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EXPONENTS OF ONE-VERTEX MAPS AND t-BALANCED MAPS

Ľubica Líšková

ABSTRACT. A Cayley map is a Cayley graph embedded in an orientable surface in such a way that the cyclic order of generators is the same at each vertex. An exponent of a Cayley map is a number e with the property that (loosely speaking) the Cayley map is isomorphic to its 'e-fold rotational image'.

Cayley maps for the trivial group are important since each Cayley map is a regular lift of a one-vertex Cayley map. In our contribution we present results on exponents of one-vertex maps, with emphasis on maps exhibiting certain types of symmetry in the distribution of generators and their inverses.

1. Introduction

Highly symmetric maps have been in the center of interest in algebraic and topological graph theory for over a century. The smallest and the largest 'level of symmetry' is obtained in the case of Cayley maps and regular maps, respectively. First results concerning regular balanced Cayley maps (in slightly different terms) can be found in [1]. The results were later extended in the series of articles for balanced maps [12], antibalanced maps [11], t-balanced maps [8], k-rotational maps [3], and for arbitrary regular Cayley maps [4, 10].

Every Cayley map is a regular lift of a one-vertex map. The one-vertex quotients are determined by the cyclic order of generators and by their distribution of inverses. They carry all the information we need about the lifted Cayley map. Exponents [9] were introduced as a tool for classification of regular maps with a given underlying graph. It is therefore natural to study exponents of one-vertex Cayley maps in more detail.

The aim of this paper is to present results on automorphisms and exponents of one-vertex t-balanced maps and some related results.

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2. Basic definitions

A *graph* K consists of a set of vertices, denoted by $V(K)$, and a set of edges, $E(K)$. We allow graphs to contain loops, semiedges or multiple edges. Let $E'(K)$ be the set of all semiedges of K. Any edge from $E(K)\backslash E'(K)$ endowed by one of the two possible orientations is called a *dart*. A semiedge admits just one orientation; therefore by definition, a semiedge gives rise to a unique dart. The number of edges incident with a vertex $v \in V(K)$ is called the valence of v and denoted by $\mathrm{val}(v)$; note that each loop contributes to the valence by 2. Let $D(K)$ be the dart set of K; thus $\left|D(K)\right| = 2\left|E(K)\right| - \left|E'(K)\right|$.

A *surface* is a 2-dimensional manifold. A surface will be called *closed* if it is compact and has no boundary. By an orientation at some point of a surface we understand an orientation assigned to a disc neighbourhood of the point. If orientation is preserved along each closed walk, the surface is said to be *orientable*. Throughout this paper we shall consider only orientable surfaces.

2.1 Cayley graphs

Let G be a finite group with identity element 1, and let $\Omega = x_0, x_1, \ldots, x_{n-1}$ be a finite sequence of elements of G. Further, let τ be an involutory permutation of the set $[n] = \{0, 1, \ldots, n-1\}$ such that $x_i^{-1} = x_{i\tau}$ for each $i \in [n]$. The permutation τ is called a *distribution of inverses.* A *Cayley graph* $C(G, \Omega, \tau)$ for G, Ω and τ is an undirected graph with G as the vertex-set and for each $g \in G$ and each $i \in [n]$ there is a dart (g, i) with initial vertex g. The dart (g, i) is a semiedge if $x_i = 1_G$ and $i = i\tau$; it is a directed loop at g if $x_i = 1_G$ and $i \neq i\tau$; and it is a directed edge incident with two distinct vertices if $x_i \neq 1_G$. In the last two cases, the terminal vertex of the dart (g, i) is the vertex gx_i, and the *dart's reverse* is the dart $(gx_i, i\tau)$. A pair of mutually reverse darts (g, i) and $(gx_i, i\tau)$ forms an edge in $C(G, \Omega, \tau)$. We note that the Cayley graph $C(G, \Omega, \tau)$ is connected if and only if the elements of Ω generate G. We will always assume that our Cayley graphs are connected.

An *automorphism* of a Cayley graph is a permutation of its dart set which preserves the incidence between darts and vertices.

