first and j-th level of the second factor and we shall designate it as $y_{ij.}$. Then, regarding to (5) (6) and (7) we have

$$l_{ij} = y_{ij.} - y_{i..} - y_{.j.} + y_{...} \tag{8}$$

In the model (1) it still remains to estimate the random error ε_{ijk}, which we can express as the difference between the empiric values of the resulting sign and its mathematical expectation E $[y_{ijk}]$, which is equal the sum of the first four members of the model (1). Thus

$$\varepsilon_{ijk} = y_{ijk} - E\,[y_{ijk}] = y_{ijk} - \tau_i - \gamma_j - \lambda_{ij} - \mu$$

We do not know the value of the mathematic expectation E $[y_{ijk}]$ and so have to estimate it by the regressive value

$$\bar{y}_{ijk} = m + t_i + g_j + l_{ij}$$

which is the best estimate. Thus

$$y_{ijk} - \bar{y}_{ijk} = y_{ijk} - m - t_i - g_j - l_{ij}$$

or regarding to (5) (6) (7) and (8)

$$y_{ijk} - \bar{y}_{ijk} = y_{ijk} - y_{ij.} = e_{ijk} \tag{9}$$

is the best estimation of the random error ε_{ijk}. Then we can express observations y_{ijk} as follows:

$$y_{ijk} = m + t_i + g_j + l_{ij} + e_{ijk}$$

If we raise to the second power both sides and sum through all i, j, and k, and consider to (4), we have:

$$\sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y^2_{ijk} = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} m^2 + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} t^2_i + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} g^2_j + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} l^2_{ij} +$$

$$+ \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} + e^2_{ijk}$$

Regarding the validity of relations (5), (6), (7), (8) and (9) we can write the last equation as follows:

$$\sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y^2_{ijk} - \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y^2_{...} = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{i..} - y_{...})^2 + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{.j.} - y_{...})^2 +$$

$$+ \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ij.} - y_{i..} - y_{.j.} + y_{...})^2 + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ijk} - y_{ij.})^2$$

and after adjustment

$$\sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ijk} - y_{...})^2 = \sum_{i}^{p} n_{i..} (y_{i..} - y_{...})^2 + \sum_{j}^{q} n_{.j.} (y_{.j.} - y_{...})^2 +$$

$$+ \sum_{i,\,j}^{p,\,q} n_{ij.} (y_{ij.} - y_{i..} - y_{.j.} + y_{...})^2 + \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ijk} - y_{ij.})^2 \qquad (10)$$

For the simplification of the record we introduce a new designation of the sums:

$$\sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y_{ijk} = Y_{...};\ \sum_{i}^{p} y_{ijk} = Y_{i..};\ \sum_{j}^{q} y_{ijk} = Y_{.j.}$$

$$\sum_{i,\,j}^{p,\,q} y_{ijk} = Y_{ij.}$$

Then we can express the individual sums of squares in a shape, which is suitable for calculation.

$$S_0 = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ijk} - y_{...})^2 = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y^2_{ijk} - \frac{Y^2_{...}}{N} \qquad (11)$$

$$S_1 = \sum_{i}^{p} n_{i..} (y_{i..} - y_{...})^2 = \sum_{i}^{p} \frac{Y^2_{i..}}{n_{i..}} - \frac{Y^2_{...}}{N} \qquad (12)$$

$$S_2 = \sum_{j}^{q} n_{.j.} (y_{.j.} - y_{...})^2 = \sum_{j}^{q} \frac{Y^2_{.j.}}{n_{.j.}} - \frac{Y^2_{...}}{N} \qquad (13)$$

$$S_{12} = \sum_{i,\,j}^{p,\,q} n_{ij.} (y_{ij.} - y_{i..} - y_{.j.} + y_{...})^2 = \sum_{i,\,j}^{p,\,q} \frac{Y^2_{ij.}}{n_{ij.}} - \sum_{i}^{p} \frac{Y^2_{i..}}{n_{i..}} - \sum_{j}^{q} \frac{Y^2_{.j.}}{n_{.j.}} + \frac{Y^2_{...}}{N} \qquad (14)$$

$$S_r = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} (y_{ijk} - y_{ij.})^2 = \sum_{i,\,j,\,k}^{p,\,q,\,n_{ij}} y^2_{ijk} - \sum_{i,\,j}^{p,\,q} \frac{Y^2_{ij.}}{n_{ij.}} \qquad (15)$$

