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Suboptimality of Immediate Annuitization in Private Pension Schemes[1]

Ján ŠEBO – Ľubica ŠEBOVÁ***

Abstract

Immediate annuitisation as a strategy for a welfare maximization for a payout phase in private pension schemes has been widely criticized. We examine the self-annuitisation strategies under two different consumption rates using programmed withdrawal compared to the immediate annuitization for a retired individual subject to uncertain portfolio returns and longevity risk. The aim is to examine the utility of both approaches under the existence of longevity risk on one side and bequest on the other. Results could serve as a basis for further discussion on improving the legislature on pay-out phase in Slovak private DC pension pillar.

Keywords: *annuity, programmed withdrawal, private DC pension, information asymmetry*

JEL Classification: D14, D81, E21, G18, G23

Introduction

Introduction of private defined contribution (DC) pension schemes in general means shifting the financial risk onto individuals. Obviously, financial risks can be split into two parts: investment risk occurring especially during the accumulation phase and annuity risk occurring at the moment of retirement. The investment

* Ján ŠEBO, Matej Bel University in Banská Bystrica, Faculty of Economics, Department of Public Economics and Regional Development, Tajovského 10, 975 90 Banská Bystrica, Slovak Republic; e-mail: jan.sebo@umb.sk

** Ľubica ŠEBOVÁ, Matej Bel University in Banská Bystrica, Faculty of Economics, Department of Tourism and Hospitality, Tajovského 10, 975 90 Banská Bystrica, Slovak Republic; e-mail: lubica.sebova@umb.sk

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risk can be described as a probability that lower than expected investment returns from the financial market in the accumulation phase will lead to a lower than expected accumulated wealth at retirement, leading to lower than expected pension income. The annuity risk on the other hand can be defined as a probability of lower than expected yields rates at retirement producing a higher than expected price of the annuity, leading to lower than expected pension income.

When discussing design of private DC schemes pay-out phase, the key point of the debate is the selection of suitable products for retirees. The second logical step is the decision on the retirement strategy, which in general means decision on the combination of various products during the retirement. If the immediate annuitization is the predefined option, annuity risk emerges. Buying annuity at any time is viewed as a sub-optimal choice. Timing of buying the annuity however requires having an alternative to finance the expenses until the annuity is accepted. If only two different products are allowed: annuity and programmed withdrawal, than the decision starts to be more complicated. Not only the annuity risk emerges, but additional risk should be recognized – risk of ruin (probability of outliving accumulated wealth before buying an annuity). Additionally, decision to postpone the annuity purchase is motivated by the existence of bequest.

Key research and regulatory question on defining an optimal pay-out phase strategy for rather inexperienced retirees under the above defined risks and bequest motive remains extensively discussed. It could be said that in many countries, actual pay-out phase set-up is far from optimal. Our research tries to contribute on this topic while applying current knowledge on self-annuitization strategies under the legislative conditions of Slovak 2. pillar (1bis pillar) pay-out phase implemented in 2014.

The paper is organized in order to present research finding on self-annuitization strategies by researchers in next chapter. Following chapter presents the information on Slovak 2. pillar pay-out phase regulation and thus defining the limitations for the research methodology. Then we present the methodology of our research and data for stochastic simulation. Last chapter discusses findings and recommendations for further research. In conclusion we summarize our findings and present potential steps for better regulation of pay-out options in Slovakia.

1. Review of Literature

In a number of contributions Milevsky and Robinson (1994; 1997; 2000), and Milevsky (1998), consider the ruin risk of self-annuitization. A self-constructed annuity consists of investing at retirement an initial endowment of wealth amongst the various asset categories (e.g. equity, bonds, real estate) represented by mutual funds, earning a stochastic rate of return, and withdrawing a fixed periodic amount for consumption purposes (Albrecht and Maurer, 2001). The financial risk of this strategy is that retirees can outlive their assets in the event of long-run low investment returns connected with longevity. This is in contrast to purchasing a life annuity, which is an insurance product that pays-out a life-long income stream to the retiree in exchange for a fixed premium charge. As Mitchell et al. (1999) pointed out; the main characteristic of the life annuity is that it protects retirees against the risk of under-funding in retirement by pooling mortality experience across the group of annuity purchasers. The particular advantage of the self annuitization strategy compared to the life annuity is the greater liquidity and the chance of leaving out money for their heirs in the case of an early death, but it is at the expense of running out of money before the uncertain date of death (Albrecht and Maurer, 2001).

In a well-cited paper from the public economics literature, Yaari (1965) proved that in the absence of bequest motives – and in a deterministic financial economy – consumers will annuitize all of their liquid wealth. Richard (1975) generalized this result to a stochastic environment, and Davidoff, Brown and Diamond (2003) demonstrates the robustness of the Yaari (1965) result. In practice, there are market imperfections, and frictions preclude full annuitization. Similarly, Brugiavini (1993) provides theoretical and empirical guidance on the optimal time to annuitize under various market structures.

As Milevsky and Young (2003) claim, comparing the drawdown option with the purchase of an annuity at retirement, two important points can be observed in literature: a retiree is given complete investment freedom (instead of locking the fund into bond-based assets, as is usual with annuities) and a bequest desire can be satisfied should the member die before buying the annuity (because in case of death the fund remains part of the individual's estate).

Problem of sub-optimality of immediate annuitization has been studied by Di Giacinto and Vigna (2012). Their preliminary conclusion suggests, that because of four key factors cannot be controlled (as some are linked to the financial market, some to mortality conditions, and some to personal preferences) it is evident that a pension system that imposes compulsory immediate annuitization to the whole universe of retirees is bound to be sub-optimal. Clearly, giving more flexibility to the decision maker has the effect of increasing her individual utility, and this holds in every context.

However, here they stated that, even if immediate annuitization might turn out to be optimal for the single retiree, it cannot be optimal for the universe of retirees in its globality.

Gerrard, Haberman and Vigna (2004) have dealt with the problem of managing the financial resources of a retiree after retirement, also due to the fact that life annuities are felt by policyholders as „poor value for money“ and have investigated other alternatives given to a retiree at retirement. In fact, retiree from a DC schemes takes the income drawdown option in the hope of doing better than buying an annuity at retirement. Therefore, it makes sense for them to have the wish of being able to buy a better annuity at a certain point of time after retirement than the annuity they would have purchased had they bought it at retirement. The option is thus taken with the final aim of buying a reasonably high pension and if the size of the fund allows the purchase of the high pension before the compulsory age the individual should stop investing the fund and lock it into an annuity. Therefore the existence of a finite maximum bound for the fund process would be realistic.

Milevsky and Robinson (2000) introduced the probability of lifetime ruin as a riskmetric for retirees, albeit in a static environment. As an extension of that work, Young (2004) determined the optimal dynamic investment policy for an individual who consumes at a specific rate, who invests in a complete financial market, and who does not buy annuities. The irreversibility of annuity purchases and their illiquidity creates a complex optimization environment, which renders many classical results inoperable.

Dus, Maurer and Mitchell (2005) conclude their research by presenting some interesting findings. First, they found discretionary management of accumulated assets with systematic phased withdrawals for consumption purposes offering the advantages of flexibility, bequests, and possibly higher rates of consumption than under a standard life annuity. However, they confirmed that phased withdrawal plans also require the retiree to dedicate effort to formulating asset allocation and withdrawal rules.