2.2 Cayley maps

A *Cayley map* $CM(G, \Omega, \tau)$ is a cellular embedding of the graph $C(G, \Omega, \tau)$ on a closed orientable surface S such that for each $i \in [n]$ the dart $(g, i+1)$ immediately follows the dart (g, i) in the orientation of S; here the second coordinate is taken modulo n.

Recall [2] that any cellular embedding M of a connected graph K on an orientable surface S can be completely described by a pair of permutations

L and R of the dart set $D(K)$ of K. The *dart-reversing involution* L is the involutory permutation that interchanges any dart z with its reverse z^{-1}; while the *rotation* R, for each vertex v, cyclically permutes the darts emanating from v following the orientation of S. Therefore we often write $M = (K; R, L)$. The permutations R and L are called the *rotation* and the *dart-reversing involution* of M, respectively. When $M = (K; R, L)$ is a Cayley map $CM(G, \Omega, \tau)$, we have $(g, i)R = (g, i+1)$ and $(g, i)L = (gx_i, i\tau)$ for each dart $(g, i) \in G \times [n]$. The faces of M are (in one-to-one correspondence with) orbits of the permutation LR.

A map isomorphism $f\colon M_1 \to M_2$ induces an incidence and orientation preserving homeomorphism between the supporting surface of M_1 and that of M_2. If $M_1 = M_2 = M$, then f is a map automorphism of M. Clearly, each map automorphism is also an automorphism of the underlying graph. It can be shown that for any two darts z and t of M there exists at most one map automorphism taking z to t [5].

2.3 Exponents

Let $M = CM(G, \Omega, \tau)$ be a Cayley map of valence $|\Omega| = n$ and let e be an integer such that $\gcd(e, n) = 1$. Further let Ω^e be the cyclic sequence $\Omega^e = (x_0, x_e, \ldots, x_{e(n-1)})$ and let τ_1 be defined by

$$e(i\tau_1) = (ei)\tau\,. \tag{1}$$

We say that e is an *exponent* of M if the Cayley maps M and $M^e = CM(G, \Omega^e, \tau_1)$ are isomorphic. Note that having exponent -1 means that M is isomorphic to its 'mirror image' M^{-1}.

If an integer e is the exponent of a map M with valence n and $f \equiv e \pmod{n}$, then the maps M^e and M^f are identical, and hence f is an exponent of M. Therefore we will make no distinction in notation between an exponent of M and its residue class (mod n). Further, let $\psi\colon M \to M^e$ and $\mu\colon M \to M^f$ be isomorphisms corresponding to exponents e and f, respectively. Then $\mu\psi$ is an isomorphism associated with ef. Thus ef is an exponent of M. It follows that the residue classes (modulo the valence of M) of exponents of the map M form a multiplicative group $\mathrm{Ex}(M)$, the exponent group of M. Clearly, $\mathrm{Ex}(M)$ is a subgroup of $\mathbb{Z}_n^*$, the multiplicative group of invertible elements of the ring $\mathbb{Z}_n$.

3. Exponents of one-vertex maps

In [6] the following necessary and sufficient condition for a number e to be an exponent of a Cayley map was established:

Theorem 3.1. [6] *Let $M = CM(G, \Omega, \tau)$ be a Cayley map. Then an integer e such that $\gcd(e, n) = 1$ is an exponent of M if and only if there exist two mappings $\eta\colon G \to \mathbb{Z}_n$, $\omega\colon G \to G$ and an integer k such that:*

(i) $\omega(1) = 1$ *and* $\omega(ax_i) = \omega(a)x_{\eta(a)+ei}$,

(ii) $\eta(1) = ek$ *and* $\eta(ax_i) = (\eta(a) + ei)\tau - e(i\tau)$

for each $a \in G$ *and* $x_i \in \Omega$.

An isomorphism $\psi\colon D(M) \to D(M^e)$ from M onto M^e can be defined by $\psi(a, x_l) = \big(\omega(a), x_{\eta(a)+el}\big)$. It may be useful to observe that ω maps vertices to vertices and η determines the images of the neighbours of a vertex.

