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STATISTICAL ASSESSMENT OF THE TYPOGRAM DATA OF THE PUPIN'S CLASSIFICATION OF ZIRCONS

(3 Figs., 5 Tabs.)

Abstract: The morphometrical data on zircons when processed according to Pupin's classification, yield significant data on parent rock history. The concept of TET (Trend of typological evolution) and TETM (Mean TET) are important parts of the Pupin system. This paper gives practical schemes for their calculation (including references to available computer programs written at the author's Laboratory) in a way that gives optimal results.

Резюме: Обработка морфометрических данных по методе классификации Пупина дает интересные данные о истории маточных пород. Концепции TET (тренд типологической зволюции) и TETM (средний TET) являются важной частью системы Pupina. В предлагаемой статье приводится практическая схема для их правильного вычисления. В этой схеме принимается во внимание дискретний характер морфометрических данных.

Key words: zircon, morphometry, Pupin's classification, computing.

The Pupin–Turco classification of the morphological parameters of zircons (Pupin, 1976) is a system that correlates the chemical and PT conditions of rock crystallization on the basis of the character of development of prismatic and pyramidal faces of zircon crystals separated from the investigated rocks. The history of development of this classification is briefly outlined by Matsuura–Aoki (1989).

The classification is performed by matching the morphology of the investigated zircon crystals with tabulated zircon forms. The basic classification scheme recognizes 64 main zircon types and tens of subtypes. To every zircon type field two (X-and Y-wise) numerical values ranging from 100 to 800 are ascribed. The rows are called Agpaicity indices (IA), the columns are called Temperature indices (IT). After completing the assessment of the number of zircon grains falling into the defined type fields, the numbers are summed up both row- and column-wise. These data are used for the calculation of the coordinates of the "Mean point" of the distribution of typological data. The mean point is calculated according to Pupin (1980) as:

$$\overline{IA} = \sum_{100}^{800} IA \, . \, n_{IA} \qquad (1)$$

$$\overline{IT} = \sum_{100}^{800} IT \, . \, n_{IT} \qquad (2)$$

where IA are the values of the Agpaicity index (columns) and IT are the values of the Temperature index (rows) (Tab. 1a). Here a field with higher IA value represents a more alcaline millieux, a field with higher IT values represents the zircons formed under higher temperature.

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Table 1a

Calculation of the mean point of zircon population shown in Fig. 1.

$\overline{IA}$	$\frac{1}{N}(\sum_{1}^{8} IA_i f_i) = 1/95\ (300 \times 8 + 400 \times 77 + 500 \times 10) = 402.1$
$\overline{IT}$	$\frac{1}{N}(\sum_{1}^{8} IT_i f_i) = 1/95\ (400 \times 20 + 500 \times 15 + 600 \times 58 + 7000 \times 2) = \quad = 468.4$
Note: Zero members are omitted.	

After completing the calculation of the mean point, the original Pupin technique proceeded with the calculation of the angle of the typological trend vector (TET1). For this calculation Pupin used the values of X- and Y-wise standard deviation of the typological distributions projected to X and Y axis. He used the formulae:

$$\mathrm{tg}\,\alpha = s_T/s_A \tag{3}$$

where α is the angle of the TET1. The s_T and s_A values are calculated here as:

$$s_T = ((A-\bar{A})^2/N)^{\frac{1}{2}} \tag{4a}$$

$$s_A = ((T-\bar{T})^2/N)^{\frac{1}{2}} \tag{4b}$$

where N is the number of observed zircon grains. This number should not be smaller than 100–150, but Pupin (1985) considers 50 grains as sufficient in case of "Fairly uniform populations with few different subtypes".

Since the mid eighties, Pupin started using exclusively the TET2 vector, evidently to avoid the problems in calculating s_T and s_A, though the TET1 has a definite merit in its simplicity of interpretation (Timčák, 1989) and in reflecting the properties of both (IA- and IT-wise) projected distributions.

The TET2 is calculated by row-wise assessing the IA mean values. For the example shown in Fig. 1, the $\overline{IT^*}$ calculation is shown in Tab. 1b.

Table 1b

Calculation of the IT-wise TET2

$\Sigma f_{100} = 0,\ \Sigma f_{200} = 0,\ \Sigma_{300} = 0,\ \Sigma f_{400} = 20,\ \Sigma f_{500} = 15$ $\Sigma f_{600} = 58,\ \Sigma f_{700} = 2$; values IT^*_{100} of $IT^*_{300} = 0$
$\overline{IT^*_{400}} = \frac{1}{20}(20\times400) = 400$ $\overline{IT^*_{500}} = \frac{1}{15}(300 + 14 \times 400) = 393.3$ $\overline{IT^*_{600}} = \frac{1}{58}(7 \times 300 + 41 \times 400 + 10 \times 500) = 405.2$ $\overline{IT^*_{700}} = \frac{1}{2}(2 \times 400) = 400$
Note: In our case TET2 is nearly linear as the given population is narrowly spread and the typological fields with maximal frequency are located in the same column (IA = 400). In case of a wider population spread the TET2 consists of connected line segments of different orientation.

