The Puzzle of Financial Market Distribution

Bohumil STÁDNÍK

Abstract

In this research we economically explain the observed shape of financial market distributions as this question has still not been fully answered. We suggest the explanation using market price directional dependence which can be also used as an additional one to the nowadays popular volatility dependence solutions. Volatility dependence is based on volatility clustering and does not cover observations of the departures without volatility clusters behind but in this research we also do explain such cases. The whole methodology is based on the financial system internal description therefore we eliminate the internal structure uncertainty which results from just the output/input system description. We try to identify the processes containing the direction dependence within universal model, discuss their contribution to the measured shape of the distributions, make their complex simulation based on the theory of dynamical systems, try to measure them empirically and outline appropriate mathematical description based on Markov chains.

Keywords: Dynamic Financial Market Model, departures from normality, leptokurtic distribution of returns, sharpness, skewness, feedbacks on financial market, price inertia feedback, directional dependence, Markov chain

JEL Classification: G10, G11, G12

1. Introduction

The main contribution of this study is to solve one of the core financial questions: “How can we actually economically explain measured shape of distributions of financial market returns?” Such a question has been originally formulated...
before 1965, when two possible alternative mathematical explanations were initially suggested — mixture of several normal distributions and non-stationarity of the normal distribution (Fama, 1965). Both explanations had problems with an economic interpretation. Other explanations have been proposed later but the question has still not been fully answered.

In this study we suggest an explanation using market price directional dependence which can be used as an additional solution to the nowadays popular volatility dependence models. Volatility dependence is based on volatility clustering and does not cover observations of the departures without volatility clusters behind (Figure 15 see in the Appendix C, <http://pozemstan.cz/Appendix_A_B_C.pdf>). In this research we also do provide explanation of such cases (Stádník, 2014 b).

As we observe non-normality almost in each time interval and regardless if it is a stock, currency or for example a bond future, we expect also general processes using the direction dependence which cause these departures and which are common for most of the liquid markets. In the study we try to identify these processes within a universal model; make their division into general groups; describe the mechanism of their actions and their contribution to the departed distribution. We also try to measure them empirically, make the simulation of the realistic probability distribution and finally outline appropriate mathematical description of the entire system (Appendix A, see <http://pozemstan.cz/Appendix_A_B_C.pdf>).

There is a lot of empirical evidence that in case of a price development of many liquid investment instruments we observe a not normal (not-Gaussian) returns’ probability distribution (S&P500, Euro Bund Future daily returns’ probability distribution, Figure 1). Such a distribution exhibits leptokurtic feature (characterized by fat tails at the borders and sharpness in the central area), extreme values and also skewness. There were many works performed in this area (among many others: Fama, 1965, p. 97: “Gaussian or normal distribution does not seem to be an adequate representation of distributions of stock price changes”; Peiro, 1999; Ane and Geman, 2000; market wide skewness measurements by Chang, Christoffersen and Jacobs, 2010, etc.). There were proposed some other distributions that can better describe departures from normality. Fama (1966) proposed symmetric stable distribution, Blattberg and Gonedes (1974) Student-t distribution.

When the distribution is not of a normal type, it means the process behind the market price development is not an independent random process (independent random walk) and we have also reason to expect some price volatility or direction development rule being active inside the financial market.
The serious question is then: “How to model measured distribution with its departures from normality and a realistic economic interpretation?” As the market price development is evidently a sequence of steps (minimum size of each step is given by certain market rule), we have basically two ways how to approach it. The first way is to assume price volatility dependence; the second way is to assume price direction dependence. Price volatility dependence is closely connected to the size of price steps in the given time period; the direction dependence is connected to the probability of the direction of future price steps.