Equation (10) can be written as:

$$S_0 = S_1 + S_2 + S_{12} + S_r \qquad (16)$$

The meaning of these sums of squares is as follows:

S_0 – is the total sum of squares calculated from the difference of all N observations from the total average and has (N–1) degrees of freedom.
S_1 – is a component of the total sum of squares calculated from the differences of row averages from the total average and (p–1) degrees of freedom belong to it.
S_2 – is the component of the total sum of squares calculated from the differences of column averages from the total average and has (q–1) degrees of freedom.
S_{12} – is the component of the total sum of squares calculated from the differences of the sum of averages of all the levels at a given i and j and of the total average from the sum of row and column averages. It is a component, which participates on interaction and has thus (p–1), (q–1) degrees of freedom.
S_r – it is so called residual component of the total sum of squares, calculated from deviations of N observations from p.q. averages of all the levels and has (N – pq) degrees of freedom.

Average square is

$$s^2_r = \frac{S_r}{N - p \cdot q} \tag{17}$$

It is an independent estimation of the variance σ^2 of the random error ε_{ijk} and therefore we use it for testing of the importance of the individual components of the total variance i.e. for testing of the given hypotheses (2). The testing of the hypotheses (2) will be performed by Fisher's test

$$F = \frac{s^2_i}{s^2_r} \tag{18}$$

where for s^2_i we use the average squares

$$s^2_1 = \frac{S_1}{p - 1} \quad ; s^2_2 = \frac{S_2}{q - 1} \quad ; s^2_{12} = \frac{S_{12}}{(p - 1) \cdot (q - 1)} \tag{19}$$

so that $F > 1$.

The conclusion of testing the hypotheses (2) will be done by looking for the critical F value from the tables of Fisher distribution at the chosen level of significance and the given degrees of freedom and by its comparison with the calculated F value according to (18).
If the critical value

$$F_{\alpha}, \upsilon_1, \upsilon_r > F \tag{20}$$

we accept the zero hypothesis (2) and state that the factor in its influence on dispersion of the resulting sign is statistically insignificant. In the opposite case we refuse the zero hypothesis, i.e. the factor significantly influences the variance of the values of the resulting sign.

An important condition of application of the given mathematical model is the requirement of completeness of the number of combinations of the levels of both factors. The number of repetitions (replicas) must be in some combinations of factors greater than one.

Method of calculation and interpretation of results

We used the derived mathematical model of variance analysis in considering the influence of the depth factor and factor of horizontal distribution of dykes to all ore mineralization components of the Rozália vein (Cu, Pb, Zn contents) and to its thickness. The average contents and thicknesses of mining blocks from Tab. 1. were used for calculation so, that the requirement of complete numbers of combinations of both factors would be fulfilled. Therefore in the variance analysis of Cu content and thickness, 72 mining blocks, in the variance analysis of Pb 51, and of Zn content 52 mining blocks were used. The values of element contents and thicknesses from Tab. 1 were sorted into tables, corresponding to the real position of the mining blocks in the vein. Each partial field of this table corresponds to a given combination of the levels of both factors. In these fields the values of the given resulting sign are plotted on the left and the sum of all values of the given field on the right part. In the two marginal columns and lines the row (column) frequencies $n_{i..}$ ($n_{.j.}$) and the sums $y_{i..}$ ($y_{.j.}$) are given. These tables are the basis for the calculation of the sum total of squares S_0 and their components S_1, S_2, S_{12} and S_r. The analysis of variance of Cu, Pb, Zn contents and the thickness of the Rozália vein is given in Tabs. 2, 3, 4 and 5.

