Investment Strategies in the Funded Pillar of the Slovak Pension System[1]

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Abstract

We present a dynamic model for optimal investment decisions in privately managed defined contribution (DC) pension plans. Stock prices are assumed to be driven by the geometric Brownian motion. Interest rates are modelled by means of the Cox-Ingersoll-Ross model (CIR). The model determines an optimal fraction of pensioner's savings (in time) to be invested in an equity fund, with the rest invested in a bond fund. Next, we present sensitivity analysis with respect to various relevant parameters. We also perform stress-testing of optimal investment decisions under different equity return scenarios. The entire analysis is carried out on the actual Slovak DC scheme and all model parameters are calibrated by the latest available data.

Keywords: *dynamic stochastic programming, funded pillar, utility function, risk aversion, Slovak pension system, defined contribution pension scheme, pension portfolio simulations, glide path*

JEL Classification: C15, C61, E27, G18, G11, G23

1. Introduction

In recent decades numerous OECD countries introduced privately managed defined contribution (DC) pension plans into their pension systems to complement or replace already existing public schemes. This structural change was driven primarily by the issue of aging population (especially in Europe) and thus challenging sustainability of the public pension plans. Many of these public

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plans work as pay-as-you-go (PAYG) systems, i.e. pensions of current pensioners are funded by contributions of currently active workers. On the other hand, privately managed DC schemes work on a basis of regular contributions of individual workers to their own pension accounts. The wealth accumulated via these contributions is continually managed by pension funds, which invest in the financial assets such as equities, bonds or cash. PAYG systems are favorable when compared to the private ones, in case that productivity growth of population exceeds return on pension fund's investments.

Issues of aging population and slowing productivity growth have recently become even more imminent in developed countries, particularly in Europe. This should, according to the presented logic, favor DC schemes. Some countries such as Slovakia, Poland or Hungary have, however, actually cut contribution rates in DC schemes or in some way disadvantaged the DC plans as a response to the crisis of 2008. And even those DC systems, which operate uninterrupted, invest rather conservatively, holding majority of their assets in instruments with relatively low return potential such as bonds or short-term notes (see Salou et al., 2012). The same applies to individuals, who predominantly prefer conservative investments as well. One of the aims of this paper is to emphasize the important role of equities in pension investment portfolios by means of a quantitative model.

The main goal of this paper is to analyze the level of pensions from the second pillar of the Slovak pension system according to the last legislative changes. We use the dynamic stochastic accumulation model introduced firstly in Kilianová, Melicherčík and Ševčovič (2006) and later generalized in Melicherčík and Ševčovič (2010). The model determines the optimal fraction of savings to be invested in the equity fund (with the rest in the bond fund), given specific time to retirement, level of accumulated wealth and actual short-term interest rate. Authors Melicherčík and Ševčovič (2010) assumed existence of 2 funds – the bond fund, represented by 1-year zero coupon bonds and the equity fund whose risk-return characteristics corresponded to the US stock index S&P 500 during 1996 – 2002. The stock returns were assumed to be driven by the geometric Brownian motion and bond returns were modelled by means of the Cox-Ingersoll-Ross (CIR) model.

We generalize the model from Melicherčík and Ševčovič (2010) to account for any duration of the bond fund. Next, we conduct a sensitivity analysis of the model outcomes to all relevant parameters. Most importantly, we perform stress-testing with respect to the most sensitive as well as the most unpredictable parameter-equity returns. To achieve this we utilize real historical stock index scenarios as well as artificially created ones. We present our results on the current

Slovak DC scheme and calibrate all of our models by latest available data. The achieved levels of savings are recalculated to the replacement rates using non--indexed annuities.

The paper is organized as follows: Section 2 contains the formulation of the dynamic model for pension savings management. In Section 3 we present results using the basic settings of the model and conduct the sensitivity analysis. Stress--testing is presented in Section 4. Section 5 contains recalculations of the results to the replacement rates. Conclusions can be found in the last section.

2. Model

Suppose that a future pensioner deposits once a year a τ_t-part of his/her yearly salary w_t to a pension fund with a δ-part of assets in stocks and a $(1-\delta)$-part of assets in bonds where $\delta \in [0, 1]$. Denote by s_t, $t = 1, 2, \ldots, T$, the accumulated sum at time t where T is the expected retirement time. Then the budget-constraint equations read as follows:

$$s_{t+1} = \delta\, s_t \exp(R^s(t, t+1)) + (1-\delta)\, s_t \exp(R^b(t, t+1)) + w_{t+1}\tau_{t+1} \qquad (1)$$

for $t = 1, 2, \ldots, T-1$, where $s_1 = w_1\tau_1$. $R^s(t, t+1)$ and $R^b(t, t+1)$ are the annual returns on stocks and bonds in the time interval $[t, t+1)$ respectively. When retiring, a pensioner will strive to maintain his/her living standards in the level of the last salary. From this point of view, the saved sum s_T at the time of retirement T is not precisely what a future pensioner cares about. For a given life expectancy, the ratio of the cumulative sum s_T and the yearly salary w_T, i.e. $d_T = s_T / w_T$ is of a practical importance to a pensioner. This quantity could be easily recalculated to the replacement ratio (pension payment/salary – see Section 5), which is the most important value for pensioners. Using the quantity $d_t = s_t / w_t$ one can reformulate the budget-constraint equation (1) as follows:

$$d_{t+1} = d_t \frac{\delta \exp(R^s(t, t+1)) + (1-\delta)\exp(R^b(t, t+1))}{1+\beta_t} + \tau_{t+1}$$

for $t = 1, 2, \ldots, T-1$, where $d_1 = \tau_1$ and β_t denotes the yearly wage growth: $w_{t+1} = w_t(1+\beta_t)$.