The first way is currently used by a wide range of models. For example Buckley, Saunders and Seco (2008) has used in his work the Gaussian mixture distribution. Gaussian mixture has an acceptable interpretation: financial market performs in two regimes with high and low volatility. Gaussian mixture can model many departed distributions which depend on the probability of both regimes and their parameters. If the latent regimes have a Markov law of motion, the mixture is then a hidden Markov model (Baum and Petrie, 1966), which is also known as the Markov regime switching model. There are many extensions of Markov switching model (Krolzig, 1997; etc.) Other famous works in this area were done by Bollerslev (1986) GARCH process; Engle (1995), ARCH process. Other important works in this are Boguñáa and Masoliver (2004); Palatella et al. (2004); Onnela, Kaski and Kertész (2004); Diviš and Teplý (2005); Cortinesa, Rierab and Anteneodoc (2007); Gontis, Ruseckas and Kononovičius (2010); Jianga, Lia and Caia (2008). While GARCH, ARCH and other stochastic volatility models propose statistical constructions based on volatility clustering in financial time series, they do not provide any economic explanation. The economic explanation of volatility clustering is difficult. The initial idea was the competition between more trading strategies but the simulation does not allow confirming mechanism being responsible for volatility clustering (Cont, 2005). Some economic works contain examples where switching of economic agents between two behavioral patterns leads to large volatility. Volatility clustering should arise from switching of market participants between fundamentalist and chartist behavior (Lux and Marchesi, 2000). Fundamentalists expect that the price follows the fundamental value in the long-run. Traders using technical analysis try to identify price trends or other patterns. Agents are allowed to switch between these two behaviors according to the performance of the various strategies. Chart traders evaluate their investments using historical development, whereas fundamentalists evaluate their investment opportunity according to the difference between the market price and the fundamental valuation. According to the Lux-Marchesi model the market price development follows Gaussian random walk till the moment when some chart traders using certain techniques surpass
a certain threshold value. At this moment a volatility outbreak occurs. According to Cont (2005), the origin of volatility clustering can be also caused by threshold response of investors to news arrivals.

Diebold and Lopez (1995), focus on the conditional behavior of the tails. Many of the authors focused on the left tail of the returns’ probability distribution as the left tail is usually heavier than the right one. There are two basic explanations for this effect. The first is, stock markets perform bubbles and if the bubble bursts a strong downward movement appears. There is also a second explanation (Campbell and Hentschel, 1992) using clustering of news. Different left and right tails were confirmed by Jondeau and Rockinger (2002) in “Testing for differences in the tails of stock-market returns”. Some new research in the area of volatility dependence was done by Witzany (2013), Roch (2011) or Stádník (2014b).

The second way how to explain departures from normality is to consider price development direction dependence on the past. Modeling of departures from normality in this way is not so frequent. One rationale behind the studies of directional dependence is that economic patterns may recur in the future. Also commonly used technical trading rules are based on a market price direction forecasting according to the past. We can consider Technical Analysis to be the prediction tool, but its benefit is still under discussion. Some works indicate that several technical indicators do provide a little forecasting improvement and may have some practical value (Lo, Mamaysky and Wang, 2000). We meet many other interesting detailed works or a case studies in the area of the development direction dependence but not as a universal model which is economically explaining observed form of the distribution using general mechanisms (Henriksson and Merton, 1981; Anatolyev and Gerko, 2005; study about the connection of liquidity and market crashes done by Huang and Wang, 2009; other works like Primbs and Rathinam, 2009; Lux, 2011), some works are connected to the prediction of business cycles (Birchenhall, Osborn and Sensier, 2001; Dzikevičius and Vetrov, 2012, etc.), direction of change ideas (Rydberg and Shephard, 1999). Price direction development dependence is also taking place in the basic feedback process according to the behavioral finance concept where upward trend is more likely to be followed by another upward movement (Schiller, 2003) or in other research as for example momentum studies (Pesaran and Timmermann, 1995; Chan, Jegadeesh and Lakonishok, 1996; Stankevičienė and Gembickaja, 2012), short-term trend trading strategy in futures market based on chart pattern recognition (Masteika and Rutkauskas 2012), development of the conception of sustainable return investment decisions strategy in capital and money markets (Rutkauskas, Miečiškiene and Stasytyte
2008), also in other works in the area of investment and risk management such as Málek, Radová and Štěrba (2007) or more recently Černohorská, Teplý and Vrábel (2012), Teplý (2012) or Janda and Svárovská (2013).