Below is the calculation of the analysis of variance of Cu content. From Tab. 2 we calculate first the sums

$$\sum_{i,j,k} y_{ijk} = Y_{...} = 76.66;\ Y^2 = 5876.7556;\ N = 72$$

$$\sum_{i,j,k} y^2_{ijk} = 106.8620;\ \frac{Y^2_{...}}{N} = 81.6216;\ p = 8;\ q = 4$$

$$\sum_{i}^{p} \frac{Y^2_{i..}}{n_{i..}} = \frac{7.67^2}{9} + \frac{8.55^2}{11} + \ldots + \frac{7.02^2}{8} = 84.88255$$

$$\sum_{j}^{q} \frac{Y^2_{.j.}}{n_{.j.}} = \frac{16.82^2}{15} + \frac{7.59^2}{10} + \ldots + \frac{41.14^2}{37} = 82.70807$$

$$\sum_{i,j}^{p,q} \frac{Y^2_{ij.}}{n_{ij.}} = \frac{3.11^2}{3} + \frac{1.84^2}{2} + \ldots + \frac{4.40^2}{5} = 90.70409$$

Table 2
Analysis of variance of Cu content

Horizon	Ore dyke	j = 1	j = 2	j = 3	j = 4	$n_{i..}$	$Y_{i..}$
		2	3	4	5		
i = 1	VI	0.98 1.06 1.07 3.11	0.78 1.06 1.84	1.11 0.51 1.62	0.59 0.51 1.10	9	7.67
i = 2	VII	1.27 0.44 1.13 2.84	0.68 0.71 1.39	1.27 0.50 1.77	0.50 0.67 0.80 0.58 2.55	11	8.55
i = 3	VIII	0.69 1.36 2.05	0.83 0.83	1.31 1.31	0.46 1.49 2.92 1.58 0.64 7.09	9	11.28
i = 4	IX	0.89 0.89	0.70 0.70	2.12 2.12	0.46 0.94 1.29 3.52 1.58 0.81 8.60	9	12.31
i = 5	X	0.69 0.69	0.81 0.81	0.51 0.51	0.52 1.37 1.92 2.11 0.38 6.30	8	8.31
i = 6	XI	0.68 1.71 2.39	0.94 0.94	2.30 2.30	0.85 1.05 1.76 2.20 0.76 0.66 7.28	10	12.91
i = 7	XII	1.75 1.42 3.17	0.75 0.75	0.87 0.87	1.10 1.01 0.94 0.77 3.82	8	8.61
i = 8	XIII	1.68 1.68	0.33 0.33	0.61	0.91 0.67 1.00 1.05 0.77 4.40	8	7.02
	$n_{.j.}$	15	10	10	37	72	76.66
	$Y_{.j.}$	16.82	7.59	11.11	41.14	76.66	

Table 3
Analysis of variance of thickness

Hori-zon	Ore dyke	$j = 1$	$j = 2$	$j = 4$	$j = 3$	$n_{.i.}$	$Y_{.i.}$
		2	3	4	5		
$i = 1$	VI	0.94 1.09 1.51 3.54	1.84 0.80 2.64	1.18 2.26 3.44	1.09 0.85 1.94	9	11.56
$i = 2$	VII	1.07 0.71 1.11 2.89	1.35 1.38 2.73	1.23 0.82 2.05	1.09 1.30 0.86 0.94 4.19	11	11.86
$i = 3$	VIII	0.54 1.13 1.67	0.87 0.87	1.21 1.21	0.80 1.39 1.50 0.95 1.39 6.03	9	9.78
$i = 4$	IX	1.15 1.15	0.74 0.74	1.07 1.07	0.80 1.03 1.41 1.69 0.95 1.11 6.99	9	9.95
$i = 5$	X	1.27 1.27	0.34 0.34	1.28 1.28	1.77 1.39 1.53 1.12 0.67 6.48	8	9.37
$i = 6$	XI	5.39 4.37 9.76	0.88 0.88	0.45 0.45	6.02 8.16 1.98 0.84 1.35 0.56 18.91	10	30.00
$i = 7$	XII	4.37 3.25 7.62	1.02 1.02	0.78 0.78	6.23 7.25 9.28 1.25 24.01	8	33.43
$i = 8$	XIII	0.80 0.80	0.56 0.56	8.24 8.24	8.69 9.85 7.28 1.23 1.21 28.26	8	37.86
	$n_{.j.}$	15	10	10	37	72	153.81
	$Y_{.j.}$	28.70	9.78	18.52	96.81	153.81	

