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COMPARISON OF SOME SUBFAMILIES OF FUNCTIONS HAVING THE BAIRE PROPERTY

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ABSTRACT. We prove that the family $\mathcal{Q}$ of quasi-continuous functions is a strongly porous set in the space $\mathcal{B}a$ of functions having the Baire property. Moreover, the family $\mathcal{DQ}$ of all Darboux quasi-continuous functions is a strongly porous set in the space $\mathcal{DB}a$ of Darboux functions having the Baire property. It implies that each family of all functions having the $\mathcal{A}$-Darboux property is strongly porous in $\mathcal{DB}a$ if $\mathcal{A}$ has the $(*)$-property.

1. Introduction

In this paper we will focuse on the "size" of some subsets of the metric space. Such studies have a long tradition, also in the case when a space is a family of functions. The classical example of this kind is a result of S. Banach obtained in 1931. Using the category method, he proved that the set of nowhere differentiable functions is residual in the space of continuous functions under the supremum metric.

Here, we will study the family of quasi-continuous and Darboux quasi-continuous functions in the space of functions having the Baire property and the space of Darboux functions having the Baire property, respectively. The comparison of these families of functions will be described in porosity terms.

The notion of porosity in spaces of Darboux-like functions was studied among others by J. Kucner, R. Pawlak, B. Świątek in [8], by H. Rosen in [13] and by G. Ivanova, E. Wagner-Bojakowska in [6].

We start with the definition of a porosity of a set at a point.

Let (X, d) be an arbitrary metric space,

$$B(x,r) = \{z \in X : d(x,z) < r\} \quad \text{for } x \in X \quad \text{and} \quad r > 0.$$

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Moreover, assume that $B(x,0)=\emptyset$. Fix $M\subset X$, $x\in X$ and $r>0$. Here and subsequently,

$$\gamma(x,r,M)=\sup\Big\{t\geq 0:\underset{z\in X}{\exists}B(z,t)\subset B(x,r)\backslash M\Big\}.$$

Define the porosity of M at x as

$$p(M,x)=2\limsup_{r\to 0^+}\frac{\gamma(x,r,M)}{r}.$$

Definition 1.1 ([16])**.** The set $M\subset X$ is porous (strongly porous) in X if and only if $p(M,x)>0$ $\big(p(M,x)=1\big)$ for each $x\in M$.

We finish this section with the lemma which is useful in the further part of this paper.

Lemma 1.2 ([5])**.** *Let $a,b\in\mathbb{R}$, $a<b$ and let $P\subset[a,b]$ be a set of the first category. There exists a sequence of sets $\{C_n\}_{n\in\mathbb{N}}$, $C_n\subset(a,b)$ for $n\in\mathbb{N}$, with the following properties:*

1. *C_n is closed nowhere dense set of cardinality continuum for $n\in\mathbb{N}$;*
2. *$C_n\cap C_m=\emptyset$ for $n,m\in\mathbb{N}$, $n\neq m$;*
3. *$\bigcup_{n=1}^{\infty}C_n\cap P=\emptyset$;*
4. *for each interval $(e,f)\subset(a,b)$, there exists a natural number n such that $C_n\subset(e,f)$.*

2. The quasi-continuous functions

Definition 2.1 ([7])**.** A function $f:\mathbb{R}\to\mathbb{R}$ is quasi-continuous at a point x if and only if for every open neighborhood U of x and for every open neighborhood V of $f(x)$ there exists a nonempty open set $G\subset U$ such that $f(G)\subset V$. A function $f:\mathbb{R}\to\mathbb{R}$ is quasi-continuous if and only if it is quasi-continuous at each point.

For $a,b\in\mathbb{R}$, the symbol $<a,b>$ stands for the interval $(\min\{a,b\},\max\{a,b\})$.

Definition 2.2. A function $f:\mathbb{R}\to\mathbb{R}$ is a Darboux function if and only if for each interval $(a,b)\subset\mathbb{R}$ and for each $\lambda\in<f(a),f(b)>$ there exists a point $x_0\in(a,b)$ such that $f(x_0)=\lambda$.