The term structure development is driven by the CIR model presented in Cox, Ingersoll and Ross (1985):

$$\mathrm{d}r_t = \kappa(\theta - r_t)\mathrm{d}t + \sigma^b \sqrt{r_t}\,\mathrm{d}Z_t \tag{2}$$

where

$\kappa,\ \theta,\ \sigma^b > 0$

θ – the long term interest rate,

κ – the rate of reversion,

σ^b – the volatility of the process,

Z_t – the Wiener process.

Suppose that the bond part of the portfolio has duration T_b. The corresponding return can be modelled using zero coupon bonds. Denote by $P(t,\ T_b)$ the price (at time *t*) of zero coupon bond with face value 1 and time to maturity T_b. Then $R^b(t,\ t+1) = \log P(t+1,\ T_b - 1) - \log P(t,\ T_b)$.

In CIR model (see Cox, Ingersoll and Ross, 1985) the term structure of zero coupon bonds can be expressed by explicit formula:

$$P(t,\ T_b) = P(r_t,\ t,\ T_b) = A(T_b)e^{-B(T_b)r_t}$$

where

$$A(T_b) = \left(\frac{2\gamma e^{\frac{(\kappa+\lambda+\gamma)T_b}{2}}}{(\kappa+\lambda+\gamma)(e^{\gamma T_b} - 1) + 2\gamma} \right)^{\frac{2\kappa\theta}{\sigma^2}}$$

$$B(T_b) = \frac{2(e^{\gamma T_b} - 1)}{(\kappa+\lambda+\gamma)(e^{\gamma T_b} - 1) + 2\gamma}$$

$$\gamma = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2}$$

The parameter $\lambda \in R$ stands for the so called market price of risk. Using a discretization of the short rate process (2) we have (see e.g. Yu and Phillips, 2001 or Bergstrom, 1984)

$$r_{t+1} = g(r_t,\ \Phi) = \theta + e^{-\kappa}(r_t - \theta) + \left(\sigma^b \sqrt{\frac{r_t}{2\kappa}(1 - e^{-2\kappa})} \right)\Phi \tag{3}$$

where Φ ~ N(0,1).

We shall assume the stock prices S_t are driven by geometric Brownian motion. The annual stock return $R^s(t,\ t+1) = \log(S_{t+1} / S_t)$ can be therefore expressed as: $R^s(t,\ t+1) = \mu^s + \sigma^s\Psi$, where μ^s and σ^s are the mean value and volatility of annual stock returns in the time interval $[t,\ t+1)$, Φ ~ N(0,1) is a normally distributed random variable. The random vector (Φ, Ψ) is assumed to have 2-dimensional normal distribution with correlation $\rho = E(\Phi\Psi) \in (-1, 1)$.

Suppose that each year the saver has the possibility to choose a level of stocks included in the portfolio $\delta_t(I_t)$, where I_t denotes the information set consisting of the history of bond and stock returns $R^b(t', t'+1)$, $R^s(t', t'+1)$, and wage growths $\beta_{t'}$, $t' = 1, 2, \ldots, t-1$. We suppose that the forecasts of the wage growths β_t, $t = 1, 2, \ldots, T-1$ are deterministic,[2] the stock returns $R^s(t, t+1)$ are assumed to be random, independent for different times $t = 1, 2, \ldots, T-1$ and the interest rates are driven by the Markov process (2). Then the only relevant information are the quantities d_t and the short rate r_t. Hence $\delta_t(I_t) \equiv \delta_t(d_t, r_t)$. One can formulate a problem of dynamic stochastic programming:

$$\max_{\delta} \mathrm{E}(U(d_T)) \tag{4}$$

subject to the following recurrent budget constraints:

$$d_{t+1} = F_t(d_t, r_t, \delta_t(d_t, r_t), \Phi, \Psi) \tag{5}$$

where $t = 1, 2, \ldots, T-1$, $d_1 = \tau_1$,

$$F_t(d, r, \delta, x, y) = d\,\frac{\delta e^{\mu_t^s + \sigma_t^s y} + (1-\delta) e^{rB(T_b) - \log(A(T_b)) - g(r,x)B(T_b-1) + \log(A(T_b-1))}}{1+\beta_t} + \tau_{t+1} \tag{6}$$

and the short rate process is driven by (2) and (3) with $r_1 = r_{init}$. We assume the stock part of the portfolio is bounded by a given upper barrier function $\Delta_t : 0 \le \delta_t(d_t, r_t) \le \Delta_t$. The function $\Delta_t : \{1, \ldots, T-1\} \mapsto [0, 1]$ is subject to governmental regulations. In our modeling we shall use the constant relative risk aversion (CRRA) utility function $U(d) = -d^{1-a}$, $d > 0$ where $a > 1$ is the constant coefficient of relative risk aversion. The reason of using CRRA function is to have the results scale invariant (it is meaningful to have same optimal portfolio when optimizing the level of savings in monthly or yearly salaries). The model is generalization of the one presented in Melicherčík and Ševčovič (2012), where the bond part of the portfolio was represented by zero coupon bonds with time to maturity $T_b = 1$. For the sake of brevity, we do not discuss a numerical procedure for solving the problem (4) – (6) and refer a reader to Melicherčík and Ševčovič (2010).

[2] The wage growth is in reality random since it depends on the random inflation and other random factors. This assumption is a simplification accepted in the study.

3. Baseline Scenario

3.1. The Slovak Pension System

Pensions in Slovakia are operated by a three-pillar system:

1. the public, compulsory, non-funded first pillar (PAYG),
2. the private, fully funded second pillar,
3. the private, voluntary, fully funded third pillar.