The personal risk of ruin from self-annuitization strategy is crucially dependent on the amount periodically withdrawn from the accumulated wealth (value of individual retirement account) as well as the fund asset allocation. The choice of a risk minimizing asset allocation with respect to a suitable benchmark for the amount of withdrawal still is an open question. In our paper we choose as a benchmark the amount generated by the single premium life annuity contract itself.

A phased withdrawal strategy paying the same benefit as an annuity exposes the retiree to the risk of outliving his assets while still alive. A phased withdrawal plan using a fixed withdrawal ratio avoids the risk of running out of money, since benefits fluctuate in tandem with the pension fund’s value. But the fixed benefit withdrawal rule affords lower risk than variable withdrawal rules, if one uses a mortality-weighted shortfall-risk measure. When looking at the probability of ruin and bequest, Dus, Maurer and Mitchell (2005) found that mandatory deferred annuitization with a fixed withdrawal rule can enhance expected payouts and cut expected shortfall risk but at the cost of reduced expected bequests, as compared to no annuity. For a variable withdrawal plan, a simple deferred annuitization may not reduce risk: rather, it requires optimization of the withdrawal ratio.

2. Slovak 2. Pillar Pay-out Phase Design

Legislation regulating pay-out phase in Slovak 2. pillar private DC schemes has been motivated predominantly by reducing the longevity risk to zero. To achieve such objective, the predefined option for the most of savers retiring from 2. pillar is a full immediate annuitization.

However, for a certain high-income cohort (roughly 10% of savers), programmed withdrawal as an option is available. To be eligible for programmed withdrawal, a retiree must prove that his retirement income from PAYG pillar and other retirement schemes is secured for the longevity risk and at the same time the cumulative amount of benefits paid are higher than the old-age benefit paid from the PAYG scheme calculated for a person with income higher than 1.25 of the average wage for a full (42 years) working career (equal to 561 EUR in 2015) or that the paid benefit from the PAYG pillar is higher than 4-times the living minimum for a single person (200 EUR in 2015)

A person wishing to receive benefits from 2. pillar is obliged to ask public administrator (Social Insurance company) for offers of annuities from life insurance companies licensed by National Bank of Slovakia (financial sector regulator). Once the offers are supplied via centralized offering system (CIPS – Central Information and Offering System), the offers for various annuities and other products (if eligible) are presented to a retiree on a single page for better comparison.

There are no limits or caps on fees applied for life insurance companies when calculating annuities. However, there is a legislative limit on using unisex life-tables and the only risk that can be used for calculating annuities is the age. No other individual risks (health status, occupation, etc.) can be calculated. The idea was to secure the same benefit from buying annuity for persons who are of the same age and same accumulated wealth regardless their sex, health status, occupation, residency and other individual risk factors.

First annuities from the Slovak 2. pillar private DC schemes have started to be offered on the market since January 2015. Conservative approach of life insurance companies generated average annuity rates around 4.75% which were deemed as to low considering the life expectancy of 17.91 years for a 62 years old person. This fact leaves the debate on increasing the competition by allowing substitutional products (especially programmed withdrawal) open.

Finding the optimal self-annuitization strategy for decumulation phase based on the mix of programmed withdrawal and moment of annuitization is the main incentive of our contribution. Understanding the risks associated to the designing the optimal programmed withdrawal/annuity mix, following parts of our paper discuss the probability of ruin as well as the bequest motive and the ability of an unexperienced individual to select the best available offer at a moment the decision is being made.

3. Research Methodology

In order to investigate the optimal product mix of programmed withdrawal and annuity purchase timing, several formulas for pricing annuity and defining withdrawal strategy has to be defined. Further on, probability of ruin as well as value of bequest has to be estimated. The last part is to define parameters for technical reserves´ returns (in case of annuity) and investment portfolio (in case of programmed withdrawal) returns.

First we have to estimate the value of monthly annuity benefits (S_m) for each simulation that serves as a benchmark for programmed withdrawal (B_t). Let us therefore present a simple single annuity model and consider a person of age X years. The probability that this person dies within the next year is denoted by q_x. The probability of a complementary event, i.e., that the person aged X years will survive to age (X + 1), is defined by $p_x = 1 - q_x$. The one-year probabilities of death q_x are usually known for $x \in \{0, 1, 2, \ldots\}$, given in life tables. Generally, ${}_k p_x$ denotes the probability that the person of age X will survive at least k consecutive years and is defined by (Melicherčík, Szücz and Vilček, 2015):

$$ {}_k p_x = p_x p_{x+1} \cdots p_{x+k-1} = \prod_{h=0}^{k-1} p_{x+h} = \prod_{h=0}^{k-1} \left(1 - q_{x+h}\right), \quad k = 1, 2, 3, \ldots \tag{1} $$

Basic single annuity generates a monthly payments of 1 unit as long as the policyholder lives (payments are made at the beginning of each month). The expected net present value of the aforementioned annuity payments is denoted by $\ddot{a}_x^{(12)}$. The formula is as follows (Gerber, 1997):

$$ \ddot{a}_x^{(12)} \approx \left(\sum_{k=0}^{\infty} {}_k p_x \left(1+i\right)^{-k} \right) - \frac{13}{24} \tag{2} $$

where

i – the technical interest rate per annum;

$\ddot{a}_x^{(12)}$ – the net present value of an annuity of 1 unit per year payable 12 times per year (1/12 unit per month) until the policyholder's death.

For defining the monthly nominal benefit from annuity purchase under the existence of 7 year pay-off guarantee stipulated by Slovak legislation on 2. pillar annuities (A_m), we use actuarial formula and associated conditions presented by Szücs (2015):

$$A_m \approx \frac{1}{12} \times \frac{(1-\gamma)P}{(1+\beta)\left(\ddot{a}_x - \frac{13}{24}\right) + \alpha + (1-\varepsilon)M} \tag{3}$$

where

$\alpha; \beta; \gamma; \varepsilon$ – the charges (initial costs for the first year of the contract; on-going monthly administration fees; one-off collection fee and guarantee payment costs);

M – the uncertain value of 7 year guarantee paid to the beneficiaries in case of policyholder´s death with the first 7 years of annuity purchase;

P – the value of savings (wealth) at the end of saving phase.

The path of benefits payable under a programmed withdrawal rule can be formalized as follows. Let *W(0)* be the value of the retirement assets at the beginning of retirement period before the withdrawal B_t for each month is made. A retiree can withdraw a certain sum *(*B_t*)* each month from a remaining assets using two approaches: fixed withdrawal rate set at the beginning of retirement using formula (4) or dynamically set each year using formula (5). At the beginning of period *t*, an ex-ante specified fraction *(c)* set at the beginning of the retirement is withdrawn from current wealth. Withdrawal rate can be set as fixed, hence the retiree receives a fixed sum of benefit for each period set at the beginning of the retirement:

$$B_t = \frac{cW(0)}{12} \tag{4}$$

Secondly, the withdrawal rate can be set as dynamic *(*c_t*)*, where the sum of benefit changes every year according to a formula:

$$B_t = \frac{c_t W(0)}{12} \tag{5}$$

Formally, under a self-annuitization strategy, the wealth process of the retiree using uncertain return *r* for a given period can be expressed by following equation:

$$W_{t+1} = r_{t+1}(W_t - B_t) \tag{6}$$

Hence, the ordinary differential equation is:

$$dW(t) = \left(rW(t) - c\right)dt,\ W(0) = 1 \tag{7}$$

If the retiree enters the retirement phase with wealth *W(0)* equal 1, invests at a rate of *r*, and withdraw at rate *c*, wealth increases at the expected return of portfolio minus the withdrawal rate. The solution to this ordinary differential equation is:

$$W(t) = e^{rt} - c\left(\frac{e^{rt} - 1}{r}\right),\ t \le t^* \tag{8}$$

where

*t** – the point in time at which the iteration process reaches the value of 0 (wealth is ruined).