We have to mention also works of Larrain (1991) which states that long term memory exists inside the financial market, other similar works of Hsieh (1991), Peters (1989; 1991; 1994) which focus mainly on measurement of probability diversions from normality, also using Hurst coefficient, but these theories are not solving in detail their explanation using basic processes and elements existing on real liquid financial market.

Price direction dependence has an impact on investment instruments valuation and also on the price forecasting.

Figure 1
a) 1 Day Returns Distribution of S&P500, 1963 – 2013

![1 Day Returns Distribution of S&P500, 1963 – 2013](image)

b) 1 Day Price Distribution of Euro-Bund Futures (FGBL, EUREX), 1990 – 2013

![1 Day Price Distribution of Euro-Bund Futures (FGBL, EUREX), 1990 – 2013](image)

Source: Own research.
2. Theory – Dynamic Financial Market Model

We have already mentioned there are two basic ways how to explain departures from normality in probability distributions and model observed shape. The first way is to consider price volatility dependence and the second way is to consider price development direction dependence.

We have to take into consideration also the combination of both effects within the real financial market.

To solve the problematic of the direction dependence the Dynamic Financial Model has been proposed. The model is a comprehensive realistic model putting great emphasis on realistic economic explanation. The Model is based on development direction dependence and it has three basic presumptions, which are described in detail in the text below:

1. primary random walk presumption,
2. feedback presumption,
3. incoming of economic news presumption.

The methodology of the model construction is based on system internal description, therefore the internal structure uncertainty, resulting from the output/input system description, is eliminated.

2.1. Primary Random Walk Presumption and its Impact to a Probability Distribution

The idea of a primary random walk presumption is based on an empirical observation when the two groups of buyers and sellers accept the same price range for their trades. Each of the investors comes to the market with an order and places it to the book of orders (or to the other market tool for collecting the orders) at random during a certain period of time. They can also randomly pick up an order then. The book of orders on liquid financial market works continuously during trading hour’s collects orders and generates a price development. The sequence, frequency and the volume of market orders are generally unpredictable for each investor. If we simulate this situation we get symmetric (when the groups of buyers and sellers are the same) independent random walk – “primary random walk”, with step length equals minimum price tick given by certain market rules. There are 4 such simulations in the Figure 2.

The work of the book of orders is a stable process if there is still certain amount of orders coming to bid and offer sides. This situation is usual for real financial markets. A stable situation is supported by the fact that investors are

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1 Stádník (2011).
interested first of all in a future price development, which is important for their profit or loss, and accept very well current price in a wide price range. For the investors is more important their opinion of the future development than of a current price value. It also means that for each price we find enough investors with an opposite opinion of a future price development.

The whole process is observable also in the time intervals without incoming of economic news which is the case of the simulation in the Figure 2.

*Figure 2*

**Outputs from Book of Orders Simulation 4 Independent Symmetric Random Walks**

Primary random walk can be influenced by the feedbacks (described below). The primary random walk, when not under influence of a feedback, is an independent pure random process with probability distribution of a Gaussian type.

### 2.2. Feedback Presumption and its Impact on a Probability Distribution

The idea of feedback processes is based on the observations that traders, investors and other market participants don’t only watch present or historical data but according to them they are also placing buy or sell orders\(^2\) and thus influence future development. So there is a feedback in the financial markets which also influences a future price development and cause the future direction dependence.

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\(^2\) There are many studies and empirical evidence that high percentage of investor uses for example technical analysis (75%, according to Arnold, Moizer and Noereen, 1984) or other tool which is based on prediction of future development according to the past.
The most usual examples are traders who use technical analysis. In the model we work with the feedback processes regardless whether they really help to make profit or not.