The contribution rate is currently set at 18% for the first pillar (in case a pensioner decides to stay only in the public scheme) or 14% for the first pillar and 4% for the second pillar (in case a pensioner decides to save in both pillars).[3] In addition to the mandatory rates, pensioners may decide to contribute any additional amount to the second pillar or establish a savings account in the third pillar. The focus of this paper is solely on the private, fully funded second pillar. The savings in this pillar are managed by pension asset managers. Each asset manager operating in the second pillar is obliged to manage two funds – a *Guaranteed Bond Fund*[4] and a *Non-guaranteed Equity fund* plus any number of additional funds. Savers have a possibility of holding all assets in any fund of their choice (one fund only at the same time instant) or to split the assets into two funds (one of which has to be a Guaranteed fund) by any ratio they choose. This ratio can be changed in time and is subject to the governmental regulations during the last years of a savings process.

When approaching retirement, the fraction of savings in a Guaranteed fund has to be gradually increased (see Table 2) and is required to reach 100% 3 years ahead of retirement.

3.2. Parameters and Data

Parameters of the CIR model were estimated from EURIBOR data[5] using maximum likelihood method published in Kilianová, Melicherčík and Ševčovič (2006). Parameter λ (market price of risk) was taken from Melicherčík and Ševčovič (2010); see Table 3 for specific values. It is worth to note that estimated parameters are close to ones used in Melicherčík and Ševčovič (2010), that were taken from Ševčovič and Urbánová Csajková (2005).

[3] The contribution rate to the private pillar has been recently cut from 9% to 4% with future planned increase to 6%. The development of the contribution rate according to the latest legislative changes is presented in Table 1.

[4] Guaranteed fund is obliged to deliver a non-negative performance, net of costs, during any rolling 10-year period.

[5] Daily data from period 1999 – 2012; source: <http://www.euribor-info.com/en/eonia>.

Table 1

Forecast of Interannual Gross Wage Growth in Slovakia

Year	Contributions (in %)
2013 – 2016	4.00
2017	4.25
2018	4.50
2019	4.75
2020	5.00
2021	5.25
2022	5.50
2023	5.75
2024 – 2051	6.00

The Estimated Amount of Contributions as a Percentage of a Gross Wage

Year	Wage growth (in %)
2013	4.37
2014	4.75
2015	5.20
2016 – 2020	6.40
2021 – 2025	5.90
2026 – 2030	5.60
2031 – 2035	5.20
2036 – 2040	4.90
2041 – 2051	4.50

Source: Law on Pension Savings, No. 43 (as of June 1, 2014) (left); Kvetan et al. (2007) (right).

Table 2

Legislative Restrictions on the Proportion of Savings in Equity Funds

Age of saver	Year of saving	Maximum % of stocks	Δ_t
<= 49	1. – 28.	100	1
50 – 58	29. – 37.	10 x (59 – age)	0.1 x (59 – age)
>= 59	38. – 40.	0	0

Source: Law on Pension Savings, No. 43 (as of June 1, 2014).

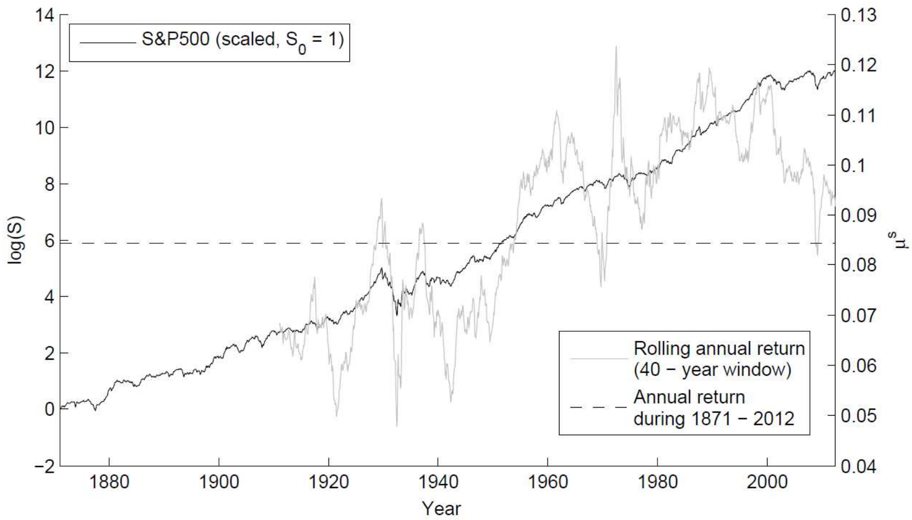
An important role plays the choice of the risk aversion coefficient *a*. There is a consensus today that the value should be between 2 and 10 (see e.g. Mehra and Prescott, 1985). In our opinion, the pension investment should be conservative. Therefore, we have used the coefficient of the relative risk aversion $a = 9$ (same as Melicherčík and Ševčovič, 2010). Our results have shown that, even with this conservative setting, the optimal investment is to invest 100% in stocks in the first 10 years of saving (see Figure 3). We have, however calculated the results also for a less conservative setting $a = 5$ (see Table 4). Nominal wage growth in Slovakia (Table 1) over the next 40 years was obtained from the most recent available forecasts. Specific values for years 2013 – 2015 are the average forecasts of the National Bank of Slovakia, Institute of Financial Policy and Slovenská sporiteľňa. Data for years 2016 – 2051 are from the Slovak Academy of Sciences publication (Kvetan et al., 2007). Legislative restrictions on the proportion of savings in equity funds can be found in Table 2.

Although it is not the aim of this work to estimate future returns of the stock markets, it is important to consider the model parameters that are not too far from reality. The basic value of the drift μ^s was estimated from historical annualized monthly returns of the U.S. stock market index S&P 500, including reinvested dividends (*total return*).[6]

Figure 1 shows that the annual return of the index at different 40-year periods ranged from 5% to 12% p.a. In our calculations we have used value $\mu^s = 8.44\%$ p.a. (estimate from the whole period 1871 – 2012).

F i g u r e 1

Annual Returns of the Stock Index S&P 500



Left axis: Historical development of the stock index S&P 500 with reinvested dividends. Logarithmic scale, $S_0 = 1$. *Right axis:* Rolling (by month) annual return of the index calculated retrospectively from the period of 40 years and the annual return of the index during the entire displayed period.

