Unemployment and Aggregate Demand in Spain[1]

*Ondřej ČÍŽEK**

Abstract

The paper contributes to the existing literature by incorporating the Keynesian principle of weak aggregate demand into the basic search-matching model of unemployment in a simple and novel way. Multiple equilibrium unemployment rates emerge as a result of this modification. It is shown that output demand not only plays short-term role but might be essential in the long-run as well. This is because the initial fall in aggregate demand may cause unemployment rate to converge to a higher (long-run) equilibrium. All these aspects are illustrated for the Spanish labour market and it is shown that the model with multiple equilibrium unemployment rates outperforms the baseline standard search and matching model in its forecasting performance as well as in its ability to describe huge persistent swings in unemployment.

Keywords: *search-matching model, unemployment rate, output demand, multiple equilibria*

JEL Classification: E24, J23, J64

Introduction

Aggregate output demand plays only a minor role in the standard search-matching Diamond-Mortensen-Pissarides[2] (DMP) model. The DMP methodology is based on a principle that supply creates its own demand – if unemployment (labour supply) increases, firms open more vacancies (i.e. labour demand is increased). However, there is empirical evidence favouring demand-oriented theories of job creation to the model of job creation proposed by DMP modelling

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[2] The work of these Nobel Prize winners is summarized by (Pissarides, 2000).

framework (Carlsson, Eriksson and Gottfries, 2006). The Nobel Prize winner in economics Joseph Stiglitz states that Europe's problem today is a lack of aggregate demand (Stiglitz, 2014). There is also an intense research in a Post Keynesian theory of unemployment which is based on a proposition that unemployment depends on output, which is itself determined by aggregate demand. Textbook treatment can be found in (Holt and Pressman, 2001; Lavoie, 2006). Famous model containing search-matching aspects as well as Post Keynesian features was formulated by Diamond (1982).

There is an extensive literature incorporating the DMP labour market model into a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework (Galí, Smets and Wouters, 2011; Blanchard and Galí, 2010; Krause and Lubik, 2007; Trigari, 2006; Walsh, 2005). Application of these approaches for the Czech economy can be found in (Hloušek, 2010; Němec, 2011; 2012). Němec (2013), and Bouda and Formánek (2014) compare Czech and Slovak economies using DSGE modelling framework. Tonner, Tvrz and Vašíček (2013) discuss a suitable way of modelling main labour market variables within the framework of the core DSGE model used by the Czech National Bank.

These models have been successfully applied to quantify the impact of aggregate output on labour market development in many above mentioned studies. Nonetheless, the main focus of these models is to describe the effects of monetary policy by (labour) market frictions. Aggregate demand plays only a short--term role in these models.

The theory of NAIRU (Non-Accelerating Inflation Rate of Unemployment) is applied in these models and this long-term unemployment rate is typically determined uniquely by the supply side of the economy. The advantage of these models is their microeconomic foundation.[3]

The unemployment rate in Spain is very persistent and can get far away from its mean value for quite a long time. This crucial economic variable reached a value of 8% at the beginning of the economic crisis in 2007. Since then, it began to soar to a high of 27% in 2013. The Spanish labour market is characterized by these large swings in unemployment not only for the periods of the current economic crisis. This challenges the hypothesis that such an evolution of unemployment rate is caused purely by search and matching frictions in the labour market and that such a huge persistent fluctuations are only short-term deviations from a unique steady state as in DSGE models.

The presented paper contributes to the existing literature by incorporating the traditional Keynesian concept of weak aggregate demand into the basic

[3] There is also an extensive empirical literature testing and estimating NAIRU, but not based on microeconomic foundations (Kárász, 2011; Jašová, 2011).

search-matching model in a novel way. The important consequence is that multiple (long-run) equilibrium unemployment rates might emerge which is illustrated for the Spanish labour market. Changes in aggregate demand might have a permanent effect on the equilibrium unemployment rate. Persistence of unemployment is thus explained by a model of multiple equilibria.