Table 4

Analysis of variance of Pb content

Hori-zon	Ore dyke	$j = 1$	$j = 2$	$j = 3$	$n_{i..}$	$Y_{i..}$
		2	4	5		
$i = 1$	VII	0.10 0.10	0.32 0.05 0.37	1.06 0.24 0.49 2.86	7	3.33
$i = 2$	VIII	0.17 0.17	0.13 0.13	0.07 0.79 1.36 0.21 0.17 2.60	7	2.90
$i = 3$	IX	0.05 0.05	0.06 0.06	0.07 0.74 0.87 1.50 0.21 0.51 3.90	8	4.01
$i = 4$	X	0.09 0.09	0.08 0.08	0.26 0.28 0.54 0.02 0.45 1.55	7	1.72
$i = 5$	XI	0.02 0.02	0.07	0.15 0.13 0.12 0.87 0.09 0.06 1.42	8	1.51
$i = 6$	XII	0.02 0.01 0.03	0.11 0.11	0.09 0.02 0.04 0.04 0.19	7	0.33
$i = 7$	XIV	0.02 0.02	0.01 0.01	0.02 0.11 0.04 0.05 0.03 0.25	7	0.28
	$n_{.j.}$	8	8	35	51	14.08
	$Y_{.j.}$	0.48	0.83	12.77	14.08	

Table 5

Analysis of variance of Zn content

Hori-zon	Ore dyke	$j = 1$	$j = 2$	$j = 3$	$n_{i..}$	$Y_{i..}$
		2	4	5		
$i = 1$	VII	0.31 0.31	0.45 0.10 0.55	1.94 0.40 0.61 0.34 3.29	7	4.15
$i = 2$	VIII	0.19 0.19	0.15 0.15	0.12 1.83 2.85 0.39 0.30 5.49	7	5.83
$i = 3$	IX	0.06 0.06	0.06 0.06	0.12 1.85 1.90 2.69 0.39 0.42 7.37	8	7.49
$i = 4$	X	0.20 0.20	0.10 0.10	0.16 1.18 0.47 0.51 0.05 2.37	7	2.67
$i = 5$	XI	0.02 0.03 0.05	**0.18** 0.18	0.17 0.21 0.19 1.32 0.08 0.05 2.02	9	2.25
$i = 6$	XII	0.03 0.03 0.06	0.06 0.06	0.10 0.06 0.06 0.10 0.32	7	0.44
$i = 7$	XIV	0.05 0.05	0.03 0.03	0.02 0.02 0.03 0.03 0.02 0.12	7	0.20
	$n_{.j.}$	9	8	35	52	23.03
	$Y_{.j.}$	0.92	1.13	20.98	23.03	