Let $\mathcal{Q},\mathcal{D},\mathcal{B}a$ denote the families of all quasi-continuous functions, Darboux functions and functions having the Baire property, respectively. By $\mathcal{DQ}$ and $\mathcal{DB}a$ we will denote the class of Darboux quasi-continuous functions and Darboux functions having the Baire property.

Let ρ be a metric in the space $\mathcal{B}a$ given in the following way:

$$\rho(f,g)=\min\Big\{1,\sup\{\mid f(t)-g(t)\mid : t\in\mathbb{R}\}\Big\}.$$

From now on, we consider any family of functions with the above metric ρ.

First, we prove that the family $\mathcal{Q}$ is a strongly porous set in the space $\mathcal{B}a$ of functions with the Baire property. For this purpose, we need some auxiliary lemmas. All of them are similar to the lemmas from [5]. However, in this paper, we consider other families of functions, so the proofs of these lemmas need some essential modifications. For the convenience of the reader, we decided to present these proofs with details.

Lemma 2.3. *Let $f:\mathbb{R}\to\mathbb{R}$. If there exist two intervals (a,b) and (A,B) such that $f^{-1}((A,B))\cap(a,b)$ is a nonempty set of the first category, then $f\notin\mathcal{Q}$.*

Proof. Let $f:\mathbb{R}\to\mathbb{R}$, (a,b) and (A,B) be such that $f^{-1}((A,B))\cap(a,b)$ is a nonempty set of the first category. Suppose on the contrary, that $f\in\mathcal{Q}$. Let $x\in f^{-1}((A,B))\cap(a,b)$. As f is quasi-continuous, there exists a nonempty open set $G\subset(a,b)$ such that $f(G)\subset(A,B)$. Hence $G\subset f^{-1}((A,B))\cap(a,b)$, which gives a contradiction. □

Lemma 2.4. *There exists a function $f:\mathbb{R}\to[0,1]$ vanishing except for some set of the first category such that*

1. *f is a Darboux function;*
2. *f has the Baire property;*
3. *$B(f,1/2)\cap\mathcal{Q}=\emptyset$;*
4. *there exists a function $h\in\mathcal{DQ}$ such that $\rho(h,f)=1/2$.*

Proof. Let $\{C_n\}_{n\in\mathbb{N}}$ be a sequence of the sets from Lemma 1.2 (for $P=\emptyset$ and [a,b]=[0,1]). For each $n\in\mathbb{N}$, there exists a function ϕ_n transforming C_n onto $[0,1]$. Put

$$f(x)=\begin{cases}\phi_n(x) & \text{for } x\in C_n,\ n\in\mathbb{N},\\ 0 & \text{for } x\in\mathbb{R}\setminus\bigcup_{n=1}^{\infty}C_n.\end{cases}$$

It is easy to check that f is a Darboux function having the Baire property.

We will prove that $B\left(f,\frac{1}{2}\right)\cap\mathcal{Q}=\emptyset$. For this purpose we will show that $B\left(f,\frac{1}{2}-\frac{1}{n}\right)\cap\mathcal{Q}=\emptyset$ for each natural number $n>2$.

Let $n>2$. Assume that $g\in B\left(f,\frac{1}{2}-\frac{1}{n}\right)\cap\mathcal{Q}$. Let $x\in(0,1)$ be such that $f(x)=1$. Hence, $g(x)>\frac{1}{2}$. As g is quasi-continuous at x, there exists a nonempty open set $G\subset(0,1)$ such that $g(G)\subset\left(\frac{1}{2},\infty\right)$. By the definition of f and the property of the sequence $\{C_n\}_{n\in\mathbb{N}}$, there exists a point $x'\in G$ such that $f(x')=0$. Hence, $g(x')<\frac{1}{2}$ and $g(x')\notin\left(\frac{1}{2},\infty\right)$, which gives a contradiction.