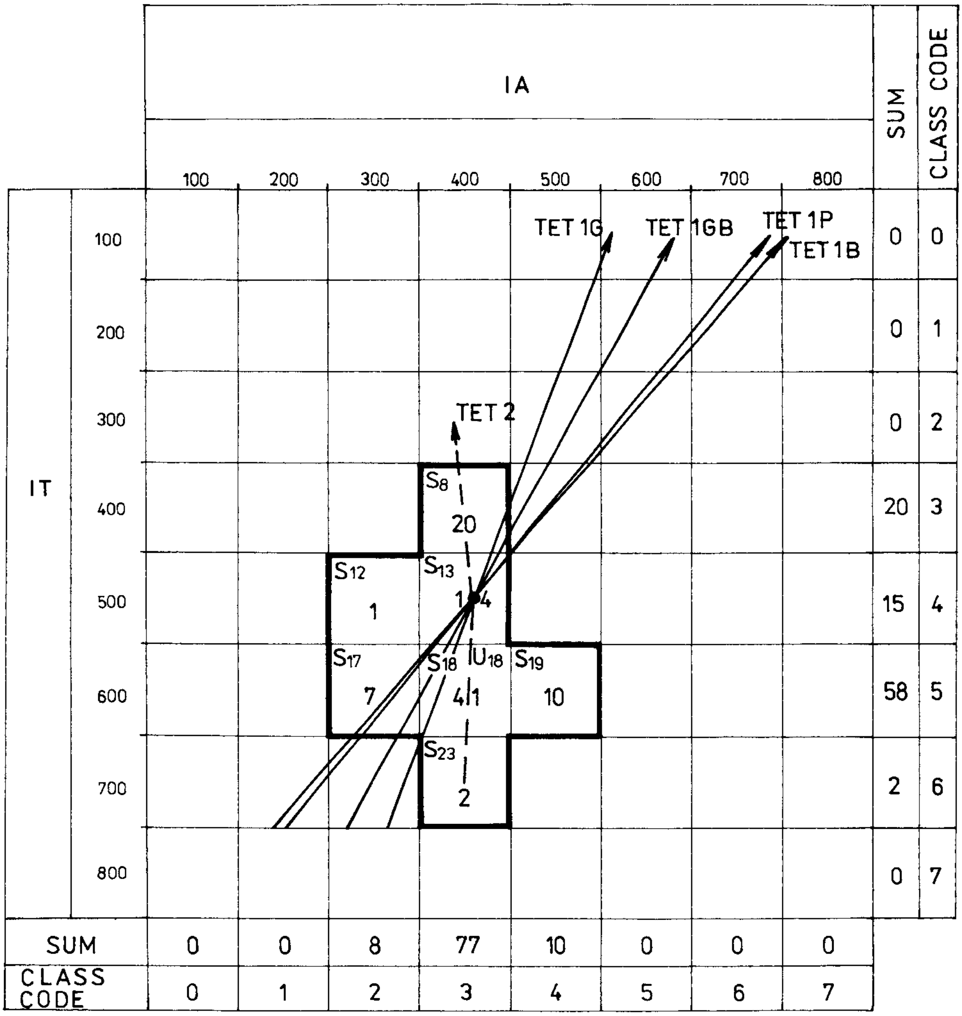

Fig. 1. A Pupin typogram for a smaller zircon population.
The frequency data, coded class numbers, mean point and TET1 vectors are shown. It can be seen that the TET1 vectors calculated assuming a Gaussian (TET1G), binomial (TET1B) and Poisson (TET1P) projected IA-IT distribution are different. From Tabs. 3b., 4b., it follows that TET1B characterizes the evolution trend better than the TET1P. The optimum result is obtained here, however if for IA data a Gaussian model is used and for IT a binomial one (TET1GB).

If one works with more data sets, two types of mean TET can be calculated (TETM1, TETM2). The TETM1 is calculated from the values of mean points ($\overline{IA}$, $\overline{IT}$) and of α's for a given set of data (say 25 zircon populations from one district). The calculation procedure is shown in Tab. 2a. Thus

$$\overline{IA}_{TETM1} = (\Sigma\ \overline{IA}_n)/n \qquad (5a)$$

$$\overline{IT}_{TETM1} = (\Sigma\ \overline{IT}_n)/n \qquad (5b)$$

$$tg\ \alpha_{TETM1} = (\overset{n}{\Sigma}\ \alpha_n^{TET})/n \qquad (6)$$

where n is the number of TETs and α^{TET} are the TET1 angles of the studied zircon populations. The starting and ending point levels of the vector is always given by the lowest and highest nonzero typological field.