Source: S&P 500, daily data, <http://finance.yahoo.com>.

The value σ^s (volatility of the stock part of the portfolio) was estimated from the same data. During 40-year periods, its value was stable. It was affected by the Great Depression (in the periods out of 1929 – 1930 the value about 12% p.a. and in the periods involving crisis the value about 19% p.a.). During the 10-year periods (out of the crisis) the value varied in the range from 10% to 14% p.a. In the crisis it has reached up to 31% p.a. We will use the estimate of the standard deviation from the whole period, $\sigma^s = 14.17\%$ p.a.

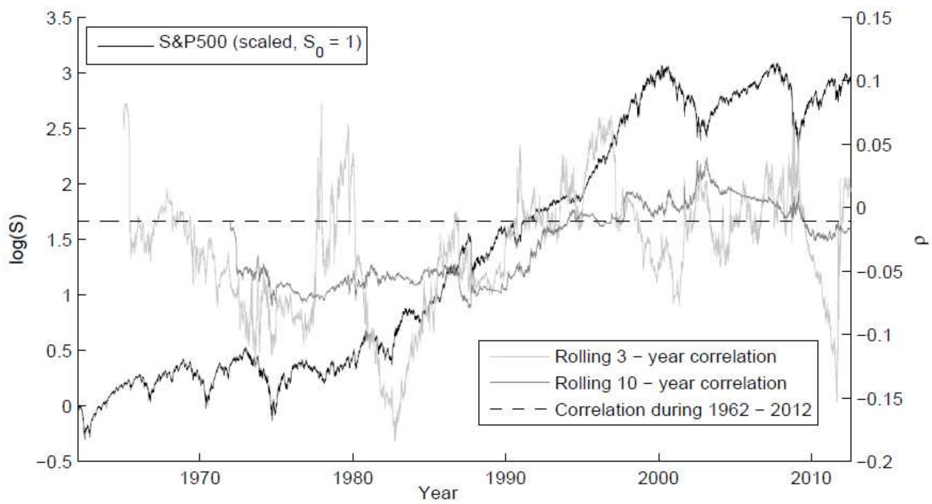
The correlation of the stock and bond parts of the portfolio was estimated using historical data.[7] Daily development of the correlation coefficient during 1962 – 2012 is in Figure 2.

[6] Monthly data; source: <http://www.econ.yale.edu/~shiller/data.htm>.

[7] The correlation coefficient can not be simply calculated as a correlation of stock and bond returns. The random variable Φ should be expressed from (8). Subsequently, the correlation with the random variable Ψ corresponding to the stock returns can be calculated.

F i g u r e 2

Correlation of Stock and Bond Returns



Left axis: Historical development of the stock index S&P 500 with reinvested dividends. Logarithmic scale, $S_0 = 1$. *Right axis:* Rolling (by day) correlation coefficient of random variables Φ and Ψ calculated retrospectively from the period of 40 years (resp. 10 years) and the value of the correlation coefficient estimated from the whole period.

Source: S&P 500, daily data, <http://finance.yahoo.com>; US short rate (Effective Federal Funds Rate), <http://research.stlouised.org/fred2/>.

One can observe values approximately between –0.18 and 0.08. In our calculations we use the estimate from the whole period, $\rho = -0.01082$.[8] This model however has some drawbacks. We have not considered historical inflation, which is one of the key parameters influencing the bond returns. Values of all parameters of equity and bond funds used in the baseline scenario can be found in Table 3.

T a b l e 3

Parameters of Equity and Bond Funds

κ	0.8993	T_b	3
θ	0.0226	μ_t^s	8.44%
σ^b	0.148	σ_t^s	0.1417
λ	0	ρ	–0.01082

Source: Our estimates.

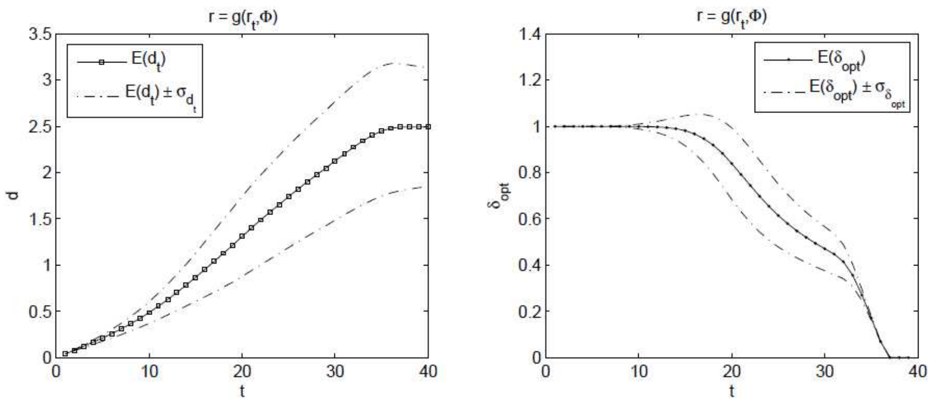
[8] S&P 500, daily data; source: <http://finance.yahoo.com/>. US short rate (*Effective Federal Funds Rate*); source: <http://research.stlouised.org/fred2/>.

3.3. Results for the Baseline Scenario

The output of the model is the function $\hat{\delta}(d_t, r_t)$. This function tells us what the optimal proportion of savings invested in equity funds is, provided that we are in the *t*-th year of saving, the current short rate is r_t and we have already saved d_t yearly salaries. The development of the average level of savings and average proportion of the stock investment with standard deviations for 100 000 Monte Carlo simulations can be found in Figure 3. Using the basic model parameters, the average terminal level of savings is relatively low (around 2.5 times of the yearly salary, see also Table 4). This is mainly due to low contributions and relatively high wage growth. The right graph shows that, at the beginning of saving, the model recommends to invest all savings in the stock fund. The reason is simple. Possible negative return of the stock fund has a small impact on future pension, since essential part of the contributions is expected in the future. Later on, return of the stock fund has higher impact on the final level of savings (the ratio of future contributions to the level of savings is lower). Therefore, the decreasing tendency of stock investments is natural. The linear decrease in the last years is due to governmental regulations. The governmental regulations supplemented with high wage growth are the reasons of stagnant level of savings in the last years before retirement.