Now, the condition 3. is obtained immediately from the equality

$$B\left(f, \frac{1}{2}\right) = \bigcup_{n=3}^{\infty} B\left(f, \frac{1}{2} - \frac{1}{n}\right).$$

Put

$$h(x) = \begin{cases} 0 & \text{for } x \in \mathbb{R} \setminus \left(-\frac{1}{2}, \frac{3}{2}\right), \\ \frac{1}{2} & \text{for } x \in [0,1], \\ \text{linear} & \text{on } \left[-\frac{1}{2}, 0\right] \text{ and } \left[1, \frac{3}{2}\right], \end{cases}$$

It is easy to see that $\rho(h, f) = 1/2$ and $h \in \mathcal{DQ}$. □

Lemma 2.5. *If $g \in \mathcal{Q}$, then for each $r \in (0,1)$ and $\varepsilon \in \left(0, \frac{r}{4}\right)$ there exists a function $h \in \mathcal{B}a$ such that*

$$B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g, r) \setminus \mathcal{Q}.$$

Moreover, if $g \in \mathcal{DQ}$, then the function h is Darboux.

Proof. Let $g \in \mathcal{Q}$. Fix r and ε such that $0 < \varepsilon < \frac{r}{4} < \frac{1}{4}$.

There are two possibilities:

1. The function g is constant, so $g(x) = c$ for some $c \in \mathbb{R}$ and any $x \in \mathbb{R}$. Let f be a function from Lemma 2.4. Putting

$$h = g - \frac{r}{2} + \varepsilon + (r - 2\varepsilon) f,$$

we obtain that h is Darboux and has the Baire property. Of course, $h \notin \mathcal{Q}$.

Let us show that $B\left(h, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q} = \emptyset$. Suppose, contrary to our claim, that there exists a function $\phi \in B\left(h, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q}$. Thus, $\rho\big(\phi, c - \frac{r}{2} + \varepsilon + (r - 2\varepsilon) f\big) < \frac{r}{2} - \varepsilon$. Obviously, $\phi - c + \frac{r}{2} - \varepsilon \in \mathcal{Q}$ and $\rho\big(\phi - c + \frac{r}{2} - \varepsilon, (r - 2\varepsilon) f\big) < \frac{r}{2} - \varepsilon$, so

$$\phi - c + \frac{r}{2} - \varepsilon \in B\left((r - 2\varepsilon) f, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q}.$$

Since $\frac{r}{2} - \varepsilon < r < 1$, we have

$$\sup_{t \in \mathbb{R}} \left\{ \left| \phi(t) - c + \frac{r}{2} - \varepsilon - (r - 2\varepsilon) f(t) \right| \right\} < \frac{r}{2} - \varepsilon.$$

Hence,

$$\frac{1}{r - 2\varepsilon} \left(\phi - c + \frac{r}{2} - \varepsilon\right) \in B\left(f, \frac{1}{2}\right).$$

Obviously,

$$\frac{1}{r - 2\varepsilon} \left(\phi - c + \frac{r}{2} - \varepsilon\right) \in \mathcal{Q},$$

which is impossible as from Lemma 2.4, $B\left(f, \frac{1}{2}\right) \cap \mathcal{Q} = \emptyset$. Thus,

$$B\left(h, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q} = \emptyset. \tag{1}$$

Moreover, using $f(\mathbb{R}) = [0,1]$, we have

$$B\left(h, \frac{r}{2} - \epsilon\right) \subset B(g, r). \tag{2}$$

By (1) and (2) we obtain:

$$B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g, r) \setminus \mathcal{Q}.$$

2. The function g is not constant.

   As $g \in Q$, there exists a point x such that g is continuous at x. Hence, we can find an interval (a_0, b_0) such that $x \in (a_0, b_0)$ and $\operatorname{diam}\big(g((a_0, b_0))\big) < 2\varepsilon$. Let

$$A_0 = \inf_{x \in (a_0, b_0)} \{g(x)\} \quad \text{and} \quad B_0 = \sup_{x \in (a_0, b_0)} \{g(x)\}.$$