The calculation of the TETM2 coordinates involves the calculation of TETM for every row (Fig. 2). The coordinate of the i-th level of the TETM2 polygon is obtained as (Σ TET2 values of ith row/number of TET2 values in that row). The IT-wise coordinate of the TETM2 points is at the middle of the given IT class. Tab. 2c gives an example of the calculation procedure. Pupin, in his earlier papers used also a third type of TETM, obtained by fitting a curve to the scattergram of mean points from the investigated region.

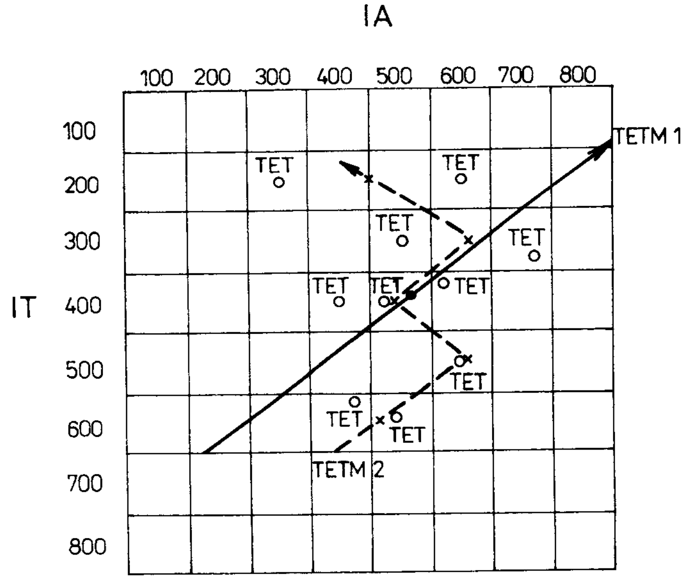

Fig. 2. Example of TETM1 and TETM2 calculations. In this case the mean points (TET) of a hypothetical population of specimens were first considered to be the TET1 mean points and subsequently to be TET2 mean points, as it was required by the calculation procedure, depending on whether the TETM1 or TETM2 was calculated. In case of real populations these points would not be identical.

The TET as well as the TETM vectors indicate the evolution trend of zircons, i.e. the change of the chemistry of the magmas as well as of the temperature and pressure conditions with time. The time "axis" is formed by the directional component latent in the scattergram (cf. Timčák, 1989). The magnitude of angle α thus indicates the speed of cooling of the melt. The greater the α, the greater is – according to this hypothesis – the cooling rate. A high cooling rate thus results also in a low IA standard deviation value (narrow projected distribution).

The practical value of the various types of TET and TETM is demonstrated by the papers of Pupin (1976, 1980, 1985 etc.) and others (eg. Caironi, 1985).

Table 2a

Calculation of variance and α for a binomial type of distribution

	$\overline{IA}_k = (\sum_1^n f_i K_i)/N = 3.0211$ where $K_i = 0.1, \ldots, 7$ $N = 95$ $n = 5$
$\overline{IA}$	$\overline{IA}_k = np$; thus $p = \overline{IA}_k/n$ $p = 3.0211/5 = 0.6042$ $q = 1 - p = 0.3958$
	$S^2_{\overline{IA}} = npq = 5 \times 0.6042 \times 0.3958 = 1.196$ $S_{\overline{IA}} = 1.094$
	$\overline{IT}_k = (\sum_1^n f_i K_i)/N$ where $N = 95$ $= 4.4421$ $n = 7$
$\overline{IT}$	$\overline{IT}_k = np$; from this we calculate p and thus $p = 4.4421/7 = 0.6346$ $q = 0.3654$
	$S^2_{\overline{IT}} = npq = 7 \times 0.634 \times 0.3654 = 1.6232$ $S_{\overline{IT}} = 1.274$
α	$tg\alpha = S_{\overline{IT}}/S_{\overline{IA}} = 1.274/1.094 = 1.165$ $\alpha = 49°21'$

Before the calculation, it is more advantageous, however, to rewrite the equations given by Pupin so that

$$\overline{IA} = (\sum_1^8 IA_i f_i)/N \tag{7a}$$

where i = 1 to 8 and N is the total number of analysed zircon grains in the investigated specimens; should one use percentual data, N = 100. IA_i is the value for the midrange (i.e. 100 to 800) and f_i is the sum of grain numbers falling into a given column. By analogy

$$\overline{IT} = (\sum_1^8 IT_i f_i)/N \tag{7b}$$

where f_i is the sum of grain numbers falling into a given row. Tab. 1a contains an example of such calculation.