The model with multiple equilibria incorporating the weak demand principle is empirically compared to the baseline search-matching model. It turns out that the basic search and matching model has difficulties in capturing persistent large swings in unemployment. The model with multiple equilibrium unemployment rates outperforms the basic search-matching model not only in matching selected statistical moments but also in its forecasting ability. This suggests that a multiple equilibria model incorporating the weak demand principle is more suitable for analysing the Spanish labour market.

There are also other models with multiple equilibria in which multiplicity is caused by various different reasons. See Mortensen (1989) for a review and discussion of this issue within the DMP framework. Hysteresis in the labour market is a closely related issue as this concept admits even an infinite number of long-term equilibrium unemployment rates and provides an alternative theoretical foundation of persistent unemployment (Blanchard and Summers, 1986; Ball, 2009; Schoder, 2016; Furuoka, 2014; Kanalici, Nargeleçekenler and Yilmaz, 2011). Hysteresis in the Czech labour market was studied by Němec and Moravanský (2006), Němec (2010) or Marjanovic, Maksimovic and Stanisic (2015).

The structure of the paper is as follows. Chapter 1 briefly presents a well-known stochastic discrete-time version of the basic DMP model with aggregate uncertainty (Hagedorn and Manovskii, 2008) (HM model, hereafter). The concept of weak aggregate output demand is then incorporated into this basic search-matching model. Econometric estimation of both these forward-looking models is based on Bayesian techniques, is performed in Dynare and is presented in chapter 2. Nonetheless, there were technical difficulties with econometric estimation of the weak demand (WD) model because of the multiplicity of equilibria as standard algorithms implemented in Dynare are based on linearization around a unique steady state. Multiplicity of equilibria of the WD model is analysed in chapter 3. Backward-looking version of the WD model is formulated, estimated and analysed in chapter 4 in order to obtain econometric estimates of the weak demand model with multiple equilibria. The subsequent chapter 5 compares the empirical performance of the baseline HM model with the backward-looking version of the WD model. The final chapter summarizes my main findings.

1. Model

The baseline model was developed by Hagedorn and Manovskii (2008) and will be briefly summarized for convenience with minor modifications in section 1.1. The second subchapter 1.2. incorporates the weak demand principle into this baseline model which is done in a simple and novel way.

1.1. The Baseline HM Model

Infinitely lived workers maximize their expected lifetime utility, $E\sum_{t=0}^{\infty}\delta^t y_t$, where y_t represents income in period *t* and $\delta \in (0,\ 1)$ is a discount factor.

Output per worker is denoted by p_t and follows the first-order autoregressive process:

$$\log(p_t) = \rho^p \cdot \log(p_{t-1}) + \varepsilon_t^1 \tag{1}$$

where

$\rho^p \in (0;1)$ and $\varepsilon_t^1 \sim N(0,\ \sigma_1^2)$ – i.i.d. productivity shock.

Flow cost c_t of posting a vacancy is assumed to change over the business cycle according to

$$c_t = c_K \cdot p_t + c_W \cdot p_t^{\xi} \tag{2}$$

Workers and firms separate with a constant[4] probability *s* per period. Employed workers are paid a wage w_t and unemployed get a flow utility *z* from leisure/non-market activity. Wages are determined by the generalized Nash bargaining solution. The bargaining power of workers is $\beta \in (0;1)$.

Let u_t denote the unemployment rate, $n_t = 1 - u_t$ the employment rate, v_t the number of vacancies and $\theta_t = v_t / u_t$ the market tightness. The number of new matches (starting to produce output at $t+1$) is given by[5]

$$m(u_t, v_t) = m_0 \cdot u_t^{\eta} \cdot v_t^{1-\eta} \cdot \exp(\varepsilon_t^2) \tag{3}$$

[4] There is empirical evidence that fluctuations in job finding probability during business cycle frequencies are substantial, while separation probability is nearly acyclic (Hall, 2005; Shimer, 2012).