Additional task is to construct retirement investment strategies based on the allocation (*A*) of wealth into two different pension funds (bond and equity pension fund). Gross returns need to be adjusted for the fee policy applied by pension funds asset managers in Slovakia. Net return for a given period after fees can be expressed as follows (Mešarová, Šebo and Balco, 2015):

$$r_t^F = \frac{r_t^s - \dfrac{F^M + F^D}{n^Y}}{1 + \left(F^P\left(\dfrac{CVPU_{t-1}\left(1 + r_t^{F^M}\right)}{\max CVPU_{t-m}} - 1\right)\right)} \tag{9}$$

where

- *CVPU* – means current value of pension unit and represents the market value of 1 pension fund unit;
- $r^F(t,\ t+1)$ – net, after management (F^M), custodian (F^D) and performance fees (F^P), returns of pension fund in the time interval $[t;\ t+1)$;
- n^Y – the number of periods (e.g. business days, months, quarters…) per year for which the returns are generated.

Gross daily returns (*r*) are generated using 96.5 years of daily historical data on equity and bond returns in US. The data for historical equity returns for Dow Jones and 3 – 5 years government bonds since January 1919 till June 2015 were retrieved from the Federal Reserve Economic Data database of Federal Reserve Bank of St. Louis (FRED, 2015).

Assuming the future returns are uncertain, we construct retirement investment strategy for self-annuitization, and present the results of our analysis. Defined retirement strategies for our research are as follows:

1. DGDF (Bond Guaranteed Pension Fund) strategy, which invests only in low-risk bond pension fund (*b*) for a whole retirement period (t_0, …., *T*);

2. INDF (Index Non-Guaranteed Pension Fund) strategy, which invests only in high-risk equity pension fund (*s*) for a whole retirement period (t_0, …., *T*);

3. EQUAL strategy, which invests equally (50:50) in both pension funds for a whole retirement period (t_0, …., *T*);

4. DYNAMIC (dynamic portfolio management) strategy, which allocates certain proportion of remaining wealth into risky equity pension fund (I_s) for the next month based on the change of the exponential moving average of equity pension fund returns for a defined period ($EMAr_s$) compared to the change of the exponential moving average of bond pension fund returns ($EMAr_b$) using following conditional equation:

$$I_s = \begin{cases} 0,5 \, if \sum_{n=1}^{60} \Delta EMAr_s > \sum_{n=1}^{120} \Delta EMAr_b \\ 0, \; otherwise \end{cases} \tag{10}$$

Probability of ruin is than given as a function of current time, wealth (*W*) at that time, benefit (*B*) paid from the remaining wealth and portfolio return (*r*). To inspect the probability of ruin from the proposed self-annuitization retirement investment strategies, we search for the time, when the wealth hits the zero value (t^*).

Life expectancy of 62 years old retiree, which defines the total time *T*, was originally set using empirical life tables taken from Slovak Statistical Office from 2014 at 17.91 years. However, we performed stress-testing, where individual life expectancy was increased by 5%, 10% and 15%, respectively. Thus the life expectancy was multiplied by 1.05, 1.1 and 1.15 and the *T* was set at 21, 22 and 23 years, respectively.

Next, we present the withdrawal strategies defining the withdrawal rate (*c*). The first strategy is based on Milevsky (2001) present value approach, where the withdrawal rate (c_t^r) is equal to the 10 year annualized returns of equity r^s and bond r^bpension fund, respectively. Thus the withdrawal rate for a given year is:

$$c_t^r = r^{s;b} \tag{11}$$

Intuitively, setting the withdrawal rate equal to long-term return of a pension fund allows for a smoothing of benefits and securing for the probability of ruin.

Second withdrawal strategy is based on the Milevsky´s (2001) sustainable retirement income (SRI) approach. Withdrawal rate is based on historical 10-year average of annual returns ($r^{s;b}$) of a pension fund used for continuing investment of remaining wealth (W) adjusted for volatility of pension fund returns (δ_t^2) calculated for the last 10 years and life expectancy of a retiree ($\frac{\ln(2)}{e_x}$) at moment of making the decision on withdrawal rate. The equation for withdrawal rate (c_t^{SRI}) is as follows:

$$c_t^{SRI} = r_t^{s;b} - \delta_t^2 + \frac{\ln(2)}{e_x} \quad (12)$$

Each Iteration process starts with the initial retirement wealth *W(0)* set at 20 000 EUR. Tables presented in the next chapter contains statistics for respective benefits *(B_t)* and final wealth defined as bequest for various longevity risks scenarios. For each investment strategy and withdrawal approach, the annual withdrawal rate (benefit ratio) can be recalculated recursively using formula:

$$c_0 = 12\frac{B_t}{W(0)} \quad (13)$$

Introducing uncertainty of equity and bond returns with the existence of correlation among them requires presenting a stochastic method. We perform simulations using historical daily data on US equity and bond returns by applying a widely used method in financial econometrics, namely the moving block bootstrap. The basic idea of the block bootstrap is closely related to the i.i.d. nonparametric bootstrap (Vogel and Shallcross, 1996). Moving block bootstrap is based on drawing observations with replacement. In the block bootstrap, instead of relying on single observations, blocks of consecutive observations are drawn. This is done to capture the dependence structure of neighbored observations. This method allowed us to overcome the problem with capturing close relations among bond and equity returns during the whole pay-out period.

It has been shown that this approach works for a large class of stationary processes (Gilbert and Troitzsch, 2005). The blocks of consecutive observations are drawn with replacement from a set of blocks. By construction, the bootstrap time series has a nonstationary (conditional) distribution. The moving blocks bootstrap is a simple resampling algorithm, which can replace the parametric time series models, avoiding model selection and only requiring an estimate of the moving block length (*l*). In our case, the block length (*l*) is defined by the stressed life expectancy of a 62 year old retiree. Thus we define the block length (*l*) based on the defined life expectancies of a 62 year old retiring individual using 2014 life tables for Slovakia presented by Výskumné a demografické centrum Infostat (VDC Infostat, 2015). For each unit of a block bootstrap, a vector of variables is defined. Pulling consecutive block of data out from the database of 96.5 years of daily data of variables, each block (k) than consists of variable observations $(X_{k-1+1}), j=1, \ldots, l$. Then the simulation is performed for each block (k).