Should the distribution be continuous and Gaussian, the calculation of the standard deviation (s_T, s_A) would have to be done according the the formulae shown below:

$$S_T = ((\sum_1^8 f_i\,(IT_i - \overline{IT})^2)/N)^{\frac{1}{2}} \tag{8a}$$

$$s_A = ((\sum_1^8 f_i\,(IA_i - \overline{IA})^2)/N)^{\frac{1}{2}} \tag{8b}$$

Table 2b

Example of TETM1 and $\bar{\bar{\alpha}}$ calculation (cf. Fig. 2)

	TET1 coordinates		TETM1 coordinates	
N	$\overline{IA}$	$\overline{IT}$	$\overline{\overline{IA}}$	$\overline{\overline{IT}}$
1	300	200	$\overline{\overline{IA}} = 5105/10 = 510.5$	$\overline{\overline{IT}} = 3860/10 = 386$
2	600	200		
3	500	300		
4	730	330		
5	400	400		
6	480	400		
7	570	370		
8	600	500		
9	430	570		
10	495	590		
Sum	5105	3860		

N	α_{TET1} values (°)	$\bar{\bar{\alpha}}_{TETM1}$
1	29	$\bar{\bar{\alpha}} = \frac{\Sigma\alpha_i}{N} = 37°06'$
2	30	
3	35	
4	43	
5	50	
6	23	
7	32	
8	41	
9	51	
10	37	
Sum	371	

Note: The α_{TET1} as well as the $\overline{IA}$ and $\overline{IT}$ values were taken from 10 typograms chosen for this demonstration.

Table 2c

Calculation of the TETM2 coordinates (cf. Fig. 2)

$\overline{\overline{IT}} = 200$	$\overline{\overline{IA}} = \frac{300 + 600}{2} = 450$
$\overline{\overline{IT}} = 300$	$\overline{\overline{IA}} = \frac{500 + 730}{2} = 615$
$\overline{\overline{IT}} = 400$	$\overline{\overline{IA}} = \frac{400 + 480 + 570}{3} = 483$
$\overline{\overline{IT}} = 500$	$\overline{\overline{IA}} = \frac{600}{1} = 600$
$\overline{\overline{IT}} = 600$	$\overline{\overline{IA}} = \frac{430 + 495}{2} = 462.5$

Note: Fig. 2 shows also the mean points of TET1s. The differences of IT coordinates of these mean points against mid-range points were not taken into consideration (they are meaningful only for TETM1 calculation).

where N is the total number of analysed grains, f_i is the number of grains in the given row (for s_T) or column (for s_A), the IT_i as well as IA_i vary from 100 to 800 (step 100) and the $\overline{IT}$ and $\overline{IA}$ values are the ones calculated according to Eqs. 7a and 7b.

The number of grains falling into the typogram fields are, however, discrete values and thus are not Gaussian, even though in case of symmetrical distributions, where the first 3 moments satisfy the conditions of normality, or when the rank correlation between the model and experimental distribution is sufficiently high, the angle α calculated from s_T and s_A obtained by formulae (8a, b) does not show a significant deviation from the α calculated for an appropriate discrete distribution (for the typogram shown in Fig. 1 the α_{Gauss} is 53°1′ to 66°42′ according to whether the projected IT distribution is deconvoluted or not (D á v i d, manuscript, 1988, cf; Tab. 2a and 4a).

In a significant proportion of cases, the distribution is too narrow or assymetric ($\beta_1 \neq 0$) or has an excess $\beta_2 \neq 3$, thus the above mentioned approximation is not applicable. Then the use of binomial, negative binomial, Poisson distribution or some other discrete distribution type is necessary.

The courses of the original and model distributions are shown in Fig. 3.

In case of the use of a binomial model, the variance (s^2) is calculated as it is shown in Tab. 2a.