Table 6

Evaluation of variance analysis

Component	Source of variability	Sum of squares S_1 S_2 S_{12} S_r S_0	Number of degrees of freedom $\nu_1 = p-1$ $\nu_2 = q-1$ $\nu_{12} = \nu_1 \cdot \nu_2$ $\nu_r = N-pq$	Average square s^2_1 s^2_2 s^2_{12} s^2_r	F	F_{crit} $\alpha = 10\,\%$
Cu	between horizons	3.2609	7	0.4658	1.15	1.87
	between dykes	1.0865	3	0.3622	—	
	interaction	4.7351	21	0.2255	—	
	residual	16.1579	40	0.4039		
	total	25.2404	71			
Pb	between horizons	1.6428	6	0.2738	2.20	1.98
	between dykes	0.8869	2	0.4435	3.55	2.49
	interaction	0.5163	12	0.0430	—	
	residual	3.7400	30	0.1247		
	total	6.7860	50			
Zn	between horizons	5.7431	6	0.9572	1.98	1.98
	between dykes	2.6301	2	1.3151	2.72	2.49
	interaction	1.4516	12	0.1210	—	
	residual	14.9853	31	0.4834		
	total	24.8101	51			
Thickness	between horizons	140.5294	7	20.0756	5.37	1.87
	between dykes	23.5020	3	7.8340	2.10	2.23
	interaction	81.0838	21	3.8611	1.03	1.61
	residual	149.4515	40	3.7363		
	total	394.5667	71			

The calculation of the total sum of squares and its components is performed according to formulae (11) to (15)

$$
\begin{aligned}
S_0 &= 106.8620 - 81.6216 = 25.2404 \\
S_1 &= 84.8825 - 81.6216 = 3.2609 \\
S_2 &= 82.7081 - 81.6216 = 1.0865 \\
S_{12} &= 90.7041 - 84.8825 - 82.7081 + 81.6216 = 4.7351 \\
S_r &= 106.8620 - 90.7041 = 16.1579
\end{aligned}
$$

Then (16) must be valid

$$S_0 = 3.2609 + 1.0865 + 4.7351 + 16.1579 = 25.2404$$

The analysis of variance of thickness and contents of Pb and Zn, was performed in the same way but, with a different number of observations N, because some data on ore mineralization in some mining blocks were missing and so the requirement of the mathematical model of complete number for all combinations of both factors would not be fulfilled.

The results of the analysis of variance are given in Tab. 6. For the testing of the hypotheses (2) we determined the critical value of F at the chosen level of significance = 10 % and the appurtenant number of the degrees of freedom from statistical tables (J a n k o, 1958). These critical values are given in the last column in Tab. 6.

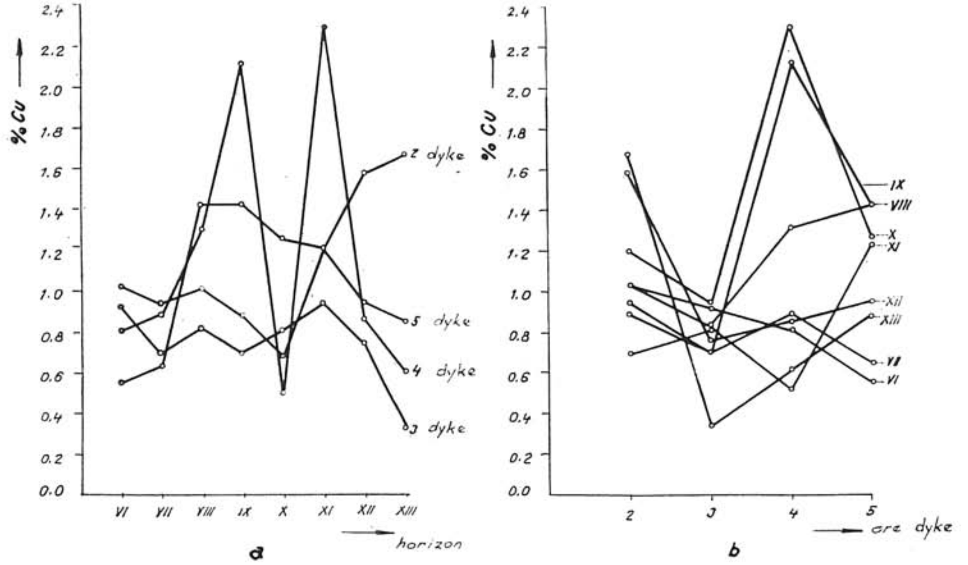

Fig. 2. Graph of vertical (a) and horizontal (b) distribution of the Cu content.