At the end, we get a 2 x 4 x 4 matrix of strategies for withdrawal rate (c_t) and investing of remaining wealth under the longevity risk scenarios, for which we inspect the probability of ruin. By performing 1,000 simulation for each combination, we get the cumulative probability of ruin and value of bequest. In total we have performed 32,000 simulations using the same blocks and simulation sequences (simulation seeds) to be able to compare various investing and withdrawal strategies. Simulations were performed in MS Excel environment using Palisade @RISK software.

4. Results and Discussion

First we present the results for the most conservative strategy, where the withdrawal rate calculated using equation (11) is set at the beginning of retirement and does not change over time (fixed withdrawal rate). Remaining wealth during retirement is invested entirely into bond pension fund (DGDF strategy). The table 1 presents selected statistics on benefits and expected value of bequest in case of death under different longevity risk.

T a b l e 1

Benefits and Bequest (DGDF/ c_t^r) Strategies – Fixed Withdrawal Rate (in EUR)

Investment / Withdrawal Strategy	**Longevity risk scenario**	**Min**	**Mean**	**Max**	***5%***	***95%***
DGDF/ c_t^r Benefit	*#1*	41.83	43.50	45.57	43.18	43.91
DGDF/ c_t^r Benefit	*#1.05*	41.40	43.51	45.32	43.18	43.93
DGDF/ c_t^r Benefit	*#1.1*	40.46	43.51	46.44	43.16	43.93
DGDF/ c_t^r Benefit	*#1.15*	41.58	43.50	46.12	43.13	43.88
DGDF/ c_t^r Bequest	*#1*	16 335.99	41 622.30	99 530.39	17 042.07	86 235.52
DGDF/ c_t^r Bequest	*#1.05*	16 179.21	43 712.24	110 548.90	16 903.56	90 921.10
DGDF/ c_t^r Bequest	*#1.1*	16 127.29	46 003.42	121 331.80	16 934.98	98 276.77
DGDF/ c_t^r Bequest	*#1.15*	16 043.44	48 354.37	123 822.80	16 919.20	104 609.50

Source: Own calculations using MikroSIM model.

None of the simulations for the DGDF/ c_t^r strategy hit zero values of final wealth. In general, the average withdrawal rate was at 2.61%, with low volatility (0.1%) which can be deemed low comparing to the offered annuity rate at

4.75%. However, average bequest reached the ratio of more than 2 compared to the initial level of savings. This combination of investment/withdrawal strategy is suitable when the bequest is preferred by a retiree. In fact, if we increase individual life expectancy the value of final wealth increases over time.

Second strategy combines investment into bond pension fund (DGDF strategy) and the withdrawal strategy based on equation (12) that is set at the beginning of retirement and does not change over time (fixed withdrawal rate). The results are presented in Table 2 below.

T a b l e 2

Benefits and Bequest (DGDF/ c_t^{SRI}) Strategies – Fixed Withdrawal Rate (in EUR)

Investment / Withdrawal Strategy	Longevity risk scenario	Min	Mean	Max	*5%*	*95%*
DGDF/ c_t^{SRI} Benefit	*#1*	68.08	93.43	166.04	68.93	143.70
DGDF/ c_t^{SRI} Benefit	*#1.05*	67.02	93.24	165.65	67.82	144.74
DGDF/ c_t^{SRI} Benefit	*#1.1*	66.07	93.08	168.42	66.74	146.49
DGDF/ c_t^{SRI} Benefit	*#1.15*	65.02	92.85	169.60	65.74	147.56
DGDF/ c_t^{SRI} Bequest	*#1*	8 580.92	18 101.63	37 540.43	8 837.09	34 268.81
DGDF/ c_t^{SRI} Bequest	*#1.05*	8 253.57	18 097.68	39 115.95	8 507.76	34 529.59
DGDF/ c_t^{SRI} Bequest	*#1.1*	7 947.65	18 118.29	41 044.85	8 228.45	35 100.59
DGDF/ c_t^{SRI} Bequest	*#1.15*	7 653.23	18 117.23	39 341.59	7 982.04	35 472.00

Source: Own calculations using MikroSIM model.

Again, this combination delivered no risk of ruin and can be considered conservative with relatively good benefits (average benefit ratio of 5.6%). However, the volatility of benefit ratio is higher (2.1%). Compared to the previous combination, this one promises higher benefits, though at the expense of lower value of bequest, which stood at the average rate of 0.9.

Further, we analyze the combination of withdrawal strategies with the investing into equity pension fund. We use both approaches (see formulas 4 and 5) for setting the withdrawal rate (fixed as well as dynamic). This is made due to the higher volatility of equity pension fund returns. The results are presented in tables and respective Figures 3 below.

Investing in equity pension fund under the annual recalculation of withdrawal rate could be viewed as an acceptable alternative for programmed withdrawal because of rather high benefit ratio (7.95%), however high volatility of annual benefits can be expected. Rather surprising result is the fact, that under both withdrawal strategies, probability of ruin under various life expectancies is zero. On the other hand, if no annual recalculation of withdrawal rates is applied, the results are significantly different (Table 4 a Figure 1 below).

T a b l e 3

Benefits and Bequest (INDF/ c_t^r) and (INDF/ c_t^{SRI}) Strategies – Dynamic Withdrawal Rates (in EUR)

Investment / Withdrawal Strategy	Longevity risk scenario	Min	Mean	Max	*5%*	*95%*
INDF/ c_t^r Benefit	*#1*	33.65	121.90	292.23	52.93	226.34
INDF/ c_t^r Benefit	*#1.05*	33.77	122.84	295.66	52.97	227.43
INDF/ c_t^r Benefit	*#1.1*	33.72	127.94	302.60	52.98	228.63
INDF/ c_t^r Benefit	*#1.15*	34.07	128.73	312.55	53.02	232.79
INDF/ c_t^r Bequest	*#1*	3 422.71	22 803.19	99 984.85	7 857.89	69 869.27
INDF/ c_t^r Bequest	*#1.05*	3 420.34	20 851.51	88 968.81	6 726.34	66 258.56
INDF/ c_t^r Bequest	*#1.1*	2 917.41	19 870.53	82 182.28	5 939.88	61 774.94
INDF/ c_t^r Bequest	*#1.15*	2 898.95	17 829.51	81 195.15	5 332.32	58 688.61
INDF/ c_t^{SRI} Benefit	*#1*	38.67	130.97	343.56	52.98	233.58
INDF/ c_t^{SRI} Benefit	*#1.05*	38.56	133.94	356.41	52.89	235.54
INDF/ c_t^{SRI} Benefit	*#1.1*	37.87	136.71	370.77	53.53	246.49
INDF/ c_t^{SRI} Benefit	*#1.15*	37.33	138.90	365.56	55.08	257.46
INDF/ c_t^{SRI} Bequest	*#1*	2 860.95	21 696.65	92 573.59	7 257.29	61 868.27
INDF/ c_t^{SRI} Bequest	*#1.05*	2 330.91	20 542.19	87 818.48	6 626.24	61 958.56
INDF/ c_t^{SRI} Bequest	*#1.1*	2 114.49	19 214.28	82 005.66	6 039.38	59 764.94
INDF/ c_t^{SRI} Bequest	*#1.15*	1 936.83	17 826.78	83 169.91	5 032.72	56 628.61

Source: Own calculations using MikroSIM model.