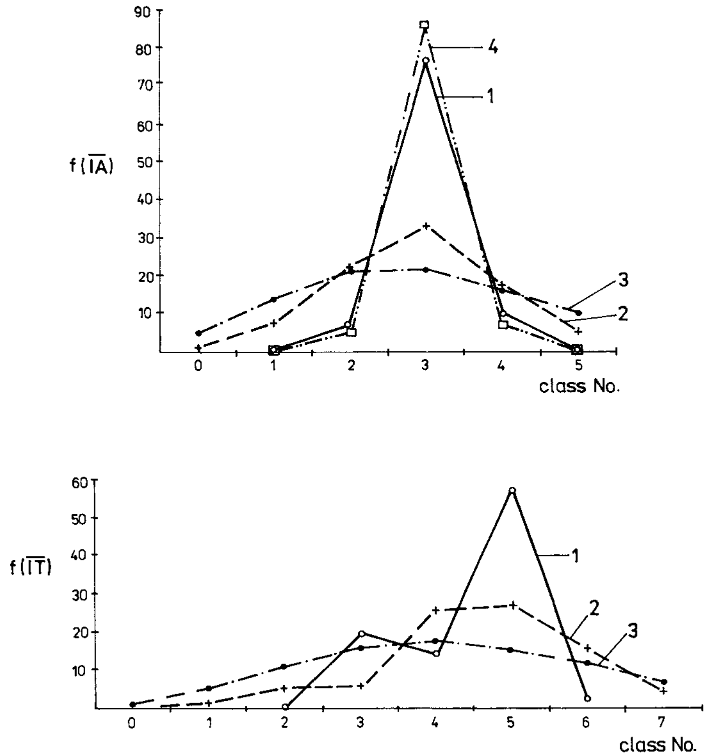

Fig. 3. Projected IA and IT value frequency distributions for data from Fig. 1 shown together with the model distributions.
1 – original distribution, 2 – equivalent binomial distribution, 3 – equivalent Poisson distribution, 4 – equivalent Normal distribution. Coded class numbers are shown on the X-axis.

Table 3a

Calculation of the original and equivalent binomial distribution

	X	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	Expected frequency for IA and IT $f(x) = Np(x)$
$\overline{IA}$	0 1 2 3 4	5 2 1 0.33 0.2	7.633 3.053 1.5265 0.5088 0.3053	$f(0) = Np(0) = 0.9228 \doteq 1$ $F(1) = 0.9228 \times 7.633 = 7.044 \doteq 7$ $f(2) = 7.044 \times 3.053 = 21.505 \doteq 22$ $f(3) = 32.83 \doteq 33$ $f(4) = 16.705 \doteq 17$ $f(5) = 5.1 \doteq 5$
$\overline{IT}$	0 1 2 3 4 5 6	7 3 1.7 1 0.6 0.33 0.14	12.157 5.21 2.895 1.737 1.04 0.58 0.248	$f(0) = Np(0) = 0.0826 \doteq 0$ $f(1) = 0.0826 \times 12.157 \doteq 1$ $f(2) = 5.2 \doteq 5$ $f(3) = 15.15 \doteq 15$ $f(4) = 26.3 \doteq 26$ $f(5) = 27.36 \doteq 27$ $f(6) = 15.87 \doteq 16$ $f(7) = 3.9 \doteq 4$
Auxiliary calculations:				
$\overline{IA}$	$f(0) = Nq^n = 95 \times q^5 = 95 \times 0.00974 = 0.9228$ where $N = 95$ a $n = 5$ $p/q = 1.5265$ (see Tab. 2a)			
$\overline{IT}$	$f(0) = Nq^n = 95q^7 = 0.0826$ where $N = 95$ a $n = 7$ $p/q = 1.5265$ (see Tab. 2a)			

The class values should, however, be changed from (100 to 800) to (0,1... to 7). The distribution is considered to terminate where the first zero frequency value occurs on its high tail.

The basic equations for the calculation of $\overline{IA}$ and $S^2_{\overline{IA}}$ are:

$$\overline{IA} = np \tag{10a}$$

$$S^2_{\overline{IA}} = npq \tag{10b}$$

where n is the number of classes (in our case for $\overline{IA}$ $n = 5$, for $\overline{IT}$ $n = 7$), p is the probability of occurrence of zircon grains; $q = 1\text{-}p$. As the data for the determination of $\overline{IA}$ are known, we can use Eq. 7a for the calculation, but instead of IA_i we should use the coded midpoint values ($K_i = 0,1,2,..,n$). The calculation of α is done as before (Tab. 2a). The correlation of the experimental and model distribution can be evaluated by rank correlation. For this, we have to calculate the expected frequency values (f_i) of the equivalent binomial distribution, for an identical N and for the calculated p (Tab. 2).

The whole procedure is illustrated in Tab. 3a-b. Tab. 3b presents the calculation of the rank correlation. In case of repeated ranks a correction is necessary (cf. Gupta–Kapoor, 1977). Thus if there are more identical f_i values, we ascribe to them an identical, mean f_i value. Thus if there are 3 equal f_i values and their order is 4, 5, 6, the mean $f_i = 5$. The next rank would be, however 7.