[5] Hagedorn and Manovskii (2008) applied another form of matching function. Specifically, they used a matching function of the form $m(u_t, v_t) = u_t \cdot v_t / \left(u_t^l + v_t^l\right)^{1/l}$ which was proposed by den Den Haan, Ramey and Watson (2000). Nonetheless, standard Cobb-Douglas matching function performed better from an empirical point of view.

The shock to matching efficiency ε_t^2 is supposed to be persistent:

$$\varepsilon_t^2 = \rho^m \cdot \varepsilon_{t-1}^2 + \tilde{\varepsilon}_t^2 \tag{4}$$

where

$\tilde{\varepsilon}_t^2 \sim N\left(0, \sigma_2^2\right)$ – i.i.d. random error.

Probability for an unemployed worker to be matched with a vacancy equals

$$f_t \equiv f\left(\theta_t\right) \equiv \frac{m\left(u_t, v_t\right)}{u_t} = m_0 \cdot \theta_t^{1-\eta} \cdot \exp\left(\varepsilon_t^2\right) \tag{5}$$

and the probability for a vacancy to be filled is

$$q_t \equiv q\left(\theta_t\right) \equiv \frac{m\left(u_t, v_t\right)}{v_t} = m_0 \cdot \theta_t^{-\eta} \cdot \exp\left(\varepsilon_t^2\right) \tag{6}$$

Evolution of employment rate is given by

$$n_{t+1} = \left(1-s\right) \cdot n_t + f_t \cdot u_t + \varepsilon_{t+1}^3 \tag{7}$$

where

$\varepsilon_{t+1}^3 \sim N\left(0, \sigma_3^2\right)$ – i.i.d. shock to the process of unemployment.

It can be shown by standard methods that the first-order conditions of the optimization problem lead to the following equilibrium condition:

$$\frac{c_t}{\delta \cdot q\left(\theta_t\right)} = E_t\left[\left(1-\beta\right) \cdot \left(p_{t+1} - z\right) - c_{t+1} \cdot \beta \cdot \theta_{t+1} + \frac{\left(1-s\right) \cdot c_{t+1}}{q\left(\theta_{t+1}\right)}\right] \tag{8}$$

This equation implicitly defines the decision variable θ_t as a function of the state variable p_t which will be denoted by $\theta_t = \theta\left(p_t\right)$. The function $\theta\left(p_t\right)$ is often called a *reaction function* as it describes the optimal reaction of firms (and workers) to the state of the economy. It is often referred to as a *job creation function* as it describes how vacancies v_t are created. From this point of view, it is an analogy to the Keynesian labour demand function.

It is also easy to show by standard methods that wages are given by

$$w_t = \beta \cdot p_t + \left(1-\beta\right) \cdot z + c_t \cdot \beta \cdot \theta_t + \varepsilon_t^4 \tag{9}$$

where

$\varepsilon_t^4 \sim N\left(0, \sigma_4^2\right)$ – i.i.d. shock added to the wage equation for the purpose of econometric estimation.

1.2. The Weak Demand Model

The basic DMP model of the previous subchapter 1.1. implicitly assumes that output produced by a worker p_t will also be sold. The principle of weak output demand is incorporated into this model by assuming that the output actually sold p_t^s depends positively on purchasing power of customers which is given by

$$\kappa_t = (1-u_t)\cdot w_t + u_t \cdot z \tag{10}$$

where

κ_t – purchasing power of customers.

The process describing the output actually sold is modelled by a generalization of a simple autoregressive process (1) as follows

$$\log\left(p_t^s\right) = \rho^p \cdot \log\left(p_{t-1}^s\right) + \gamma\cdot\left(\kappa_t - \overline{\kappa}\right) + \varepsilon_t^1 \tag{11}$$

where

p_t^s – the output actually sold,

$\overline{\kappa}$ – an arithmetic mean of the variable κ_t.