Now we shall apply this result to one-vertex maps. A one-vertex map can be represented as a Cayley map for the trivial group. Let τ be an involution on $[n]$, let $s = \big|\{i = i\tau, i \in [n]\}\big|$, and let $\langle 1_G\rangle$ be the trivial group presented in the form $\langle 1_G\rangle = \langle x_0, \dots, x_{n-1} | x_0 = x_1 = \dots = x_{n-1} = 1\rangle$; note that we trivially have $x_i x_{i\tau} = 1$. The Cayley map $\mathcal{M} = CM\big(\langle 1_G\rangle, (x_0, \dots, x_{n-1}), \tau\big)$ has one vertex, namely 1_G, s semiedges and $(n-s)/2$ loops. For any i such that $i \neq i\tau$, the two opposite darts $(1_G, x_i)$ and $(1_G, x_{i\tau})$ form a loop. Clearly, the distribution of inverses of $\mathcal{M}$ is τ.

In the paper we fix the generating sequence Ω to be $(x_0, x_1, \dots, x_{n-1})$. For one-vertex maps we may use identification between darts and generators, i.e., $(1_G, i) \equiv x_i$, and denote a one-vertex map of valence n by $\mathcal{M}(n, \tau)$. In the case of one-vertex Cayley maps Theorem 3.1 implies:

Corollary 3.2. *A one-vertex Cayley map $\mathcal{M}(n, \tau)$ has an exponent e if and only if there exists $z \in \mathbb{Z}_n$ such that $(z + ei)\tau = z + (i\tau)e \pmod{n}$ for each $i \in [n]$.*

Since an automorphism of a Cayley map is a non-trivial isomorphism $M \to M^e$ corresponding to the exponent $e = 1$, we have the following corollary:

Corollary 3.3. *The mapping $x_0 \mapsto x_z$, $z \neq 0$, is a non-trivial automorphism of $\mathcal{M}(n, \tau)$ if and only if for all $i \in [n]$ it holds:*

$$(z + i)\tau \equiv z + (i\tau) \pmod{n}.$$

4. One-vertex t-balanced maps

In this section we will consider in detail the so-called one-vertex t-balanced maps together with their automorphisms and exponents.

A Cayley map $M = CM(G, \Omega, \tau)$ with distribution of inverses τ such that $i\tau = b + it$ for all $i \in [n]$ and fixed t and b is called t-*balanced*. We first present a construction of t-balanced maps.

Theorem 4.1. *Let n and t be such that $\gcd(t,n)=1$ and $t^2 \equiv 1 \pmod{n}$. Further, let b be a solution of the congruence*

$$b(t+1) \equiv 0 \pmod{n}.$$

Then, a Cayley map M with distribution of inverses τ defined by $i\tau = b+ti$ is t-balanced.

Proof. Since $\gcd(t,n)=1$, we have $\{b+ti, i \in [n]\} = [n]$, and hence τ is a bijection on $[n]$. It remains to prove that τ is an involution. Take an arbitrary $i \in [n]$ and let $i\tau = b+ti$. Then, $(b+ti)\tau = b+t(b+ti) = b(t+1)+t^2 i = 0+i = i$, and hence $\tau^2 = \mathrm{id}$. □

4.1 Automorphisms

For one-vertex t-balanced Cayley maps Theorem 3.1 simplifies to:

Corollary 4.2. *Let $\mathcal{M}(n,\tau)$ be a one-vertex t-balanced Cayley map. Then the mapping $x_i \mapsto x_{z+i}$, $0 < z < n$, is an automorphism of $\mathcal{M}$ if and only if*

$$z(t-1) \equiv 0 \pmod{n}.$$

Proof. In fact one has to prove that the mappings $\psi\colon i \mapsto z+i$ and $\tau\colon i \mapsto b+ti$ commute if and only if $z(t-1) \equiv 0 \pmod{n}$. This immediately follows from the congruences:

$$\begin{aligned} z+(b+ti) &\equiv b+t(z+i) \pmod{n}, \\ z &\equiv tz \pmod{n}, \\ z(t-1) &\equiv 0 \pmod{n}. \end{aligned}$$

□

Note that the only regular one-vertex maps are balanced maps (t=1). To classify all the automorphisms of one vertex maps, we have to find all solutions of the congruence $z(t-1) \equiv 0 \pmod{n}$. First, we divide pairs (t,n) which satisfy $t^2 \equiv 1 \pmod{n}$ into three types (where $2^k || n$ means that $2^k | n$ but $2^{k+1} \nmid n$):

(1) n is odd.