T a b l e 4

Benefits and Bequest (INDF/ c_t^{SRI}) and (INDF/ c_t^r) Strategies – Fixed Withdrawal Rate (in EUR)

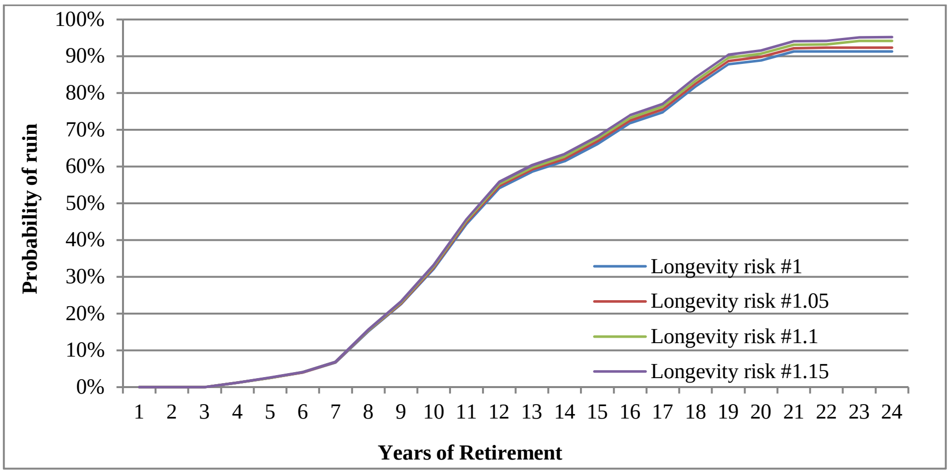
Investment / Withdrawal Strategy	Longevity risk scenario	Min	Mean	Max	*5%*	*95%*
INDF/ c_t^r Benefit	*#1*	191.15	207.32	221.19	195.15	214.44
INDF/ c_t^r Benefit	*#1.05*	189.74	207.52	222.22	194.25	214.47
INDF/ c_t^r Benefit	*#1.1*	187.22	207.47	224.11	193.74	214.87
INDF/ c_t^r Benefit	*#1.15*	185.02	207.49	224.15	191.97	214.82
INDF/ c_t^r Bequest	*#1*	–	1 385.84	58 201.55	–	6 191.45
INDF/ c_t^r Bequest	*#1.05*	–	1 365.27	74 985.20	–	3 979.73
INDF/ c_t^r Bequest	*#1.1*	–	1 314.14	78 362.22	–	1 480.92
INDF/ c_t^r Bequest	*#1.15*	–	1 105.02	81 858.28	–	114.92
INDF/ c_t^{SRI} Benefit	*#1*	204.62	215.62	225.33	212.69	218.28
INDF/ c_t^{SRI} Benefit	*#1.05*	199.76	215.52	227.45	212.25	218.17
INDF/ c_t^{SRI} Benefit	*#1.1*	194.79	215.57	224.44	212.54	218.30
INDF/ c_t^{SRI} Benefit	*#1.15*	199.12	215.57	229.77	212.50	218.32
INDF/ c_t^{SRI} Bequest	*#1*	–	1 285.94	55 106.80	–	6 191.45
INDF/ c_t^{SRI} Bequest	*#1.05*	–	1 266.93	76 313.10	–	3 979.73
INDF/ c_t^{SRI} Bequest	*#1.1*	–	1 297.16	75 773.72	–	1 480.92
INDF/ c_t^{SRI} Bequest	*#1.15*	–	1 063.63	67 676.83	–	114.92

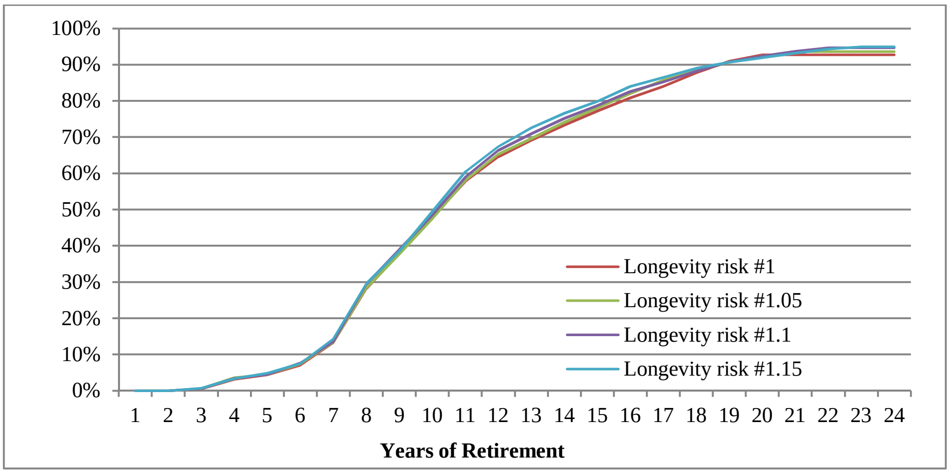
Source: Own calculations using MikroSIM model.

Ignoring annual recalculation of withdrawal rates which accept the adjustments in returns of the remaining wealth and using fixed withdrawal rate set at the beginning of retirement could lead to a false expectations on the sustainability of benefits and thus increases the risk of ruin. Figure 1 below presents the cumulative probability of ruin under this combination of strategies without annual recalculation of withdrawal rate and using fixed withdrawal rate set by using formula (4).

Figure 1

Probability of Ruin for INDF/ c_t^r and INDF/ c_t^{SRI} with Fixed Withdrawal Rate





Source: Own elaboration.

Understanding the fact, that annual recalculation of withdrawal rate adjusts the benefit from programmed withdrawal and helps minimizing the probability of ruin, further presentation of results is oriented on combination of strategies where the withdrawal rates are defined at the beginning of retirement and ignore annual recalculations (fixed withdrawal rates). In all cases, when withdrawal rates are recalculated annually (dynamically set), probability of ruin is close to zero and therefore only the value of bequest can be discussed further. At the same time we can conclude, that annual recalculation of withdrawal rates generates significantly higher volatility of benefits compared to the benefits where the withdrawal rate is set at the beginning of the retirement and does not change over time.

Considering the next investment strategy EQUAL, where the remaining wealth is equally invested into equity and bond pension fund, one would expect that the probability of ruin would decrease even if the withdrawal rate is fixed. At the same time expected benefit should be lower than in INDF strategy and higher than in DGDF strategy. Using fixed withdrawal rate formula (4) for EQUAL strategy returned initial withdrawal rates between 6.99% and 8.00% annually. The results are presented in Table 5 and Figure 2 below.