Feedback can increase or decrease the frequency of incoming buy or sell orders and therefore changes a probability of the next price step direction from 50% (in case of a pure symmetric random walk) to for example 51%. Then we can conclude that the probability of the next step direction depends on the past. A typical example could be a situation when the upward trend formation appears. The trend can be initially caused by a symmetric pure random walk. Afterwards some traders, investors start to believe in its continuation and try to use this expectation to make a speculative profit. They support the trend using their own buy orders (to open speculative long positions) thus the probability of the next price step is slightly increasing in the trend direction. Feedback is triggered or cancelled according to the past or current circumstances. Feedback can work together or against another feedback. Feedbacks influence and deform the primary random walk.

According to the Dynamic Financial Market Model’s presumptions we expect more mechanisms which use feedbacks and which are involved in the following groups: price development limits, technical analysis, trend stabilizer, price inertia, trading techniques, different up/down movements, market price manipulations, market regulations, round numbers, logarithmic correction of a price, etc.

The groups of the feedbacks which are directly observable within the financial markets and which have the most significant impact on the distribution are: price inertia, trend stabilizer, trading techniques and different up/down movements’ dynamic. These groups are described in the following text.

The price inertia is a basic negative feedback which helps to keep price unchanged. Feedback works in all periods of time as a minute, hour or day. If there are not any economic news, primary random walk is forced by traders towards the level (as it is shown in the Figure 3a) which is adequate to the previous economic news level or to the other levels. The other level can be previous day closing price, day opening price, support or resistance levels given by technical analysis, etc. Especially a day closing price is considered to be reflecting all the economic news during a day. Over a long time period traders prefer to close long positions above the level or open long positions below this level. Analogically they prefer close short positions below the level and open short position above the level. Some of them also believe that the price has a tendency to return to the adequate level and support this idea by their own orders. Later in the discussion we will conclude that if only approximately 1% of traders participate in the feedback then the probability distribution is deformed in the same way as we observe in reality.
The trend stabilizer is a positive feedback which is stabilizing trends and keeps the price development in a trend direction. Feedback works in all periods of time. When a trend formation appears, trend stabilizing is triggered. The principle is shown in the Figure 3a. Trend stabilizing (supporting) has an origin in the psychology of investors. Trend stabilizer feedback can work against price inertia feedback and try to distribute price from the level. A good example of a price trend stabilizing is a creation of market bubbles or a price momentum.

**Figure 3**

Price Inertia, Trend Stabilizer Feedbacks and Impact of Important Economic News

![Diagram](attachment:image.png)

*Source: Own research.*

The trading techniques are commonly used on financial markets. For example a daily gap trading is a popular trading technique for many liquid investment instruments. Traders believe gaps opened in the morning will be closed during a trading day. They place orders to support this idea. This technique is actually a price inertia feedback on daily basis. Many techniques are also based on level-level trading. It means if any level is broken; the movement will keep the direction. This technique is actually a trend stabilizer feedback on the daily basis. If the level is not broken the market price will return back or keep the trend. Many levels are represented with round numbers (10, 15 ... 100, 200).

Trading techniques can be recognized not only on daily basis but also during other time periods (on daily basis are the most significant).

Empirical experiences indicate that the dynamics of the market downward movements is different than in case of market upward movements. Downward movements are quicker and less frequent. This effect is probably caused by similar processes as for example a stronger downward movement during a financial crisis or when market price bubble bursts.
In case of a price inertia feedback we expect impact on sharpness of the distribution, in case of trend stabilizer we expect impact mainly on the fat tails and extreme values.

On a real financial market we can observe these effects in the chart. There can be differences recognized visually among three charts in the Figure 4a, which can be explained using feedbacks 4b where the arrows symbolize the direction of the feedback according to the Figure 3a, b (the most feedbacked price development is the second one in the figure and note “normal” in the figure means independent random walk). Charts under the influence of feedbacks are more “staircase-like” than the chart of the random walk which is well represented in the Figure 5 (the most feedbacked development is the second one in the figure; the first one is an independent random walk).