F i g u r e 3

The Development of the Average level of Savings | **Average Proportion of the Stock Investment with Standard Deviations**



Source: Our calculations.

3.4. Sensitivity Analysis

It is difficult to forecast the model parameters exactly. Therefore, we have performed simulations for the following modifications of the baseline scenario:

(M0) Baseline scenario.
(M1) Contributions $\tau_t = 4\%$ during the entire saving period.
(M2) Contributions $\tau_t = 9\%$ during the entire saving period.
(M3) No governmental regulations for the stock fund, i.e. $\Delta_t = 1$ during the entire saving period.
(M4) Lower aversion to risk *a* = 5.
(M5) Higher duration of the bond fund $T_b = 5$.
(M6) Lower wage growth: $\tilde{\beta}_{4-39} = \beta_{4-39} - 1\%$.
(M7) Lower drift of the stock returns: $\mu_t^s = 5\%$.
(M8) Linear growth of the drift of the stock returns: $\mu_t^s = 2\% + 0.25(t-1)\%$
(M9) Higher volatility of the stock returns: $\sigma_t^s = 20\%$.
(M10) Forbidden mixing of stock and bond funds, i.e. $\delta_t \in \{0, 1\}$.

Expected values of the final level of savings $\mathrm{E}(d_T)$, standard deviations $\sigma(d_T)$, catastrophic scenarios represented by 5% quantiles $Q_{5\%}(d_T)$ and certainty equivalents (*CE*) defined as $U^{-1}[\mathrm{E}(U(d_T))]$ (i.e. a certain value having the same utility as the random result of the strategy) can be found in Table 4. One can observe that final level of savings is most of all sensitive to the contribution rate[9] and the drift of the stock returns.

T a b l e 4

Sensitivity Analysis – Comparison with the Baseline Scenario

Modification	$\mathrm{E}(d_T)$	$\sigma(d_T)$	$Q_{5\%}(d_T)$	*CE*
(M0)	2.4947	0.6441	1.6226	1.9304
(M1)	1.7922	0.4747	1.1454	1.3591
(M2)	4.0357	1.0757	2.5808	3.0676
(M3)	2.8063	0.8028	1.7302	2.0361
(M4)	2.9284	1.1535	1.5875	2.2103
(M5)	2.4984	0.6487	1.6195	1.9266
(M6)	2.9597	0.7774	1.8997	2.2569
(M7)	1.6873	0.2326	1.3415	1.5550
(M8)	2.2122	0.5093	1.5049	1.7900
(M9)	2.1803	0.4912	1.4893	1.7719
(M10)	2.0326	0.4924	1.4054	1.6857

Columns contain the mean expected value, the standard deviation, the 5% quantile and the certainty equivalent of the final level of savings respectively.

Source: Our calculations.

[9] It is worth to note that the model does not consider the part of the pension received from the first pillar. The pension from the first pillar decreases when increasing contributions to the second pillar (assuming the same total amount of pension contributions). The conclusion applies only to the level of savings from the second pillar.

4. Stress-testing

4.1. Scenarios and Strategies

The estimates of model parameters associated with asset returns (especially drifts of the stock returns) are usually unreliable. Therefore, we have tested selected strategies against a set of different models for the equity fund returns. The model for the bond fund was the same as the one used in the previous section.

We have considered the following drift scenarios μ_t^s:

(SC1) $\mu_t^s = 11\%$ during the entire saving period.

(SC2) $\mu_t^s = 9\%$ during the entire saving period.

(SC3) $\mu_t^s = 7\%$ during the entire saving period.

(SC4) $\mu_t^s = 5\%$ during the entire saving period.

(SC5) Linear growth of the drift from 2% to 11.5%:

$$\mu_t^s = 2\% + 0.25(t-1)\% .$$

(SC6) S&P 500 (1900 – 1939): growth scenario with depression at the end.[10]

(SC7) S&P 500 (1915 – 1954): scenario of stagnation, boom, recession and recovery.

(SC8) S&P 500 (1950 – 1989): long-term healthy growth scenario.

(SC9) S&P 500 (1929 – 1968): scenario of recession, recovery and growth.

(SC10) S&P 500 (1880 – 1919): scenario of stagnation and modest growth.

(SC11) Nikkei 225 (1991 – 2012, 1949 – 1967): scenario with long-term recession and recovery.[11]

Scenarios (SC6) – (SC11) based on historical returns of the stock indices are summarized in Figure 4. We have tested 15 strategies (ST1) – (ST15) against the set of 11 scenarios (SC1) – (SC11). Strategies (ST1) – (ST11) are the optimal ones according to our dynamic model. (ST12) and (ST13) invest all the savings to bond and equity funds respectively. Strategy (ST14) begins with the investment in the equity fund and each year linearly moves the savings into the bond fund.