Put

$$J = \left[\frac{A_0 + B_0}{2} - \frac{r}{2} + \varepsilon, \frac{A_0 + B_0}{2} + \frac{r}{2} - \varepsilon\right].$$

We have

$$\varepsilon < \frac{r}{2} - \varepsilon, \quad \text{so} \quad (A_0, B_0) \subset J.$$

Let $\{C_n\}_{n \in \mathbb{N}}$ be a sequence of the sets from Lemma 1.2 (for $P = \emptyset$ and $[a, b] = [a_0, b_0]$). For each $n \in \mathbb{N}$, there exists a function ϕ_n transforming C_n onto J. Put $C = \bigcup_{n=1}^{\infty} C_n$ and

$$h(x) = \begin{cases} g(x) & \text{for } x \in \mathbb{R} \setminus C, \\ \phi_n & \text{for } x \in C_n,\ n \in \mathbb{N}. \end{cases}$$

Clearly, h has the Baire property. Moreover, using $h(C) \subset J$ and $g(C) \subset (A_0, B_0)$, we obtain

$$\rho(h, g) < \frac{r}{2}, \tag{3}$$

so $B\left(h, \frac{r}{2} - \epsilon\right) \subset B(g, r)$.

What is more, $B\left(h, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q} = \emptyset$. Indeed, let $s \in B\left(h, \frac{r}{2} - \varepsilon\right)$. Suppose, contrary to our claim, that $s \in \mathcal{Q}$. Let $x \in (a_0, b_0)$ be such that $h(x) = \max J$. Then, $s(x) > \frac{A_0 + B_0}{2}$. The function s is quasi-continuous at x, so there exists a nonempty open set $G \subset (a_0, b_0)$ with $s(G) \subset \left(\frac{A_0 + B_0}{2}, \infty\right)$. By the construction of h and the properties of the sequence $\{C_n\}_{n \in \mathbb{N}}$, there exists $x' \in G$ such that $h(x') = \min J$. Therefore, $s(x') < \frac{A_0 + B_0}{2}$. It means, $s(x') \notin (\frac{A_0 + B_0}{2}, \infty)$, which gives a contradiction.

As s was chosen arbitrarily, we have $B\left(h, \frac{r}{2} - \varepsilon\right) \cap \mathcal{Q} = \emptyset$.

Finally, $B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g, r) \setminus \mathcal{Q}$.

Now, we prove the second part of the lemma.

If g is a Darboux function, then h constructed above is also Darboux. Indeed, let $(c_0, d_0) \subset \mathbb{R}$ be such that $h(c_0) \neq h(d_0)$. We will show that for any $Z \in (C_0, D_0)$, where

$$(C_0, D_0) = < h(c_0), h(d_0) >,$$

there exists a point $z \in (c_0, d_0)$ such that $h(z) = Z$. Fix $Z \in (C_0, D_0)$.

If $[c_0, d_0] \cap (a_0, b_0) = \emptyset$, then $h(x) = g(x)$ for $x \in [c_0, d_0]$. Since $g \in \mathcal{D}$, one can find a point $z \in (c_0, d_0)$ such that $Z = g(z) = h(z)$.

If $(c_0, d_0) \cap (a_0, b_0) \neq \emptyset$, then put $(s, t) = (c_0, d_0) \cap (a_0, b_0)$. There are two possibilities:

(i) $Z \in J$. Lemma 1.2 implies that there exists $k \in \mathbb{N}$ such that $C_k \subset (s, t)$. Since $h(C_k) = \phi_k(C_k) = J$, there exists a point $z \in C_k \subset (c_0, d_0)$ such that $h(z) = Z$.

(ii) $Z \notin J$. We consider only the case when $Z \in (\sup J, +\infty)$ and $D_0 = h(c_0)$ (in other cases, the proof runs similarly, so we can omit it). It is easy to see that $c_0 \notin (a_0, b_0)$, so $c_0 < a_0$.