**Figure 4**

**Developments with a Different Kurtosis**

![Diagram a) and b)](image_url)

*Source: Own research.*

An impact to returns’ probability distribution is described in the Figure 6. The most deformed distribution (depends on the intensity of feedbacks) is the black one in the figure. Price inertia feedback keeps price near the center area of a distribution and trend stabilizer feedback distributes the price to the borders of a distribution. Due to the fractal structure it doesn’t depend on the time period in which we measure the distributions (vertical lines in the Figure 5). So for example price inertia feedback causes sharpness in the distribution regardless of the starting point of the time interval for the distribution and its length. We can observe the described effects in one hour or one day distributions.
From a logical point of view, different up/down movements’ feedback must influence skewness and cause different left and right tail behavior of the distribution.

2.3. Incoming of Economic News Presumption and Its Impact on a Probability Distribution

Instead of feedbacks it is empirically evident that the final price development is under the influence of a random incoming of economic news.

Impact of important economic news is figured in the Figure 3b. As an impact we expect longer steps which consist of a certain number of minimum price ticks. Parameters should be set according to the empirical observations for each market. There is a possibility to assume the length and frequency of these steps to be independent or we can build news clustering or some other volatility dependence in. Into this presumption we involve also unexpected steps whose length is more than 1 minimum price tick and such steps are not determined by any economic news. Important economic news can support fat tails and extreme values in the distribution.

An impact on a price development in case of less important economic information is similar to the trend stabilizer feedback. In this case we expect an impact on the probability of a next price step direction.

3. Realistic Probability Distribution Simulation\(^3\) (Monte Carlo Style)

In order to simulate realistic daily returns’ probability distribution according to the figure 1, using the Dynamic Financial Market Model, we have used software simulations (Monte Carlo style, each simulation simulates 100 000 price developments) and we have used the following model’s assumptions:
1. primary random walk,
2. price inertia feedback (previous day closing price and levels given by incoming of economic news during a day are used as the price inertia feedback level),
3. trend stabilizer feedback,
4. different up/down movements,
5. incoming of economic news,
6. logarithmic correction of price.

All the parameters (number of steps of primary random walk during a day, frequency and step length of incoming important economic news and its distribution, intensity of a feedback, etc.) are initially set according to empirical observations and then were calibrated according to the realistic distribution.

All six effects have been active at the same time.

As an output of the simulation we have obtained realistic distributions (Figure 7a, also Figure 15 in the Appendix B, <http://pozemstan.cz/Appendix_A_B_C.pdf>) and also probability of a next price step direction if price inertia feedback is triggered and pushes a price toward to the level. The probability of a step direction towards the level has changed from 50.00% to 50.92% during pushing of the price. Fat tails and extreme values are caused by incoming of important economic news together with trend stabilizer feedback. Skewness and heavier left tail are caused by different up/down movements’ feedback. The normal distribution in the figures belongs to the primary random walk.

**Figure 7**

a) Simulated Daily Returns’ Probability Distribution of S&P 500

[Image of a graph showing probability distributions]

**Source:** Own research.

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b) Compared to Its Real Distribution

![Graph showing distribution]

Source: Own research.

Measured probability means that only approximately 1 trader from 100 always supports a feedback. So the intensity of this feedback is quite small and we cannot expect a strong help in predictability of a future price development or to make a speculative profit.

4. Empirical Tests\(^4\) of Price Inertia Feedback

A feedback should increase the value of probability of the future market price development direction from 50% to a higher value. Due to this fact we can use a feedback for speculative investments with the probability of success also higher than 50%. According to the Dynamic Financial Market Model price inertia feedback is pushing the market price toward the level which is adequate to the previous economic news level or to another important level. This causes sharpness in the distribution regardless of the starting point of the time interval for the distribution and its length.

The principle of empirical testing is back tests on US stock market and also on Euro Bund Futures\(^5\). Back tests are based on speculative buys/sells using a price inertia feedback. As a price inertia feedback level was used the closing price of the previous day and speculative positions were opened always for 1, 2 and 3 days and then were closed. The short positions were opened if the opening

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\(^4\) Existence of the price inertia feedback is based mainly on its direct observation within financial markets. The tests were done indicatively and cover US stock market and European bond futures market.