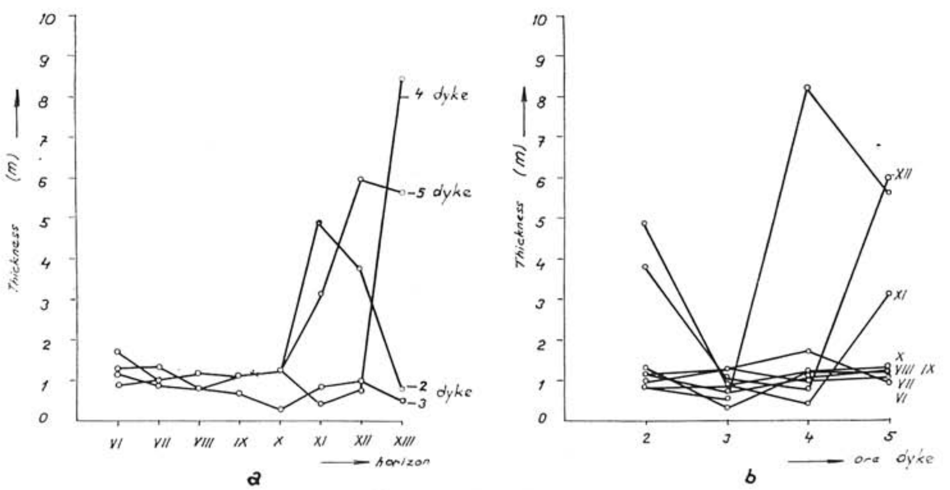

Fig. 3. Graph of vertical (a) and horizontal (b) distribution of the thickness.

From the comparison of the calculated F values with the critical values we can deduce these conclusions:

a) the depth factor has a considerable influence on the variance of Pb, Zn contents and on vein thickness,

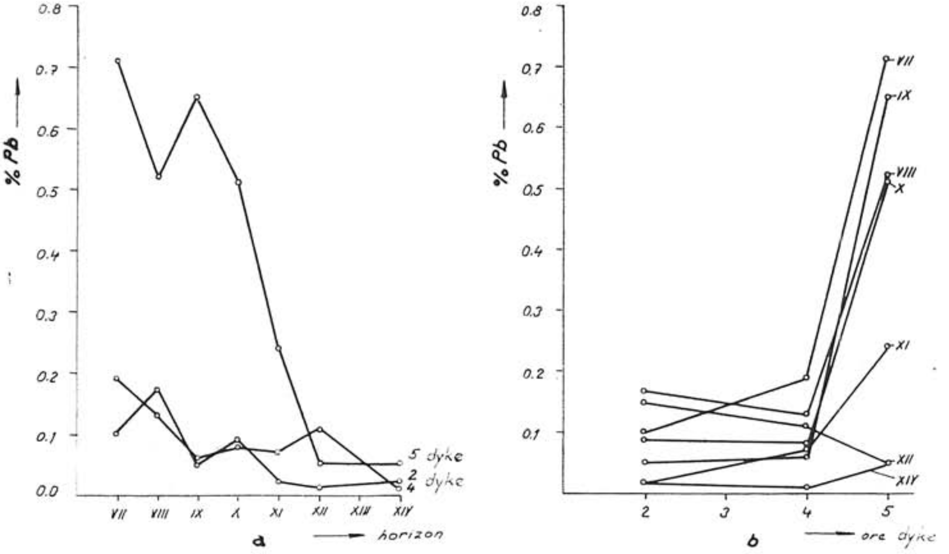

Fig. 4. Graph of vertical (a) and horizontal (b) distribution of the Pb content.