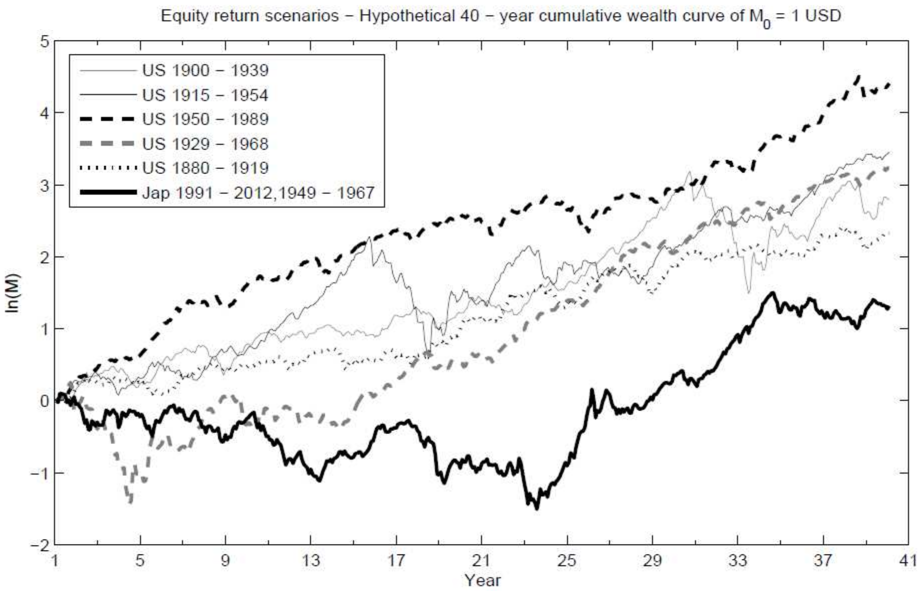
The last one follows a popular rule: „invest (100-age)% to stocks”. Complete list of strategies can be found in Table 5.

[10] Historical annual returns of U.S. stock index S&P 500 with reinvested dividends (total return). Source: <http://www.econ.yale.edu/~shiller/data.htm>.

[11] The scenario is created artificially by combining 22-year period of the recent Japanese stock market decline completed with its previous period of growth. Historical annual returns of the Japanese stock index NIKKEI225. Source: <http://indexes.nikkei.co.jp/en/nkave/archives/data>.

F i g u r e 4

Scenarios (SC6) – (SC11) Used in Stress-testing. Monthly Data



Source: S&P 500, daily data: <http://finance.yahoo.com/>. Historical annual returns of the Japanese stock index NIKKEI225: <http://indexes.nikkei.co.jp/en/nkave/archives/data>.

T a b l e 5

Investment Strategies for Stress-testing

Strategy	Description
(ST1)-(ST11)	Optimal $\hat{\delta}$ for corresponding scenario
(ST12)	$\delta = 0$
(ST13)	$\delta_t = \min\{\Delta_t,\ 1\}$
(ST14)	$\delta_t = \max\{0,\ 1-(t-1)/36\}$
(ST15)	$\delta_t = \min\{\Delta_t,\ 1-(t+22)/100\}$

Source: Our calculations.

4.2. Stress-testing: The Outcome

For each pair (strategy *i*, scenario *j*) 100 000 Monte Carlo simulations have been performed supposing that strategy *i* is applied and scenario *j* takes place. Using the simulations, values of three different indicators have been calculated. We have used the following indicators: certainty equivalent *CE* (see Section 4 for the definition), mean value of the final level of savings $\mathrm{E}(d_T)$ and $Q_{5\%}(d_T)$ (5% quantile of the final level of savings). Results are presented in Tables 6 – 8. Concerning the certainty equivalent indicator, strategies (ST6) – (ST11) achieve high values in the case when the corresponding scenarios (SC6) – (SC11) take place. On the other hand, they are not as flexible as the other strategies in the case a different scenario occurs. The mean value indicator prefers the most risky investment strategies. Exceptions are strategies (ST6) – (ST11) but again only in the case of occurrence of the corresponding scenarios.

T a b l e 6

Certainty Equivalents *CE* Using Various Strategies and Scenarios

	(SC1)	**(SC2)**	**(SC3)**	**(SC4)**	**(SC5)**	**(SC6)**	**(SC7)**	**(SC8)**	**(SC9)**	**(SC10)**	**(SC11)**
(ST1)	2.40	2.00	1.71	1.48	1.76	1.87	1.81	1.94	2.85	1.72	2.64
(ST2)	2.37	2.01	1.73	1.50	1.78	1.83	1.87	2.01	2.72	1.77	2.58
(ST3)	2.28	1.99	1.74	1.54	1.77	1.82	1.94	2.05	2.48	1.77	2.30
(ST4)	2.05	1.87	1.70	1.56	1.70	1.76	1.90	1.95	2.13	1.70	1.92
(ST5)	2.29	1.99	1.73	1.52	1.79	1.80	1.97	2.02	2.62	1.76	2.58
(ST6)	1.91	1.72	1.57	1.44	1.60	3.78	1.33	1.61	2.03	1.74	1.86
(ST7)	1.90	1.73	1.57	1.45	1.61	1.73	4.67	1.68	1.92	1.43	2.49
(ST8)	2.04	1.81	1.62	1.46	1.66	1.83	1.85	3.11	2.51	2.00	1.72
(ST9)	2.01	1.79	1.61	1.46	1.67	1.45	1.45	1.83	4.39	1.41	3.11
(ST10)	1.89	1.72	1.57	1.45	1.60	1.88	1.81	1.93	2.09	2.69	1.38
(ST11)	1.82	1.66	1.53	1.42	1.59	1.45	1.70	1.63	2.26	1.25	6.98
(ST12)	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40
(ST13)	2.38	1.99	1.69	1.46	1.76	1.90	1.76	1.91	2.89	1.68	2.58
(ST14)	1.95	1.78	1.64	1.53	1.63	1.73	1.77	1.82	2.05	1.67	1.87
(ST15)	2.16	1.90	1.70	1.53	1.76	1.73	1.88	1.96	2.44	1.68	2.40

Source: Our calculations.