\(^5\) Euro Bund Futures contract is traded on EUREX (interest rates derivatives).
price was above the level; long positions if the opening price was below the level. For 2 and 3 days speculations we recognize 2 and 3 different days when the first speculation starts (start-1, start-2, and start-3 in the Tables 1 and 2).

Back testing results of Price Inertia Feedback on 2 553 US stocks over 10 and 5 years’ time periods are in the Tables 1 and 2. The most important result is that the win ratio is always more than 50% (50.04 – 51.99). The value is in accordance with a software simulation which gives value approximately around 51%

Higher fluctuation of win ratio is caused by the correlation between US stocks. Other items in the table mean: “n. of stock all” – number of stocks available for testing, “number of stocks tested” – number of stocks under the test (some of them do not have 10 years history), “n. of B” – number of buys, “n. of S.” – number of sells, “P/L” – profit/loss (each speculation with 1 stock), “stocks in plus” – number of stocks in profit, “average deviation” – standard deviation for a case of an independent binomial process.

The win ratio result in this test is not under the influence by upward/downward long-term trend.

The results from the same back tests on the selected stocks from US stock market are in the Table 3. The back tests were done over 10 years period from 2002 – 2012.

Measured Euro Bund Futures’ win ratio was 51.29% over 32 years’ time period (Table 3). The result indicates that the same processes act on both US stock market and European bond market.

Results of these simple tests support the direct empirical observations that market participants cause the feedback by their own behavior.

**Table 1**

**Results of Empirical Tests of Price Inertia Feedback on 2553 US Stocks over 10 Years Time Period**

<table>
<thead>
<tr>
<th>Time period</th>
<th>Intraday</th>
<th>2 days (start-1)</th>
<th>2 days (start-2)</th>
<th>2 days (start-1)</th>
<th>2 days (start-2)</th>
<th>3 days (start-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n. of stocks all</td>
<td>2 553</td>
<td>2 553</td>
<td>2 553</td>
<td>2 553</td>
<td>2 553</td>
<td>2 553</td>
</tr>
<tr>
<td>n. of stocks tested</td>
<td>2 196</td>
<td>2 196</td>
<td>2 196</td>
<td>2 196</td>
<td>2 196</td>
<td>2 196</td>
</tr>
<tr>
<td>n. of B</td>
<td>2 899 445</td>
<td>1 622 926</td>
<td>1 643 180</td>
<td>1 095 378</td>
<td>1 081 266</td>
<td>1 093 327</td>
</tr>
<tr>
<td>n. of S</td>
<td>3 154 594</td>
<td>1 675 002</td>
<td>1 652 304</td>
<td>1 104 856</td>
<td>1 118 871</td>
<td>1 106 904</td>
</tr>
<tr>
<td>n. of B + S</td>
<td>6 054 039</td>
<td>3 297 928</td>
<td>3 295 484</td>
<td>2 200 234</td>
<td>2 200 137</td>
<td>2 200 231</td>
</tr>
<tr>
<td>win ratio</td>
<td>0.51992</td>
<td>0.51132</td>
<td>0.50672</td>
<td>0.50552</td>
<td>0.51126</td>
<td>0.50944</td>
</tr>
<tr>
<td>P/L</td>
<td>49 935</td>
<td>40 961</td>
<td>8 116</td>
<td>27 946</td>
<td>35 875</td>
<td>38 293</td>
</tr>
<tr>
<td>stocks in plus</td>
<td>0.63964</td>
<td>0.65257</td>
<td>0.54485</td>
<td>0.60282</td>
<td>0.65648</td>
<td>0.65962</td>
</tr>
<tr>
<td>average deviation</td>
<td>0.00020</td>
<td>0.00028</td>
<td>0.00028</td>
<td>0.000034</td>
<td>0.000034</td>
<td>0.000034</td>
</tr>
</tbody>
</table>

Source: Own research.
### Table 2
**Results of Empirical Tests of Price Inertia Feedback on 2553 US Stocks over 5 Years Time Period**