T a b l e 5

Benefits and Bequest (EQUAL/ c_t^r) and (EQUAL/ c_t^{SRI}) Strategies – Fixed Withdrawal Rate (in EUR)

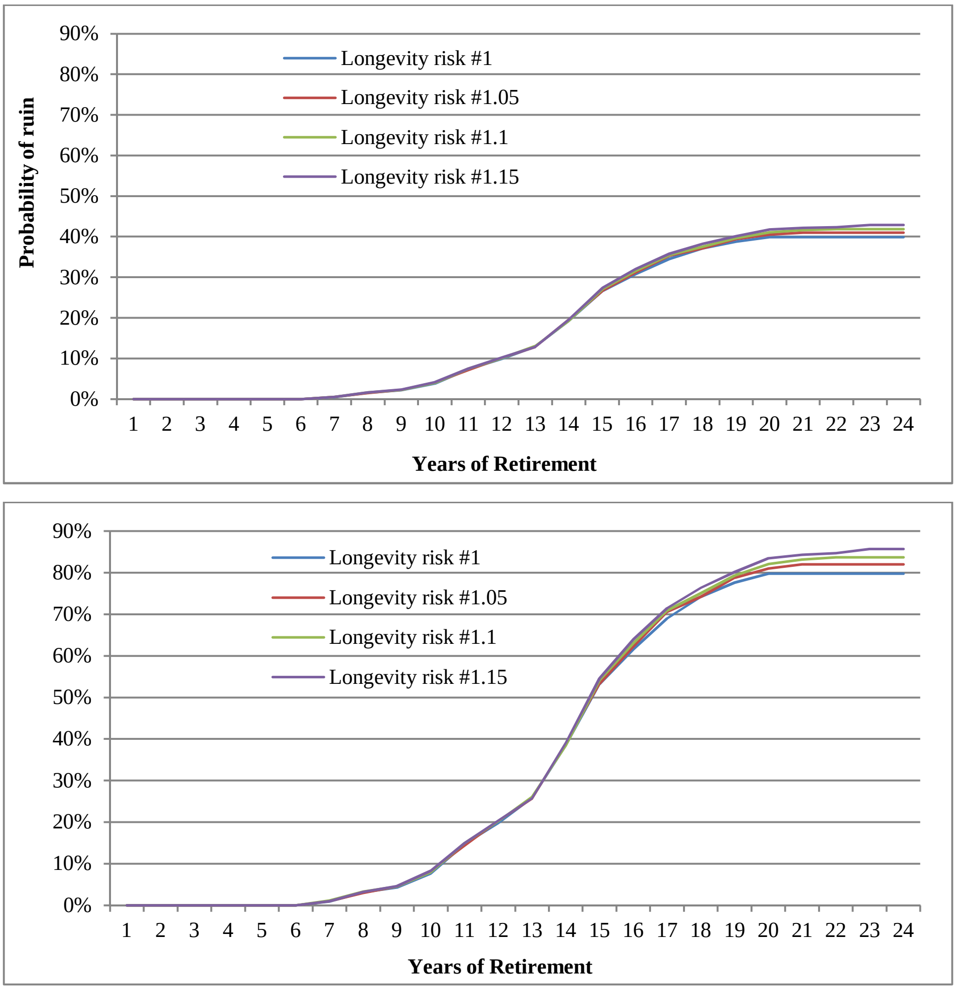
Investment / Withdrawal Strategy	Longevity risk scenario	Min	Mean	Max	*5%*	*95%*
EQUAL/ c_t^r Benefit	*#1*	116.49	125.41	133.38	119.17	129.18
EQUAL/ c_t^r Benefit	*#1.05*	115.57	125.52	133.77	118.72	129.20
EQUAL/ c_t^r Benefit	*#1.1*	113.84	125.49	135.28	118.45	129.40
EQUAL/ c_t^r Benefit	*#1.15*	113.30	125.50	135.14	117.55	129.35
EQUAL/ c_t^r Bequest	*#1*	–	21 504.07	78 865.97	3 521.04	46 213.49
EQUAL/ c_t^r Bequest	*#1.05*	–	22 538.76	92 767.05	3 451.78	47 450.42
EQUAL/ c_t^r Bequest	*#1.1*	–	23 658.78	99 847.01	3 467.49	49 878.85
EQUAL/ c_t^r Bequest	*#1.15*	–	24 729.70	102 840.54	3 459.60	52 362.21
EQUAL/ c_t^{SRI} Benefit	*#1*	152.68	158.11	162.90	156.52	159.53
EQUAL/ c_t^{SRI} Benefit	*#1.05*	150.17	158.06	163.98	156.36	159.49
EQUAL/ c_t^{SRI} Benefit	*#1.1*	147.67	158.08	162.58	156.52	159.49
EQUAL/ c_t^{SRI} Benefit	*#1.15*	150.39	158.08	165.09	156.54	159.49
EQUAL/ c_t^{SRI} Bequest	*#1*	–	3 457.47	56 699.51	–	24 246.88
EQUAL/ c_t^{SRI} Bequest	*#1.05*	–	3 311.29	67 194.55	–	24 821.94
EQUAL/ c_t^{SRI} Bequest	*#1.1*	–	3 282.69	65 480.06	–	26 444.41
EQUAL/ c_t^{SRI} Bequest	*#1.15*	–	3 083.12	65 361.90	–	23 447.38

Source: Own calculations using MikroSIM model.

Following Figure 2 presents probability of ruin for both withdrawal strategies under the EQUAL investment strategy.

F i g u r e 2

Probability of Ruin for EQUAL/ c_t^r and EQUAL/ c_t^{SRI} with Fixed Withdrawal Rate



Source: Own elaboration.

Using present value approach defined in formula (11) for withdrawal rate generates lower probability of ruin, however the benefits are on average 25% lower than using sustainable retirement income (SRI) approach defined in formula (12). SRI approach on the other hand promises a very high benefit ratio (9.48%). Both withdrawal strategies are not able to secure that bequest can be expected. However, if we consider the worst 5% of return scenarios, present value approach is able to deliver the bequest ratio of 0.15.

The last investment strategy (DYNAMIC) uses trends in returns and market timing. Logically, this strategy requires active approach. Simulation results are presented in Table 6 and Figure 3.

Table 6

Benefits and Bequest (DYNAMIC/ c_t^r) and (DYNAMIC/ c_t^{SRI}) Strategies – Fixed Withdrawal Rate (in EUR)

Investment / Withdrawal Strategy	Longevity risk scenario	Min	Mean	Max	5%	95%
Discretive/ c_t^r Benefit	#1	60.25	77.45	89.32	67.71	86.03
Discretive/ c_t^r Benefit	#1.05	64.28	78.16	88.83	68.89	86.42
Discretive/ c_t^r Benefit	#1.1	65.11	78.86	88.52	69.91	86.72
Discretive/ c_t^r Benefit	#1.15	65.12	79.43	89.11	71.11	86.88
Discretive/ c_t^r Bequest	#1	–	32 984.26	144 298.50	7 799.92	84 573.33
Discretive/ c_t^r Bequest	#1.05	–	33 914.77	139 604.50	6 950.38	91 619.56
Discretive/ c_t^r Bequest	#1.1	–	35 031.89	153 262.70	6 224.85	98 597.84
Discretive/ c_t^r Bequest	#1.15	–	36 349.54	158 794.80	4 932.96	108 730.90
Discretive/ c_t^{SRI} Benefit	#1	114.97	130.19	140.70	121.70	137.72
Discretive/ c_t^{SRI} Benefit	#1.05	118.35	130.80	140.30	122.68	138.10
Discretive/ c_t^{SRI} Benefit	#1.1	119.26	131.41	139.94	123.65	138.27
Discretive/ c_t^{SRI} Benefit	#1.15	119.23	131.91	140.58	124.65	138.40
Discretive/ c_t^{SRI} Bequest	#1	–	11 201.78	107 478.30	–	50 338.02
Discretive/ c_t^{SRI} Bequest	#1.05	–	10 879.83	101 239.10	–	54 563.45
Discretive/ c_t^{SRI} Bequest	#1.1	–	10 764.44	113 031.30	–	55 927.86
Discretive/ c_t^{SRI} Bequest	#1.15	–	10 836.57	113 865.50	–	59 690.02

Source: Own calculations using MikroSIM model.