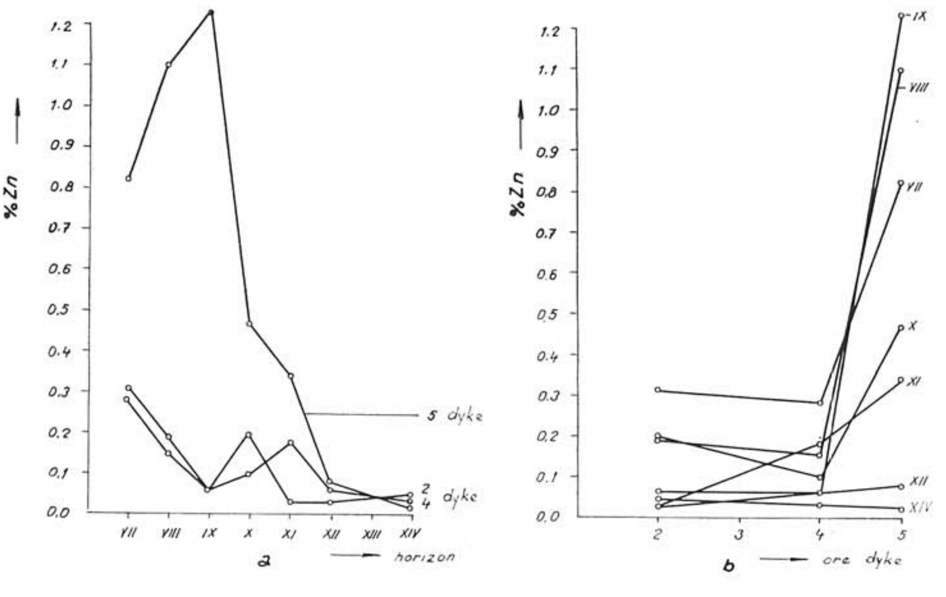

Fig. 5. Graph of vertical (a) and horizontal (b) distribution of the Zn content.

b) the horizontal factor of distribution (ore dykes) largely participates in dispersion of Pb and Zn contents,

c) the interaction of both factors is statistically insignificant for all components of ore mineralization as well as for thickness.

These important calculations are also confirmed by the graphs of vertical and horizontal distribution of contents and thickness — Figs. 2, 3, 4 and 5.

Fig. 2 is the polygon of vertical (a) and horizontal (b) distribution of Cu content. The polygons are random, without indication of any trend and confirm the random character of the vertical and horizontal distribution of Cu content in the ore filling of the Rozália vein.

In Fig. 3 is the polygon of vertical (a) and horizontal (b) distribution of the thickness of the Rozália vein. The polygon of vertical distribution (a) shows a distinct trend of increasing vein thickness with depth whilst the polygon of horizontal distribution (b) is without a distinct trend.

In Fig. 4 is the polygon of vertical (a) and horizontal (b) distribution of Pb content. Both polygons show a trend and confirm the influence of both factors on Pb variance. With depth the Pb content falls and with transition from the 2-th to the 4-th and 5-th ore dyke the Pb content increases.

In Fig. 5 is the polygon of vertical (a) and horizontal (b) distribution of Zn content. Both graphs are similar to the one for Pb and the same conclusions as in the foregoing case are valid here.

Conclusion

The derived mathematical model of the variance analysis in classification according to two factors takes fully into account horizontal and vertical distribution of mining blocks, the average content and thicknesses of which were the basis of the variance analysis. Besides them, the mathematical model makes possible to evaluate the influence of the effects of the main factors, as well as their interaction on the resulting sign and provides so much more information on distribution of ore mineralization components than the simple model of the variance analysis in sorting according to one factor. The only but important requirement of its application is the neccessity of full numbers for combinations of the levels of both factors, of which some must have the number of observations greater than one. In variance analysis of components of ore mineralization of the Rozália vein at the significance level 10 % it has been proved that the depth factor has a significant share in the Pb, Zn content and vein thickness. The second factor — horizontal distribution in ore dykes, is significant for the Pb and Zn contents. The interaction of factors is in all cases statistically insignificant. We suppose that we have to take into account these conclusions in projecting of further investment exploitation works.

Translated by J. Pevný

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