<table>
<thead>
<tr>
<th>Time period</th>
<th>Intraday</th>
<th>2 days (start-1)</th>
<th>2 days (start-2)</th>
<th>3 days (start-1)</th>
<th>3 days (start-2)</th>
<th>3 days (start-3)</th>
</tr>
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<tr>
<td>n. of stocks all</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
</tr>
<tr>
<td>n. of stocks tested</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
<td>2553</td>
</tr>
<tr>
<td>n. of B</td>
<td>1303941</td>
<td>710046</td>
<td>707570</td>
<td>467798</td>
<td>475179</td>
<td>475302</td>
</tr>
<tr>
<td>n. of S</td>
<td>1391948</td>
<td>711042</td>
<td>711446</td>
<td>480102</td>
<td>472631</td>
<td>470352</td>
</tr>
<tr>
<td>n. of B+S</td>
<td>2695889</td>
<td>1421088</td>
<td>1419016</td>
<td>947900</td>
<td>947810</td>
<td>645654</td>
</tr>
<tr>
<td>win ratio</td>
<td>0.50914</td>
<td>0.50968</td>
<td>0.50677</td>
<td>0.50481</td>
<td>0.51108</td>
<td>0.50704</td>
</tr>
<tr>
<td>P/L stocks in plus</td>
<td>0.64055</td>
<td>0.64936</td>
<td>0.59062</td>
<td>0.58561</td>
<td>0.68944</td>
<td>0.64026</td>
</tr>
<tr>
<td>average deviation</td>
<td>0.000305</td>
<td>0.000419</td>
<td>0.00042</td>
<td>0.00051</td>
<td>0.00051</td>
<td>0.00062</td>
</tr>
</tbody>
</table>


### Table 3
**Results of Empirical Tests of Price Inertia Feedback on the Selected Stocks and FGBL**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Type</th>
<th>n. of B + S</th>
<th>Win ratio</th>
<th>Average deviation</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>Apple Inc.</td>
<td>stock</td>
<td>2629</td>
<td>0.5170</td>
<td>0.00098</td>
<td>10 years</td>
</tr>
<tr>
<td>AGU</td>
<td>Agrium Inc.</td>
<td>stock</td>
<td>2361</td>
<td>0.5263</td>
<td>0.0103</td>
<td>10 years</td>
</tr>
<tr>
<td>BA</td>
<td>The Boeing Company</td>
<td>stock</td>
<td>2219</td>
<td>0.4598</td>
<td>0.0106</td>
<td>10 years</td>
</tr>
<tr>
<td>DELL</td>
<td>Dell Inc.</td>
<td>stock</td>
<td>2593</td>
<td>0.5226</td>
<td>0.00098</td>
<td>10 years</td>
</tr>
<tr>
<td>GE</td>
<td>General Electric Company</td>
<td>stock</td>
<td>2487</td>
<td>0.5074</td>
<td>0.0100</td>
<td>10 years</td>
</tr>
<tr>
<td>IBM</td>
<td>Int. Business Machines Corp.</td>
<td>stock</td>
<td>2499</td>
<td>0.5159</td>
<td>0.0100</td>
<td>10 years</td>
</tr>
<tr>
<td>INJ</td>
<td>Johnson &amp; Johnson</td>
<td>stock</td>
<td>2446</td>
<td>0.5372</td>
<td>0.0101</td>
<td>10 years</td>
</tr>
<tr>
<td>MSFT</td>
<td>Microsoft Corporation</td>
<td>stock</td>
<td>2612</td>
<td>0.5062</td>
<td>0.0098</td>
<td>10 years</td>
</tr>
<tr>
<td>XOM</td>
<td>Exxon Mobil Corporation</td>
<td>stock</td>
<td>2478</td>
<td>0.4951</td>
<td>0.0100</td>
<td>10 years</td>
</tr>
<tr>
<td>YHOO</td>
<td>Yahoo! Inc.</td>
<td>stock</td>
<td>2614</td>
<td>0.5279</td>
<td>0.0098</td>
<td>10 years</td>
</tr>
<tr>
<td>FGBL</td>
<td>Euro Bund Futures</td>
<td>futures</td>
<td>5308</td>
<td>0.5129</td>
<td>0.0096</td>
<td>32 years</td>
</tr>
</tbody>
</table>

*Source: Own research.*

If we do the same back tests on indices the win ratio must be smaller than 50% (Table 4) as there are specific situations decreasing the whole win ratio. For example in case of trends we observe situations which lead to positive results on stocks and a negative result on their index. In the Figure 8, there is an example of two stocks and their index.