If $d_0 \leq b_0$, then $(s, t) = (a_0, d_0)$. As C is of the first category, there exists a point $w \in (a_0, d_0) \setminus C$. Obviously, $h(w) = g(w) \in J$. As $h(c_0) = g(c_0)$, we have

$$\big[\sup J, h(c_0)\big) = \big[\sup J, g(c_0)\big) \subset \big(g(w), g(c_0)\big),$$

and $Z \in \big(g(w), g(c_0)\big)$. Since g is a Darboux function, there exists a point $z \in (c_0, w)$ such that $g(z) = Z$. We have $Z \notin J$ and $g((a_0, b_0)) \subset J$, so $z \notin (a_0, b_0)$. Thus, $h(z) = g(z) = Z$.

If $d_0 > b_0$, then $(C_0, D_0) = \big(h(d_0), h(c_0)\big) = \big(g(d_0), g(c_0)\big)$. Hence, $Z \in \big(g(d_0), g(c_0)\big)$. The function g is Darboux, so one can find a point $z \in (c_0, d_0)$ such that $g(z) = Z$. As $Z \notin J$ and $g((a_0, b_0)) \subset J$, $z \notin (a_0, b_0)$. Consequently, $h(z) = g(z) = Z$. □

Theorem 2.6. *The set $\mathcal{Q}$ is strongly porous in $(\mathcal{B}a, \rho)$.*

Proof. Let $r > 0$. Without lose of generality, we can assume that $r \in (0, 1)$. Fix $g \in \mathcal{Q}$. Lemma 2.5 implies that for any $\varepsilon \in \left(0, \frac{r}{4}\right)$ there exists $h \in \mathcal{B}a$ such that

$$B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g, r) \setminus \mathcal{Q}.$$

Thus,

$$\gamma(g, r, \mathcal{Q}) = \frac{r}{2}$$

and

$$p(\mathcal{Q}, g) = 2 \limsup_{r \to 0^+} \frac{\gamma(g, r, \mathcal{Q})}{r} = 1.$$

□

3. The classes of Darboux functions

Theorem 3.1. *The set $\mathcal{DQ}$ is strongly porous in $(\mathcal{DB}a, \rho)$.*

Proof. Let $r > 0$. Without lose of generality, we can assume that $r \in (0,1)$. Fix $g \in \mathcal{DQ}$. Lemma 2.5 implies that for any $\varepsilon \in (0, \frac{r}{4})$ there exists $h \in \mathcal{DB}a$ such that

$$B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g,r) \setminus \mathcal{Q}.$$

Obviously,

$$B\left(h, \frac{r}{2} - \varepsilon\right) \subset B(g,r) \setminus \mathcal{DQ},$$

so

$$\gamma(g,r,\mathcal{DQ}) = \frac{r}{2},$$

and

$$p(\mathcal{DQ}, g) = 2 \limsup_{r \to 0^+} \frac{\gamma(g,r,\mathcal{DQ})}{r} = 1.$$

□

From the last theorem, it follows that any subfamily of $\mathcal{DQ}$ is strongly porous in $\mathcal{DB}a$. In particular, we have this property for the classes of functions connected with some modifications of Darboux property (see [2], [3], [11]). As, according to the last theorem, $\mathcal{DQ}$ is strongly porous in $\mathcal{D}$ and each strongly porous set is nowhere dense, $\mathcal{DQ}$ is nowhere dense in $\mathcal{D}$.

In 1995, A. Maliszewski [11] investigated a class of functions with a property being some modification of Darboux property. He considered a family of functions with so-called strong Świątkowski property.

Definition 3.2 ([11])**.** A function $f : \mathbb{R} \to \mathbb{R}$ has the strong Świątkowski property if and only if for each interval $(a,b) \subset \mathbb{R}$ and for each $\lambda \in < f(a), f(b) >$ there exists a point $x_0 \in (a,b)$ such that $f(x_0) = \lambda$ and f is continuous at x_0.