T a b l e 7

Mean Values $\mathrm{E}(d_T)$ Using Various Strategies and Scenarios

	(SC1)	**(SC2)**	**(SC3)**	**(SC4)**	**(SC5)**	**(SC6)**	**(SC7)**	**(SC8)**	**(SC9)**	**(SC10)**	**(SC11)**
(ST1)	3.69	2.94	2.38	1.96	2.44	2.58	2.64	2.95	4.33	2.44	3.85
(ST2)	3.26	2.70	2.25	1.90	2.29	2.38	2.56	2.79	3.68	2.30	3.37
(ST3)	2.76	2.39	2.07	1.81	2.09	2.20	2.41	2.52	3.00	2.09	2.74
(ST4)	2.24	2.04	1.85	1.69	1.84	1.96	2.13	2.16	2.34	1.85	2.15
(ST5)	2.89	2.48	2.14	1.85	2.21	2.22	2.53	2.57	3.32	2.17	3.30
(ST6)	2.47	2.16	1.92	1.71	1.95	5.91	1.54	1.99	2.65	2.21	2.39
(ST7)	2.41	2.12	1.88	1.69	1.91	2.07	7.79	2.06	2.42	1.66	3.24
(ST8)	2.72	2.33	2.01	1.76	2.05	2.35	2.38	4.75	3.41	2.62	2.13
(ST9)	2.67	2.30	2.00	1.76	2.07	1.82	1.70	2.38	7.60	1.72	4.38
(ST10)	2.37	2.10	1.88	1.69	1.90	2.35	2.32	2.41	2.63	3.73	1.56
(ST11)	2.26	2.03	1.83	1.66	1.90	1.73	2.12	1.98	2.91	1.42	11.17
(ST12)	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40
(ST13)	4.33	3.23	2.49	1.99	2.57	2.97	2.60	3.01	5.43	2.50	4.10
(ST14)	2.15	1.93	1.76	1.61	1.73	1.85	1.92	1.99	2.25	1.78	1.99
(ST15)	2.67	2.28	1.98	1.74	2.04	2.03	2.23	2.35	3.07	1.96	2.93

Source: Our calculations.

A natural question arises, which strategy can be regarded as the best under all circumstances. The answer to this question obviously depends on how we define an evaluation criterion for the strategies. For some savers it could be e.g. a strategy that has the highest mean value of the final level of savings averaged from all scenarios, i.e. *Max-Mean* approach. Risk-takers would prefer the strategy with the highest value of the indicator for the best scenario, i.e. *Max-Max* criterion. Risk averse investors would probably use *Max-Min* approach (maximizing the value for the worst strategy). In addition we have used the *Max-Median* (maximizing the median of scenarios) and *Max-E[U]* (maximizing the mean expected utility with the same probability for all scenarios) criteria. The best strategies using mentioned criteria are presented in Table 9. One can e.g. observe that the stock investment strategy (ST13) dominates for the mean value indicator. For the certainty equivalent indicator and *Max-Min* criterion, strategy (ST4) is optimal. For the same indicator and *Max-Mean* approach, (ST11) should be used. This is mainly due to high value of the indicator in the case scenario (SC11) occurs. It could be also seen from the fact, that (ST11) is the winning strategy for the *Max-Max* criterion as well.

T a b l e 8

5-percent Quantiles $Q_{5\%}$ of the Final Wealth d_T Using Various Strategies and Scenarios

	(SC1)	(SC2)	(SC3)	(SC4)	(SC5)	(SC6)	(SC7)	(SC8)	(SC9)	(SC10)	(SC11)
(ST1)	2.06	1.69	1.42	1.22	1.47	1.56	1.51	1.63	2.44	1.44	2.25
(ST2)	2.01	1.69	1.45	1.25	1.49	1.53	1.57	1.70	2.30	1.48	2.18
(ST3)	1.91	1.67	1.46	1.30	1.49	1.53	1.63	1.72	2.09	1.49	1.93
(ST4)	1.75	1.60	1.47	1.34	1.47	1.51	1.62	1.67	1.82	1.46	1.63
(ST5)	1.93	1.67	1.45	1.28	1.50	1.51	1.66	1.71	2.20	1.47	2.18
(ST6)	1.59	1.43	1.31	1.20	1.34	3.27	1.12	1.34	1.69	1.45	1.55
(ST7)	1.58	1.44	1.32	1.22	1.35	1.45	4.07	1.41	1.60	1.21	2.08
(ST8)	1.70	1.51	1.35	1.22	1.38	1.53	1.54	2.64	2.10	1.67	1.44
(ST9)	1.68	1.49	1.34	1.22	1.39	1.20	1.22	1.52	3.87	1.18	2.61
(ST10)	1.57	1.44	1.32	1.22	1.34	1.57	1.51	1.61	1.74	2.26	1.18
(ST11)	1.52	1.39	1.28	1.20	1.33	1.21	1.42	1.36	1.88	1.06	6.09
(ST12)	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37
(ST13)	2.03	1.67	1.40	1.21	1.46	1.59	1.46	1.59	2.53	1.40	2.19
(ST14)	1.67	1.55	1.44	1.35	1.43	1.50	1.54	1.57	1.76	1.46	1.63
(ST15)	1.80	1.60	1.43	1.30	1.48	1.45	1.58	1.64	2.04	1.42	2.01

Source: Our calculations.

T a b l e 9

Best Strategies for Different Indicators and Criteria Using the Results Presented in Tables 6 – 8

	CE	$E(d_T)$	$Q_{5\%}(d_T)$
Max-Min	(ST4) 5%	(ST13) stock	(ST12) bond
Max-Mean	(ST11) recession	(ST13) stock	(ST11) recession
Max-Median	(ST5) 2 to 12	(ST13) stock	(ST5) 2 to 12
Max-E[U]	(ST3) 7%	(ST13) stock	(ST3) 7%
Max-Max	(ST11) recession	(ST11) recession	(ST11) recession

Source: Our calculations.