(2) n is even $2^k || n$, $t \equiv \pm 1 \pmod{2^k}$.

(3) n is even $2^k || n$, $k \geqslant 3$, $t \equiv 2^{k-1} \pm 1 \pmod{2^k}$.

Theorem 4.3. [8] *The set of solutions of the congruence $z(t-1) \equiv 0 \pmod{n}$ is a subgroup of $\mathbb{Z}_n$ of order $\gcd(t-1,n)$, generated by*

(i) $t+1$ *if (t,n) is of type 1 or 3,*

(ii) $\gcd(t+1,n)/2$ *if (t,n) is of type 2.*

The following corollary shows that one-vertex t-balanced maps have at least one non-trivial automorphism (with one exception). It is the main result of this section.

Corollary 4.4. *A one-vertex t-balanced Cayley map $\mathcal{M}(n,\tau)$ has at least one non-trivial automorphism, except the case when $t=-1$ and n is odd.*

Proof. By Corollary 4.2 it suffices to show that there is a non-trivial solution of the congruence $z(t-1) \equiv 0 \pmod{n}$.
(i) For $t \neq -1$ we have a non-trivial solution $t+1$.
(ii) If n is even, then $t-1$ is even and $z=n/2$ is a non-trivial solution of the congruence.
(iii) In the case when $t=-1$ and n is odd, the corresponding one vertex map has no automorphism because it has just one semiedge. □

Moreover, Corollary 4.4 yields that only one-vertex antibalanced maps with odd valence have exactly one semiedge.

If $t=3$, then there is exactly one solution $z=n/2$. Hence 3-balanced Cayley maps of even valence have only one non-trivial automorphism. For example, the 3-balanced Cayley map of valence $n=8$ with distribution of inverses $i\tau = 3i$ has just one automorphism $x_i \mapsto x_{4+i}$. The one-vertex 4-balanced Cayley map of valence $n=15$ with distribution of inverses $i\tau=4i$ for $i \in [15]$ has two non-trivial automorphisms, namely, $x_i \mapsto x_{i+5}$ and $x_i \mapsto x_{i+10}$.

For later use we state another two results. These results also hold for general Cayley maps and their rotary automorphisms [3], i.e., automorphisms of Cayley maps fixing the vertex 1_G.

Proposition 4.5. *Let a one-vertex map $\mathcal{M}(n,\tau)$ have the automorphisms $x_i \mapsto x_{a+i}$ and $x_i \mapsto x_{b+i}$. Then for an arbitrary k and l the map $\mathcal{M}$ also has the automorphism $x_i \mapsto x_{(ka+lb)+i}$.*

Theorem 4.6. *Let $\mathcal{M}(n,\tau)$ have the automorphisms $x_i \mapsto x_{a+i}$, $x_i \mapsto x_{b+i}$ and let $\gcd(a,b)=z$. Then $\mathcal{M}$ also has the automorphism $x_i \mapsto x_{z+i}$.*

Proof. If $\gcd(a,b)=z$, then we can write $z = ka+lb \pmod{n}$ for some k and l. By Proposition 4.5 there is an automorphism $x_i \mapsto x_{ka+lb+i} = x_{z+i}$. □

4.2 Exponents

By [9], if e is coprime with the valence of a (general) map M, then the groups $\mathrm{Aut}(M)$ and $\mathrm{Aut}(M^e)$ are identical. Consequently, we have:

Corollary 4.7. *If $\mathcal{M}(n,\tau)$ has the automorphism $x_i \mapsto x_{z+i}$, then $\mathcal{M}^e(n,\tau_1)$ has the automorphism $y_i \mapsto y_{z+i}$, that is, $x_i \mapsto x_{e(z+i)}$, $i \in [n]$.*