Alternatively, the output actually sold p_t^s is modelled as a decreasing function of unemployment as unemployment is negatively correlated with purchasing power of customers[6]

$$\log\left(p_t^s\right) = \rho^p \cdot \log\left(p_{t-1}^s\right) - \gamma\cdot\left(u_t - \overline{u}\right) + \varepsilon_t^1 \tag{12}$$

where

$\overline{u}$ – an arithmetic mean of the unemployment rate u_t.

It can be shown by standard methods that all the equations from the previous subchapter 1.1. remains the same except the equations (8) and (9) which are slightly modified in the following manner:

$$\frac{c_t}{\delta\cdot q(\theta_t)} = E_t\left[(1-\beta)\cdot\left(p_{t+1}^s - z\right) - c_{t+1}\cdot\beta\cdot\theta_{t+1} + \frac{(1-s)\cdot c_{t+1}}{q(\theta_{t+1})}\right] \tag{13}$$

$$w_t = \beta\cdot p_t^s + (1-\beta)\cdot z + c_t\cdot\beta\cdot\theta_t + \varepsilon_t^4 \tag{14}$$

where

$c_t = c_K\cdot p_t^s + c_W\cdot\left(p_t^s\right)^{\xi}$.

[6] Similar assumption is common in literature. Aggregate purchasing power is modeled by the number of unemployed workers in the famous model formulated by Diamond (1982).

2. Econometric Estimation

Firstly, data is described in the first section 2.1. Econometric estimation of the baseline search and matching model as well as modified weak demand model is discussed in subsequent chapters 2.2. and 2.3.

2.1. Data

The source of the data is OECD database. All data is seasonally adjusted. The first observable variable is the standardized unemployment rate in Spain u_t (relating to all ages of workers) which is measured monthly from 1986 M4 to 2016 M8. The second variable market tightness θ_t calculated as a ratio of number of unfilled vacancies to number of unemployed persons from 1986 M4 to 2005 M4. The third observable variable is productivity p_t which is measured as a relative deviation from a linear trend of an industrial production index in manufacturing. It is also measured in monthly frequency from 1986 M4 to 2016 M8. The last observable variable relates to wages w_t. The variable w_t is measured as a relative deviation from a linear trend of an index of (real) hourly earnings in manufacturing. This measure of w_t implies that its mean value equals approximately to one as in the case of p_t. Therefore, monthly rate of change $\left(w_t - w_{t-1}\right) / w_{t-1}$ is used as an observable variable. Index of hourly earnings in manufacturing was transformed from quarterly frequency into a monthly frequency by cubic spline. Data ranges from 1986 M4 to 2016 M4 after this transformation.

2.2. The Baseline HM Model

Firstly, the baseline HM model is estimated. Econometric estimation is based on Bayesian econometric techniques and is performed using the Matlab toolbox Dynare (version 4.4.3). Priors for the baseline model are reported in the following table together with a short parameter description.

In most cases, prior means for parameters are values typically used in literature. Therefore, standard deviations of the prior densities are chosen to be relatively high. The discount factor of 0.99 is a value typically used in literature for quarterly data (Němec, 2013). Prior means for ρ^p and ρ^m reflect the fact that a productivity and a matching process are persistent. Mean values for vacancy cost parameters are based on calibration in Hagedorn and Manovskii (2008). The value $\beta = 0.5$ is considered to be the most plausible by Pissarides (2000). Elasticity of matching η ranges from 0.2 to 0.8 in empirical studies, the results of which were surveyed by Petrongolo and Pissarides (2001). Therefore, the value 0.5 was chosen as a prior mean. Matching efficiency m_0 was not estimated. Instead, the parameter m_0 was made a function of the coefficient η according to $m_0 = mean\left(f_t / \theta_t^{1-\eta} \right)$, where f_t was calculated by the method described by Shimer (2012). The separation probability was calculated according to $s = mean\left(1-\left(n_{t+1} - f_t \cdot u_t\right)/ n_t\right)$. The value of $z = 0.4$ is typically used in empirical studies.