Table 3b

Rank correlation of the results shown in Tab. 3. a (Binomial distribution)

$\overline{IA}$

class	f_1	f_2	RC1	RC2	d	d^2	
0	0	1	5	5	0	0	
1	0	7	5	4	1	1	n = 5
2	8	22	3	2	1	1	RC1, RC2 — rank
3	77	33	1	1	0	0	coefficient
4	10	17	2	3	−1	1	
					Sum	3	

$\overline{IT}$

class	f_1	f_2	RC1	RC2	d	d^2	
0	0	0	6	7	−1	1	
1	0	1	6	6	0	0	
2	0	5	6	5	1	1	
3	20	15	2	4	−2	4	n = 7
4	15	26	3	2	1	1	
5	58	27	1	1	0	0	
6	2	16	4	3	1	1	
					Sum	8	

$r_{IA} = 1 - 6(\sum_{1}^{n} d_i^2 + m')/n\,(n^2 - 1)$

$= 0.825$

$m = 2$
$m' = m(m^2 - 1)/12$
$= 0.5$

$r_{IT} = 1 - 6\,(8 + 2)/(7(49 - 1))$

$= 0.821$

$m = 3$
$m' = 3\,(9 - 1)/12$
$= 2$

Should the rank correlation coefficient have a value lower than 0.7, the Poisson model should be tested. Here

$$\overline{IA} = \lambda \tag{11a}$$

$$\frac{s^2}{\overline{IA}} = \lambda \tag{11b}$$

where λ is calculated according to the scheme given in Tab. 4a. The calculation of the Poisson distribution equivalent to the experimental one is shown in Tab. 4b. A test of goodness of fit is given in Tab. 4c. As it is shown in Fig. 3, for our data the binomial distribution appears to be a better equivalent.

In case the distribution modus is markedly shifted to the left, it is possible to model the distribution by a negative binomial one (Tab. 5a-d). The goodness of fit can be tested eg. by rank correlation. Note that Tab. 5 contains input data different from the previous ones.

Table 4a

Calculations of the equivalent Poisson distribution and of α

	X	$\frac{\lambda}{x+1}$	p(x)	Expected Np(x)	frequency N=95
$\overline{\text{IA}}$	0	3.021	0.0488	$4.6 \doteq 5$	
	1	1.5105	0.1474	$13.99 \doteq 14$	
	2	1.007	0.222705	$21.15 \doteq 21$	
	3	0.7553	0.22426	$21.3 \doteq 21$	
	4	0.6042	0.1694	$16.09 \doteq 16$	
			0.10235	$9.5 \doteq 10$	
$\overline{\text{IT}}$	0	4.442	0.01177	$1.1 \doteq 1$	
	1	2.221	0.0523	$4.95 \doteq 5$	
	2	1.4807	0.1161	$11.03 \doteq 11$	
	3	1.1105	0.172	$16.331 \doteq 16$	
	4	0.8884	0.191	$18.1 \doteq 18$	
	5	0.74	0.1697	$16.12 \doteq 16$	
	6	0.635	0.126	$11.97 \doteq 12$	
			0.079	$7.6 \doteq 8$	

$\bar{X}_{IA} = (\Sigma f_i x_i)/N = 3.021$ $p(x) = (e^{-\lambda}\lambda^x)/x!$ $p(x+1) = (\lambda/(x+1))p(x)$	$\lambda = 3.021$ $n = 5$ $p(0) = e^{-\lambda} = 0.0488$ $p(1) = 3.021 \times 0.0488 = 0.1474$
$\bar{X}_{IT} = 4.442$ $p(0) = 0.01177$ $p(1) = 4.442 \times 0.01177 = 0.0523$	$\lambda = 4.442$ $n = 7$

Calculation of α for the original distribution assuming that it is of Poisson type:

$S_{\overline{IT}} = (\lambda_{IT})^{0.5}$ $\qquad$ $S_{\overline{IA}} = (\lambda_{IA})^{0.5}$

$\text{tg}\ \alpha = S_{\overline{IT}} / S_{\overline{IA}}\ (\lambda_{IA})^{0.5} = 2.108/1.738 = 1.2126$

$\alpha = 50°29'$

Table 4b

Rank correlation of the original and equivalent Poisson distribution

	class	f_1	f_2	RC1	RC2	d	d^2		
IA	0	0	5	5	6	−1	1		n = 6
	1	0	14	5	4	1	1		
	2	8	21	3	1.5	1.5	2.25		
	3	77	21	1	1.5	−0.5	0.25		
	4	10	16	2	3	−1	1		
	5	0	10	5	5	0	0	Sum	5.5
IT	0	0	1	6	8	−2	4		n = 7
	1	0	5	6	7	−1	1		
	2	0	11	6	5	1	1		
	3	20	16	2	2.5	−0.5	0.25		
	4	15	18	3	1	2	4		
	5	58	16	1	2.5	−1.5	2.25		
	6	2	12	4	4	0	0	Sum	12.5
	$r_{IA} = 1 - 6(5.5 + 2 + 0.5)/(6(36 - 1))$ $=$							$m_1 = 3$ $m_1' = 2$	$m_2 = 2$ $m_2' = 0.5$
	$r_{IT} = 0.738$							$m_1 = 3$ $m_1' = 2$	$m_2 = 2$ $m_2' = 0.5$