Moreover, if M and M^e are Cayley maps, they have the same type of balance:

THEOREM 4.8. *If a Cayley map $M = CM(G, \Omega, \tau)$ is t-balanced, then $M^e = CM(G, \Omega_1, \tau_1)$ is t-balanced as well.*

P r o o f . The definition of t-balanced map and (1) yield:

$$i\tau_1 = e^{-1}(ei)\tau = e^{-1}[b + tei] = e^{-1}b + ti\,.$$

The above computation implies that M^e is t-balanced with distribution of inverses $i\tau_1 = e^{-1}b + ti$, $i \in [n]$. ☐

Moreover, one-vertex Cayley maps $\mathcal{M}$ and $\mathcal{M}^e$ have the same exponent groups:

THEOREM 4.9. *Let $\mathcal{M}(n, \tau)$ and $\mathcal{M}^e(n, \tau_1)$ be one-vertex Cayley maps. Then* $\mathrm{Ex}(\mathcal{M}) = \mathrm{Ex}(\mathcal{M}^e)$.

P r o o f . Assume that f is an exponent of $\mathcal{M}$. Then from Corollary 3.2 we have that there is z such that for all $i \in [n]$ it holds that

$$(z + fi)\tau = z + (i\tau)f \pmod{n}.$$

Substituting $z = ez_1$ and $i = ej$ yields

$$(ez_1 + fej)\tau = ez_1 + \big((ej)\tau\big)f \pmod{n}.$$

From (1) we have

$$e(z_1 + fj)\tau_1 = ez_1 + e(j\tau_1)f \pmod{n},$$

$$(z_1 + fj)\tau_1 = z_1 + (j\tau_1)f \pmod{n}.$$

The last congruence implies that f is an exponent of $\mathcal{M}^e(n, \tau_1)$ as well. ☐

In the previous section we have seen that one-vertex t-balanced Cayley maps have at least one automorphism (with one exception). Next we will see that such maps have all theoretically admissible exponents. Before we show this, we need a result from [8].

THEOREM 4.10. [8] *Let t and n be such that* $\gcd(t, n) = 1$ *and* $t^2 \equiv 1 \pmod{n}$. *Further, let* $\gcd(e, n) = 1$ *and let b be a solution of* $b(t + 1) \equiv 0 \pmod{n}$. *Then the congruence*

$$(e - 1)b \equiv (t - 1)z \pmod{n} \tag{2}$$

has exactly $\gcd(t - 1, n)$ *solutions in* $\mathbb{Z}_n$.

THEOREM 4.11. *If $\mathcal{M}(n,\tau)$ is t-balanced, then every integer e such that $\gcd(e,n)=1$ is an exponent of $\mathcal{M}$.*

Proof. Using the definition of a t-balanced map and Corollary 3.2 we subsequently have

$$\begin{aligned}(z+ei)\tau &\equiv z+e(i\tau)\,,\\ b+tz+tei &\equiv z+eb+eti\,,\\ (t-1)z &\equiv (e-1)b\,.\end{aligned}$$

By Theorem 4.10 there exists at least one solution z of the last congruence. □

We show that the converse is true for prime valences.

THEOREM 4.12. *Let $\mathcal{M}(p,\tau)$ be a one-vertex Cayley map of prime valence p. Then $\mathrm{Ex}(\mathcal{M})=\mathbb{Z}_p^*\cong\mathbb{Z}_{p-1}$ if and only if $\mathcal{M}$ is t-balanced where $t=\pm1$.*

Proof. The condition $\mathrm{Ex}(\mathcal{M})=\mathbb{Z}_{p-1}$ is equivalent to the existence of an exponent ξ of $\mathcal{M}$ which is a primitive element of $\mathbb{Z}_p^*$. Setting $e=\xi$ in Corollary 3.2 we have

$$(z+\xi i)\tau \equiv z+\xi(i\tau)\,.$$

(i) First, let us assume that $z=0$, that is, $(\xi i)\tau\equiv\xi(i\tau)$. Let $(\xi)\tau=t\xi$. Then $(\xi^{i+1})\tau=\xi(\xi^i\tau)=t\xi^{i+1}$, and hence $i\tau=ti$ which implies that $\mathcal{M}$ is t-balanced.