From now on, we will use the symbol $\mathcal{D}_s$ to denote the class of functions with strong Świątkowski property (for more details on strong Świątkowski functions, see [11], [12], [14]). In [4], it is proved that

$$\mathcal{D}_s \subset \mathcal{DQ}.$$

Thus

Corollary 3.3. *The family $\mathcal{D}_s$ is strongly porous in $(\mathcal{DB}a, \rho)$.*

The next modification of Darboux property is $\mathcal{A}$-Darboux property. This notion was introduced in [4]. Let $\mathcal{A} \subset \mathcal{P}(\mathbb{R})$, where $\mathcal{P}(\mathbb{R})$ is the power set of $\mathbb{R}$.

Definition 3.4 ([4])**.** We will say that $f : \mathbb{R} \to \mathbb{R}$ is $\mathcal{A}$-continuous at a point $x \in \mathbb{R}$ if and only if for each open set $V \subset \mathbb{R}$ with $f(x) \in V$ there exists a set $A \in \mathcal{A}$ such that $x \in A$ and $f(A) \subset V$. We will say that $f : \mathbb{R} \to \mathbb{R}$ is $\mathcal{A}$-continuous if and only if f is $\mathcal{A}$-continuous at each point $x \in \mathbb{R}$.

Definition 3.5 ([4])**.** We will say that $f : \mathbb{R} \to \mathbb{R}$ has the $\mathcal{A}$-Darboux property if and only if for each interval $(a, b) \subset \mathbb{R}$ and each $\lambda \in < f(a), f(b) >$ there exists a point $x \in (a, b)$ such that $f(x) = \lambda$ and f is $\mathcal{A}$-continuous at x.

Let $\mathcal{D}_{\mathcal{A}}$ denote the family of all functions having the $\mathcal{A}$-Darboux property.

Let $A \subset \mathbb{R}$. The set A is of the first category at a point x if and only if there exists an open neighborhood G of x such that $A \cap G$ is of the first category (see [9]). $D(A)$ will denote the set of all points x such that A is not of the first category at x.

Definition 3.6 ([4])**.** We will say that the family $\mathcal{A} \subset \mathcal{P}(\mathbb{R})$ has (*)-property, if and only if

1. $\mathcal{A}$ contains the Euclidean topology,
2. each set $B \in \mathcal{A}$ has the Baire property,
3. $A \subset D(A)$ for each $A \in \mathcal{A}$.

It is not difficult to see that a wide class of topologies has (*)-property. For example, the Euclidean topology, $\mathcal{I}$-density topology, topologies constructed in [10] by E. Łazarow, R. A. Johnson, W. Wilczyński or the topology constructed by R. Wiertelak in [15]. Also, some families of sets which are not topologies have (*)-property, for example, the family of semi-open sets. On the other hand, the density topology does not have this property.

In [4], it is proved that if the family $\mathcal{A}$ has (*)-property, then

$$\mathcal{D}_{\mathcal{A}} \subset \mathcal{DQ}.$$

From this fact and Theorem 3.1, we obtain

Corollary 3.7. *The family $\mathcal{D}_{\mathcal{A}}$ is strongly porous in $(\mathcal{DB}a, \rho)$ for each family $\mathcal{A}$ having (*)-property.*

It should be added that A. Maliszewski in [11] proved that $\mathcal{D}_s$ is dense in $\mathcal{DQ}$. So, we have that $\mathcal{D}_s$ is not strongly porous in $\mathcal{DQ}$. Moreover, the family of all continuous functions $\mathcal{C}$ is strongly porous in $\mathcal{D}_s$ (see [1]). Taking into account these facts and Theorem 3.1, we have

Remark 3.8. $\mathcal{C} \subset \mathcal{D}_s \subset \mathcal{DQ} \subset \mathcal{DB}a$ and $\mathcal{C}$ is strongly porous in $\mathcal{D}_s$, $\mathcal{DQ}$ is strongly porous in $\mathcal{DB}a$, but $\mathcal{D}_s$ is not strongly porous in $\mathcal{DQ}$.

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