Table 5a

An example of projected IA and IT distributions that can be modelled by negative binomial distribution

IA	100	200	300	400	500	600	700	800	Sum
K_i	0	1	2	3	4	5	6	7	
f_i	70	15	10	1	4	0	0	0	100

Note: K_i are the coded midranges, f_i are the numbers of grains in the columns of the typogram

IT	100	200	300	400	500	600	700	800	Sum
K_i	0	1	2	3	4	5	6	7	
f_i	0	70	10	10	6	4	0	0	100

Table 5b–1

Calculation of variance and the parameters of the negative binomial distribution for IA values taken from Tab. 5a

First moment (mean)	$\mu'_1 = \Sigma f_i K_i / \Sigma f_i = 54/100 = 0.54$ Further $\mu'_1 = rq/p$ where $q = 1 - p$
Second moment around the origin	$\mu'_2 = (\Sigma f_i (z_i + \mu'_1)^2)/N$ where $z_i = x_i - \bar{x}$ and $\bar{x} = A + \mu'_1$ where in our case $A = 0$ $\mu'_2 = (\Sigma f_i K_i^2)/N = 128/100 = 1.28$
Second moment around the mean (variance)	$\mu_2 = \mu'_2 - \mu_1'^2 = 1.28 - 0.292 = 0.988$ Further $\mu_2 = rq/p^2$
Probability p	$(rq/p) \times (1/p) = \mu'_1/p$ thus $p = \mu_1/\mu_2 = 0.5/0.988 = 0.546$
Probability q	$q = 1 - p = 0.4537$
Value of r	$r = p\,\mu'_1/q$ $r = 0.6503$

It may happen that the distribution of IA and IT is different, thus eg. IA may follow a binomial and IT a negative binomial distribution. In this case α should be calculated from the standard deviations calculated for those model data.

Sometimes, from the 8 IA or IT classes, only 3 or less contain nonzero values. This seems to happen most often in cases where the number of analysed grains is low, though it may also reflect real crystallization phenomena. In these cases the reliability of the TET1 is lower and it is better to use the TET2. In case the zircons are grouped into one row or column, the standard deviation has a zero value for IT or IA projected distribution. The α is 0° or 90° in these cases.

To have a truly reliable s_A and/or s_T value (ie. a robust α and TET1) it is necessary for the projected IA and IT distributions to have at least 5 nonzero adjacent classes each. In case that

Table 5b–2

Calculation of variance, standard deviation, and the parameters of the equivalent negative binomial distribution for IT values given in Tab. 5a and the calculation of α for IA and IT from Tab. 5a under the same assumption

First moment (mean)	$\mu_1' = 44/100 = 0.44$ Further $\mu_1' = rq/p$
Second moment around the origin	$\mu_2' = (\Sigma f_i K_i^2)/N = 168/100 = 1.68$
Second moment around the mean (variance)	$\mu_2 = \mu_2' - \mu_1'^2 = 1.68 - 0.19 = 1.49$
Probability p	$p = 0.44/1.49 = 0.295$
Probability q	$q = 0.705$
Value of r	$r = 0.295 \times 0.44/0.705 = 0.184$
Angle α for data from Tab. 5a (μ_2 values were taken from this and the preceding Table)	$tg\alpha = s_{IT}/s_{IA}$ $s_{IA} = (\mu_2^{IA})^{0.5} = 0.994$ $s_{IT} = (\mu_2^{IT})^{0.5} = 1.22$ $tg\alpha = 1.22/0.994 = 1.228$ $\alpha = 50°50'$

Table 5c

Calculation of the frequency values of the equivalent negative binomial distribution for the projected IA frequency values (the calculation for the IT values is analogous)

f_i	$\frac{(r + (i - 1))\, qf}{(i - 1) + 1}\,(i - 1)$	$(\Sigma f_i)\, f_i$	Rounded frequency values of the equivalent distribution
f_0	$p^r = 0.675$	67.5	68
f_1	0.199	19.9	20
f_2	0.029	2.9	3
f_3	0.012	1.2	1
f_4	0.05	0.5	1

Note: (1) The expression in the 2nd column appears to be complicated, so note that for the 2nd row it is $((r + 0)/1) \times qf_0$; Σf_i is 100 in this case.

Table 5d

Rank correlation of the projected IA and equivalent negative binomial distribution

class	X1	X2	RC1	RC2	d	d^2
0	70	68	1	1	0	0
1	15	20	2	2	0	0
2	10	3	3	3	0	0
3	1	1	5	4	1	1
4	4	1	4	5	−1	1
$r = 1 - ((6\Sigma d_i^2)/(n(n^2 - 1))$ $r = 1 - 0.000012 = 0.999$					Sum	2

the first analysed set of grains does not yield such distribution, the analysis of additional sets of grains may bring the desired result. Pupin (1985) states that unaltered zircons belonging to different size classes may be used for complementary measurements, without causing a difference in $\overline{IA}$ and $\overline{IT}$ greater than the half width of a class. If the increment of the analysed grain number still does not change the width of the projected IA and IT distribution, the TET2 or 3TET (Timčák, 1989) may be used instead of TET1.