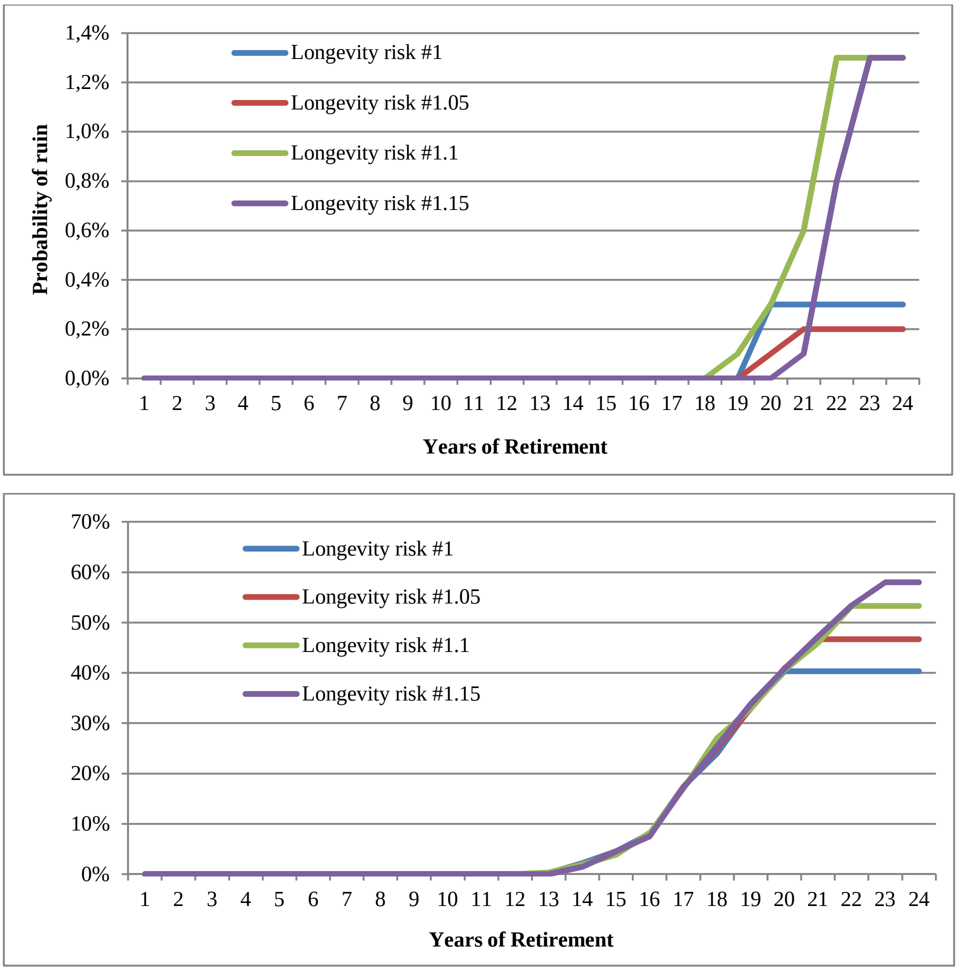
Both withdrawal approaches (present value as well as sustainable retirement income) that uses fixed withdrawal rates under the DYNAMIC investment strategy are not able to secure certain bequest. However, when considering present value approach, DYNAMIC investment strategy is able in 95% of simulations to deliver bequest ratio of 0.35, e.g. the expected bequest in case of death after 20 years could be higher than 35% of initial wealth.

Probability of ruin under the DYNAMIC investment strategy and present value approach for fixed withdrawal rates is significantly low. This combination promises rather high bequests even under the stressed scenario of long life expectancy. At the same time, probability of ruin starts occurring after 19 years, which is fairly late. The shortfall is the relatively low benefit ratio (only 4.68%).

Finally, we looked at the probability of ruin if withdrawal rates are set discretionally at the beginning of retirement. The idea is motivated by having the same benefit ratio and annuity rate, so an individual can investigate, how much risk of ruining his wealth will be transferred onto him and what kind of reward in form of bequest can be expected. We compared all investment strategies under various discretionally set fixed withdrawal rates using the longest life expectancy of 23 years.

Figure 3

Probability of Ruin for DYNAMIC/ c_t^r and DYNAMIC/ c_t^{SRI} with Fixed Withdrawal Rate



Source: Own elaboration.

Intuitively, riskier investment strategies could provide higher bequests with lower probability. DYNAMIC strategy can provide higher probability as well as level of bequest than conservative bond strategy. One could argue, that the probability of ruin should be investigated in a way how it evolves over time during retirement (Figure 4). As it can be seen on Figure 4, DYNAMIC strategy delivers interestingly low probability of ruin, while maintaining relatively high values of bequest.

The results of the bequests and respective benefits under various longevity risk scenarios are presented in Table 7 and Figure 4 below.

T a b l e 7

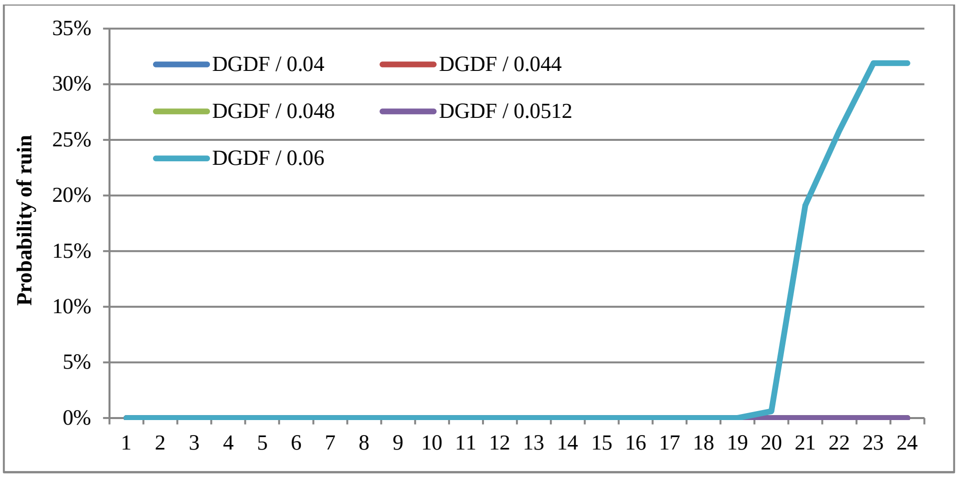
Bequest of Strategies under Defined Fixed Benefit Ratios