### Table 4
**Results of Empirical Tests of Price Inertia Feedback on Indices**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Type</th>
<th>n. of B + S</th>
<th>Win ratio</th>
<th>Average deviation</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>DAX Index</td>
<td>index</td>
<td>5973</td>
<td>0.4285</td>
<td>0.0065</td>
<td>24 years</td>
</tr>
<tr>
<td>SP500</td>
<td>S&amp;P500 Index</td>
<td>index</td>
<td>6232</td>
<td>0.3657</td>
<td>0.0063</td>
<td>62 years</td>
</tr>
</tbody>
</table>

*Source: Own research.*
We measure positive P/L (“+PL”) on each stock while negative P/L (“−PL”) on their index. Other items in the picture mean: “S” – selling price, “B” – buying price, “close” – closing price.

**Figure 8**

**Testing of Price Inertia Feedback on Indices**

![Diagram](image)

*Source: Own research.*

**Conclusion**

The problematic of measured shape of the financial market distributions, price direction dependence and its impact on the distribution has been solved within the proposed model – the Dynamic Financial Market Model.

The Dynamic Financial Market Model is the model of liquid financial markets putting an emphasis on a realistic economic interpretation. The model is based mainly on the precise description of an internal structure and internal mechanism on the lowest level of the system.

Dynamic Financial Market Model considers feedback processes within financial markets which cause the price step direction dependence on the previous development. The model also expects a mix of an independent primary random walk process, influenced by the feedbacks, with random process of incoming economic news. All the processes together cause not-Gaussian observations in returns’ probability distributions. S&P500 index daily returns’ probability distribution in the Figure 1 is a good example of the diversion from normality. Such distribution was simulated in the Figure 7 using the model.

The idea of feedback processes is based on the observations that investors and other market participants not only watch present or historical data but according to them they are placing buy or sell orders and thus influence future development. Feedback can increase or decrease a frequency of incoming buy or sell orders and therefore changes the probability of the next price step direction from 50% (in case of pure symmetric random walk) to another value. Then we can conclude that the probability of the next step direction depends on the past.
The system of feedbacks includes price development limits, technical analysis, trend stabilizing, price inertia, trading techniques, different up/down movements, market price manipulations, market regulations, round numbers, logarithmic correction of a price, etc.

From the Monte Carlo simulation of the real distribution, using the model we have obtained probability of the next price step direction towards the price inertia level, if this feedback is triggered. The value is close to 51%.

In this study we were also trying to confirm the existence of a price inertia feedback within the liquid financial market. Empirical tests of the price inertia feedback according to the model have supported its existence. Empirically obtained probability of a future price development direction varies approximately from 50% to 52% (50.04 – 51.99%). Back tests were done on approximately 2500 US stocks over 10 years’ time period. These values of a deformed probability of future market price development direction due to the price inertia feedback are in good accordance with the values obtained from the simulation. Similar tests were done on Euro Bund Future on daily basis (2009 – 2012) and the measured probability of the direction to the previous closing price is approximately 50.4%. Numerical calculations (refer to the Appendix see <http://pozemstan.cz/Appendix_A_B_C.pdf>) using more dimensional Markov chain have confirmed these results.

Alternatively for modeling of the real distribution’s shape with its abnormalities we can use models with volatility dependence. We have to take into consideration also the combination of both effects present in the real financial market.

We can conclude that the empirical measurements support the correctness of the Dynamic Financial Market Model with its feedback processes. We can also state that the departures from normality in returns’ probability distributions can be caused by the market price direction dependence on the past.

Directional dependence has a good realistic economic explanation and explains also the shape of financial market distributions, especially in the cases where we do not observe volatility clustering.

References