As it was already hinted at, the third (μ_3) and fourth (μ_4) moments around the mean may be used to characterize the closeness of the projected IA and IT distribution to the course of a Gaussian or other distribution. They can also be used as measures of similarity of various zircon populations. These moments can be used also for a one-number characterization of the chemical and PT conditions of the crystallization of the most significant part of zircon grains. For a Normal type of distribution

$$\mu_r = (1/N \sum_{100}^{800} f_i \, (IA_i - \overline{IA})^r \qquad (12)$$

where $N = \sum_{100}^{800} f_i$ and f_i is the number of

zircon grains falling into the individual classes, r is the order of the moment. The measure of skewness $\beta_1 = \mu_3^2/\mu_2^3$. The measure of kurtosis $\beta_2 = \mu_4/\mu_2^2$. (13) (14)

For a binomial distribution the

$$\mu_3 = npq\ (q-p) \qquad (15)$$

and the measure of skewness is

$$\beta_1 = (1-2p)^2/npq \qquad (16)$$

$$\mu_4 = npq\ (1+3\ (n-2)\ pq) \qquad (17)$$

and the measure of kurtosis is

$$\beta_2 = (3+(1-6pq))/npq. \qquad (18)$$

As it was mentioned above, for a symmetrical distribution $\beta_1 = 0$, for a Normal distribution $\beta_2 = 3$. If the modus of the distribution is shifted towards the origin, β_1 is positive, if it is shifted to the opposite direction, β_1 is negative. If the distribution is flatter than the normal (ie. is platycurtic), $\beta_2 < 3$, if its peakedness is greater than that of the Normal distribution (ie. is leptocurtic), $\beta_2 > 3$.

Thus if for a projected IA distribution $\beta_1 > 0$, the majority of zircons have crystallized under relatively acid conditions. If $\beta_1 < 0$ for the projected IT distribution, the crystallization of the majority of zircon grains had occurred under higher PT conditions. A $\beta_2 << 3$ for the projected IA distribution indicates a quick cooling rate and a $\beta_2 > 3$ indicates a slow cooling rate. A $\beta_2 < 3$ for the projected IT distribution indicates considerable changes in the PT conditions during crystallization and a $\beta_2 >> 3$ indicates relatively constant PT conditions of cooling. The β_1 and β_2 coefficients may be used also for simple numerical assessing of the similarity of the sets of typograms.

For an easy calculation of the mean point coordinates, α, TET2 and 3TET points and of the equivalent model distributions, the PUP1 and PUP3 computer programs were developed.

Translated by I. Timčáková

REFERENCES

CAIRONI, V., 1985: Characterization of different granitic facies in the Baveno-Mottarone pluton by means of the typologic study of zircon populations. *Rc. Soc. Ital. Mineral. Petrologia Soc. mineral. ital.*, 40, pp. 341–352.

GUPTA, S. C. – KAPOOR, V. K., 1977: Fundamentals of mathematical statistics, Sultan Chand, N. Delhi.

MATSUURA, A. – AOKI, Y., 1989: A new method for quantitative representation of zircon morphology. *Neu. Jb. Mineral.,* Mh. (Stuttgart), 7, pp. 309–319.

PUPIN, J. P., 1976: Signification des caractères morphologiques du zircon commun des roches en petrologie. *Thése des Dr. Sc.,* Univ. Nancy.

PUPIN, J. P., 1980: Zircon and granite petrology. *Contr. Mineral. Petrology* (Berlin–New York), 73, pp. 207–220.

PUPIN, J. P., 1985: Magmatic zoning of Hercynian granitoides in France based on zircon typology. *Schweiz. mineral. petrogr. Mitt.* (Zürich), 65, pp. 29–56.

ROZLOŽNÍK, L. et al., 1982: Studies of zircons and biotites in Štiavnica granitoids Pt. 1: *Zircons. Proc. Symp. on geochemistry of endogenous and exogenous processes. Geol. Inst. of Slov. Acad. Sci.*, Bratislava, pp. 52–78.

TIMČÁK, G. et al., 1985: Numerical assessing of geochemical and geological data. VÚ ŠPZV II-4-6/8, Final. Rept., KGaM BF VŠT Košice (in Slovak).

TIMČÁK, G. M., 1989: Typological evolution trend vectors of the Pupin classification of zircons. *European Jour. of Miner.* (in print).

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