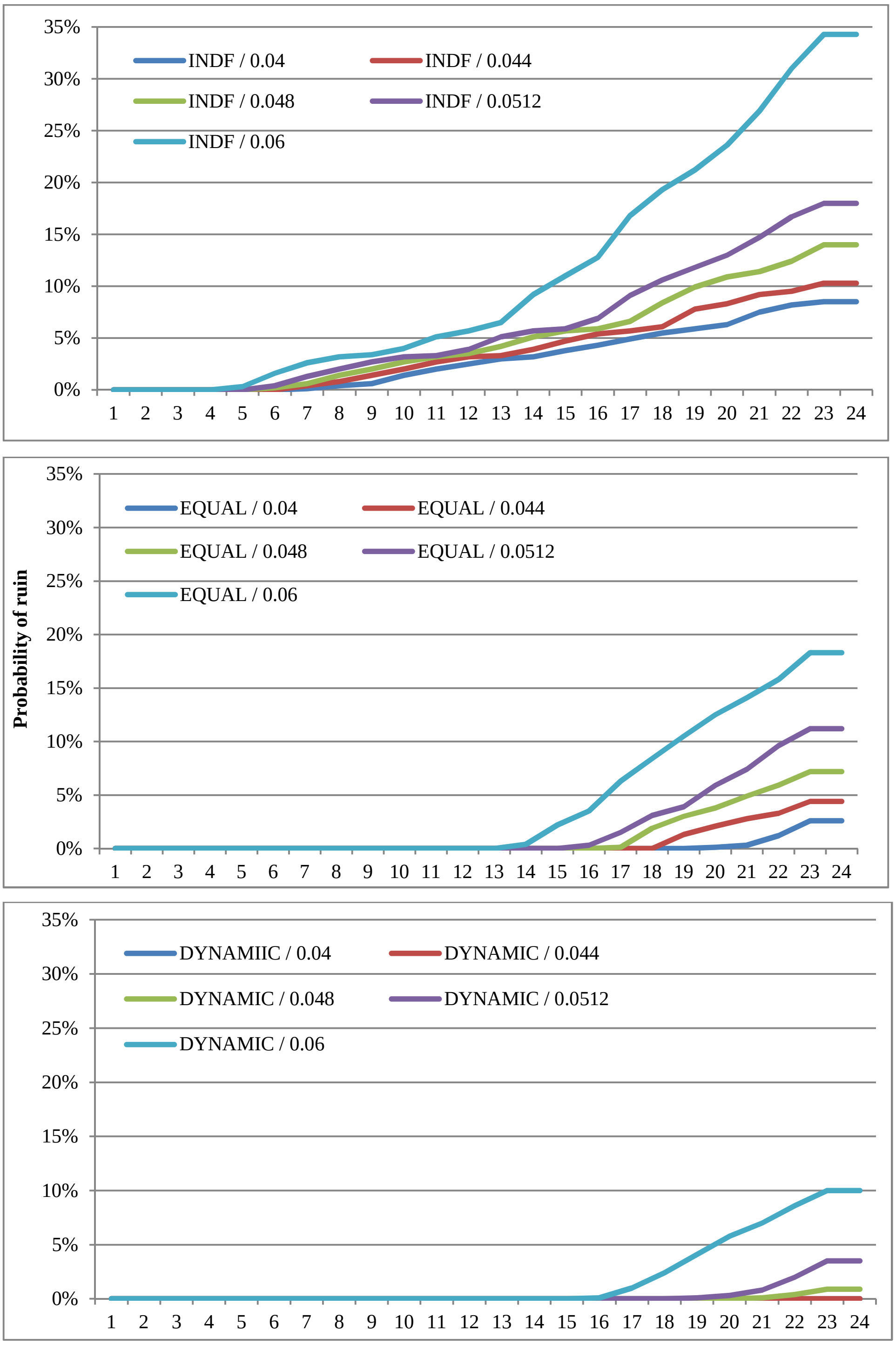
Investment Strategy	Withdrawal rate	Min	Mean	Max	*5%*	*95%*
DGDF Bequest	*0.04*	8 126.46	35 324.77	105 912.90	8 631.17	85 484.61
DGDF Bequest	*0.044*	5 842.47	31 574.64	100 768.80	6 237.84	79 997.93
DGDF Bequest	*0.048*	3 504.02	27 824.50	95 624.74	3 799.96	75 020.29
DGDF Bequest	*0.0512*	1 536.11	24 824.39	91 509.46	1 887.97	70 649.41
DGDF Bequest	*0.06*	–	17 367.19	80 192.45	–	59 058.92
INDF Bequest	*0.04*	–	50 402.90	227 641.40	–	156 579.40
INDF Bequest	*0.044*	–	45 971.84	220 409.30	–	148 903.00
INDF Bequest	*0.048*	–	41 649.70	213 177.20	–	140 670.50
INDF Bequest	*0.0512*	–	38 318.36	207 391.50	–	132 888.70
INDF Bequest	*0.06*	–	29 996.80	191 480.80	–	115 336.20
EQUAL Bequest	*0.04*	–	42 561.38	156 563.00	3 354.29	118 708.80
EQUAL Bequest	*0.044*	–	38 425.91	150 720.40	644.47	112 068.60
EQUAL Bequest	*0.048*	–	34 352.28	144 877.70	–	105 679.50
EQUAL Bequest	*0.0512*	–	31 183.32	140 203.60	–	100 039.20
EQUAL Bequest	*0.06*	–	22 763.02	127 349.80	–	86 301.59
DYNAMIC Bequest	*0.04*	3 736.58	49 906.92	142 767.00	11 635.04	111 551.70
DYNAMIC Bequest	*0.044*	209.11	44 808.79	137 089.10	8 126.91	105 373.80
DYNAMIC Bequest	*0.048*	–	39 955.08	129 419.30	4 502.45	97 377.96
DYNAMIC Bequest	*0.0512*	–	35 922.48	120 143.00	1 605.86	91 792.39
DYNAMIC Bequest	*0.06*	–	25 984.12	101 454.90	–	78 398.47

Source: Own calculations using MikroSIM model.

F i g u r e 4

Probability of Ruin for Various Investment Strategies and Defined Benefit Ratio





Source: Own elaboration.

Further research should be oriented on investigating the stopping function, which defines the critical point in time or „point of no return“. This point defines, that if the value of wealth crosses below certain value at certain point, retiree would be better-off if all remaining wealth is used to by a single premium life annuity. If he would continue with programmed withdrawal, the ruin will certainly occur before he dies. At the same time, bequest could be valued using present value approach and thus better reflect its value.

We came close to the Dus, Maurer and Mitchell (2005) conclusions, that an immediate annuitization can be viewed suboptimal in general and also in individual circumstances. However, understanding the stopping function might help retirees to better manage retirement savings and maximize the utility function while minimizing probability of ruin due to the individual longevity risk and ability to maximize utility from the existence of a bequest.

Conclusions

Our paper focuses on proclaimed suboptimality of immediate annuitization and investigates possible investment and withdrawal strategies as an alternative to life annuity. Using stochastic simulations of uncertain equity and bond pension funds returns under the existence of fee policy an uncertainty in life expectancy, we have shown that a programmed withdrawal strategy paying the same benefit as an annuity exposes the retiree to the risk of outliving his assets while still alive. A programmed withdrawal using a dynamic withdrawal rate that corresponds to the past returns and adjust the paid benefits on an annual basis helps avoiding the risk of running out of money, since benefits fluctuate in tandem with the pension fund’s returns. We have constructed investment strategy, which respects trend in returns and using timing, retiree dynamically manages the portfolio in order to minimize the probability of ruin.

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