(ii) Let $z\neq0$ and $b=0\tau$. Since ξ is an exponent of $\mathcal{M}$, we have $(z+\xi k)\tau\equiv z+\xi(k\tau)$ for each $k\in\mathbb{N}$. For $k=0$ we obtain $z\tau=z+\xi b$. Define t by $z+\xi b=b+tz$; since $z\neq0$ we have $t=(z+\xi b-b)z^{-1}$. Note that if $b=0$, then $t=1$. By induction we prove that $\big[(1+\xi+\cdots+\xi^k)z\big]\tau=b+t\big[z(1+\xi+\cdots+\xi^k)\big]$. Clearly, it is sufficient to focus just on the induction step. We have

$$\begin{aligned}\big[(1+\xi+\cdots+\xi^{k+1})z\big]\tau &= \big[z+\xi\big((1+\xi+\cdots+\xi^k)z\big)\big]\tau\\ &= z+\xi\big[(1+\xi+\cdots+\xi^k)z\big]\tau\\ &= z+\xi\big[b+tz(1+\xi+\cdots+\xi^k)\big]\\ &= z+\xi b+\xi tz(1+\xi+\cdots+\xi^k)\\ &= b+tz+tz(\xi+\cdots+\xi^{k+1})\\ &= b+tz(1+\xi+\cdots+\xi^{k+1})\,.\end{aligned}$$

Since $t^2\equiv1 \pmod p$, it follows that $t=\pm1$. This completes the proof. □

There are no similar results for 'non-balanced' maps, that is, for maps which are not t-balanced for any t. A one-vertex non-balanced Cayley map $\mathcal{M}(p,\tau)$ of prime valence has no automorphism. However, it may have an exponent. For example, the map $\mathcal{M}_1(7,\tau_1)$ has exponents $e = 2,4$ if $\tau_1 = (0)(13)(26)(45)$. The map $\mathcal{M}_2(7,\tau_2)$ has no exponent for $\tau_2 = (0)(12)(34)(56)$.

If n is not prime, then all possible combinations of outcomes can occur. For instance, the maps $\mathcal{M}_i(8,\tau_i)$, $i \in \{3,4\}$, have no exponents if $\tau_3 = (01)(2)(3)\dots(8)$ or $\tau_4 = (0)(12)(3)(45)(6)(78)$ but $\mathcal{M}_3$ is without automorphism and $\mathcal{M}_4$ has automorphisms $x_i \mapsto x_{3+i}$ and $x_i \mapsto x_{6+i}$. Finally, we give a few examples of maps with exponents. The map $\mathcal{M}_5(14,\tau_5)$ with distribution of inverses $\tau_5 = (04)(12)(3)(59)(67)(8)(10,14)(11,12)(13)$ has automorphisms $x_i \mapsto x_{5+i}$ and $x_i \mapsto x_{10+i}$ and exponent 4. The map $\mathcal{M}_6(8,\tau_6)$ $\tau_6 = (02)(1)(3)(4)(5)\dots(8)$ has exponents $e = -1,3,5$ and has no automorphism. The map $\mathcal{M}_6(8,\tau_6)$ has $\mathrm{Ex}(\mathcal{M}_6) = \mathbb{Z}_8^*$. This fact implies that Theorem 4.12 does not hold if n is not prime.

Note that Theorem 4.12 cannot be extended to arbitrary Cayley maps. By Theorem 2.3 from [LL6] we can construct a non-balanced Cayley map M of prime valence p with exponent ξ, where ξ is a primitive element of $\mathbb{Z}_p^*$. Thus, the map M is non-balanced and has exponent group $\mathrm{Ex}(M) = \mathbb{Z}_p^*$.

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Department of Mathematics
Faculty of Civil Engineering
Slovak University of Technology
Radlinského 11
SK–813 68 Bratislava
SLOVAKIA
E-mail: ls@math.sk