5. Annuities from the Second Pillar

In this section we recalculate the savings represented by the number of yearly salaries to a non-indexed perpetual annuity. Consider a person of age x years. The probability that this person dies within the next year is denoted by q_x. The probability of complementary event, i.e., that the person aged x years will survive to age $(x+1)$, is defined by $p_x = 1 - q_x$. One-year probabilities of death q_x are usually known for $x \in \{0, 1, 2, \ldots\}$, given in life tables. Generally, ${}_k p_x$ denotes the probability that the person of age x will survive at least k consecutive years and is defined by

$$ {}_k p_x = p_x p_{x+1} \cdots p_{x+k-1} = \prod_{h=0}^{k-1} p_{x+h} = \prod_{h=0}^{k-1} (1 - q_{x+h}), \quad k = 1, 2, 3, \ldots $$

As we mentioned in Section 3, future pensioners after reaching the retirement age will use accumulated savings to buy an additional part of the pension in a commercial insurance company, typically in a form of life annuity. Let us define the basic whole life annuity-due which provides for annual payments of 1 unit as long as the beneficiary lives (payments are made at the beginning of each year). Denote by $\ddot{a}_x$ the expected net present value of the annuity payments:

$$ \ddot{a}_x = \sum_{k=0}^{\infty} {}_k p_x (1+i)^{-k} $$

where i represents the technical interest rate per annum. In real life, pension benefits are not paid annually, but usually with a monthly frequency. In this case one has

$$ \ddot{a}_x^{12} \approx \left(\sum_{k=0}^{\infty} {}_k p_x (1+i)^{-k} \right) - \frac{11}{24} $$

where $\ddot{a}_x^{(12)}$ represents the net present value of an annuity of 1 unit per year payable 12 times per year (1/12 unit per month) until the policyholders death (see Gerber, 1997).

Consider the ratio d_T of accumulated sum and the yearly salary at retirement time T and the annual annuity payment M payable monthly. Based on the assumption of net premium principle, we have the following relationship:

$$ d_T = M \ddot{a}_x^{(12)} \quad \Rightarrow \quad M = \frac{d_T}{\ddot{a}_x^{(12)}} \approx \frac{d_T}{\sum_{k=0}^{\infty} {}_k p_x (1+i)^{-k} - \frac{11}{24}} . $$

In Table 10 we present annual amounts of annuity payments (payable monthly) in case of various levels of savings d_T and technical interest rate i. These values are usually called replacement rates and represent the ratio of the last yearly salary to the yearly pension. Within our calculations we applied static probabilities of death drawn from the unisex life tables of the Statistical Office of the Slovak Republic applicable for year 2012 and we did not consider the dynamics of mortality and the potential longevity of future pensioners.[12]

To illustrate the calculated levels of replacement rates let us consider a person contributing to the second pillar 6% of the gross wage[13] (i.e. 1/3 of old-age contributions). This future pensioner will receive 2/3 of the pension from the first pillar designed for 50% replacement rate. Therefore, the saving pillar is efficient for this person if it delivers at least 17% replacement rate. Using Table 10 one can see, that such a replacement rate needs at least 2.5 – 3 yearly salaries saved (depending on the technical interest rate). Recall that the average level of savings using the baseline scenario was 2.5 times the yearly salary. Considering the risk associated with saving one can conclude that reaching the benchmark replacement rate is quite questionable.

T a b l e 10

Annual Amounts of Annuity Payments Expressed in Yearly Salary (Replacement Rates)

	Technical interest rater per annum (i)						
Accumulated sum in yearly salaries	0.00%	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
1.0	0.05	0.06	0.06	0.06	0.07	0.07	0.07
1.5	0.08	0.08	0.09	0.09	0.10	0.10	0.11
2.0	0.11	0.11	0.12	0.13	0.13	0.14	0.15
2.5	0.13	0.14	0.15	0.16	0.16	0.17	0.18
3.0	0.16	0.17	0.18	0.19	0.20	0.21	0.22
3.5	0.19	0.20	0.21	0.22	0.23	0.24	0.25
4.0	0.21	0.23	0.24	0.25	0.26	0.28	0.29

Source: Our calculations.

Conclusions

We have extended the dynamic stochastic accumulation model introduced firstly in Kilianová, Melicherčík and Ševčovič (2006) and later generalized in Melicherčík and Ševčovič (2010). As in the previous versions of the model, the

[12] Statistical Office of the Slovak Republic [Online.] Mortality Tables. [Cit. 22. 04. 2014]. URL <http://portal.statistics.sk/showdoc.do?docid=33032>.

[13] The average contribution rate from Table 1 is 5.63%. We have used close value of 6% for clearer illustration.

stock returns were modelled using the geometric Brownian motion, the interest rates followed the CIR model. The last legislative changes in Slovakia allow the pension asset managers to increase the duration of the bond fund. Therefore, we have generalized the model to account for any duration. As a result, the model may be, in addition to the Slovak scheme, utilized in any other DC scheme. Furthermore, the decrease of the contributions to the funded pillar in Slovakia from 9% to 4% also induced a necessity of new calculations. For better understanding of the results, we have recalculated the final savings to the replacement rate. Comparing to Kilianová, Melicherčík and Ševčovič (2006) and Melicherčík and Ševčovič (2010) our calculated estimates of the level of pensions from the funded pillar are lower. The achievement of the benchmark first pillar replacement rate is not certain.

Our results show that equities still play an important role in a pension investment. Especially at the beginning of saving, our model recommends to invest all savings in the stock fund. Later on, it is optimal to decrease the equity investments.

Since it is very difficult to estimate the parameters of the model, we have performed a sensitivity analysis for various parameter settings. We analyze sensitivity to the contribution rates, the equity return's drift and volatility, the pensioner's risk aversion, the duration of the bond fund, the wage growth and the absence of governmental regulations. The final level of savings is most of all sensitive to the contribution rate and the drifts of the stock returns.

The estimates of the drifts of the stock returns are usually unreliable. Notwithstanding, they are a crucial component of the model and can alter results significantly. Therefore, we have considered several strategies which have been tested against a set of scenarios of the drifts. The optimal strategy is not exclusive under all considered conditions. For a particular investor, the optimal strategy depends on the preferred criterion and indicator.

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