T a b l e 1

Parameter Description and Prior Densities for the Baseline HM Model

Parameter	Description	Density	Mean	Std. Dev.
δ	Discount factor	Fixed	$0.99^{1/3}$	–
s	Separation probability	Fixed	0.014	–
ρ^p	AR coef. in productivity process	Beta	0.80	0.20
ρ^m	AR coef. in matching process	Beta	0.80	0.20
c_K	Vacancy cost	Beta	0.47	0.20
c_W	Vacancy cost	Beta	0.11	0.20
ξ	Vacancy cost	Beta	0.45	0.20
β	Workers' bargaining power	Beta	0.50	0.20
η	Elasticity of matching	Beta	0.50	0.20
z	Value of non-market activity	Beta	0.40	0.20
$\sigma_{1,2,3,4}$	Std. dev. of shocks	Inv. gamma	0.01	1

Source: Own calculations.

The estimation results are presented in the form of posterior means together with 90% confidence intervals:

T a b l e 2

Parameter Estimates of the Baseline HM Model

Parameter	Posterior mean	90% confidence interval
ρ^p	0.830	(0.822; 0.839)
ρ^m	0.958	(0.940; 0.977)
c_K	0.974	(0.951; 0.996)
c_W	0.981	(0.961; 1.000)
ξ	0.842	(0.705; 0.978)
β	0.014	(0.006; 0.021)
η	0.973	(0.948; 0.997)
z	0.853	(0.9846; 0.9861)
σ_1	0.025	(0.024; 0.027)
σ_2	0.117	(0.108; 0.127)
σ_3	0.0030	(0.0028; 0.0032)
σ_4	0.015	(0.014; 0.016)

Source: Own calculations.

The Table 2 illustrates that posterior distribution differs from a prior distribution and that the length of the posterior confidence intervals is relatively short in most case. These facts suggest that the data is informative in this case.

Estimated posterior means of the parameters z and β are 0.9853 and 0.014. This is an interesting result as these values are close to the calibrated values $z = 0.955$, $\beta = 0.052$ used in the paper by Hagedorn and Manovskii (2008). These authors showed that their calibration strategy of z close to one and β close to zero generates volatilities of unemployment and market tightness that are very close to those observed in U.S. data. Estimation results thus suggest that their calibration strategy might be appropriate not only for U.S. data.

2.3. The Weak Demand Model

The only additional parameter in this modified model is the coefficient γ. Prior distribution for other parameters is the same as in the baseline HM model. Econometric estimation of the WD model with Dynare was much more difficult than estimation of the baseline HM model. There were lots of technical problems due to which an estimation was not performed at all in many cases. The cause of these technical problems turned out to be an existence of multiple equilibrium unemployment rates in the WD model.

Technical problems were associated with the Blanchard-Kahn condition which was not satisfied in many cases. This result proved to be quite robust to the change of the priors. In some cases, it was possible to overcome these problems by a sophisticated choice of initial values of variables serving as a starting point for an algorithm searching for a steady state.

Nonetheless, the obtained results were in these cases very close to the case of multiple equilibria when a steady state analysis of the estimated model was performed.

Specifically, the relation (12) will now be considered in order to illustrate these results.[7] This equation is replicated here for convenience

$$\log\left(p_t^s\right) = \rho^p \cdot \log\left(p_{t-1}^s\right) - \gamma \cdot \left(u_t - \bar{u}\right) + \varepsilon_t^1 \tag{12}$$

where

p_t^s – output actually sold,

$\rho^p \in (0; 1)$, $\bar{u}$ – an arithmetic mean of unemployment rate,

$\varepsilon_t^1 \sim N\left(0, \sigma_1^2\right)$ – i.i.d. shock.

[7] The results are robust to the choice of specific functional form. Similar results were obtained when the relation (11) was used instead of the equation (12).

The prior for the additional parameter γ is described in the Table 3.

T a b l e 3

Prior Density for the Additional Parameter in the Modified WD Model

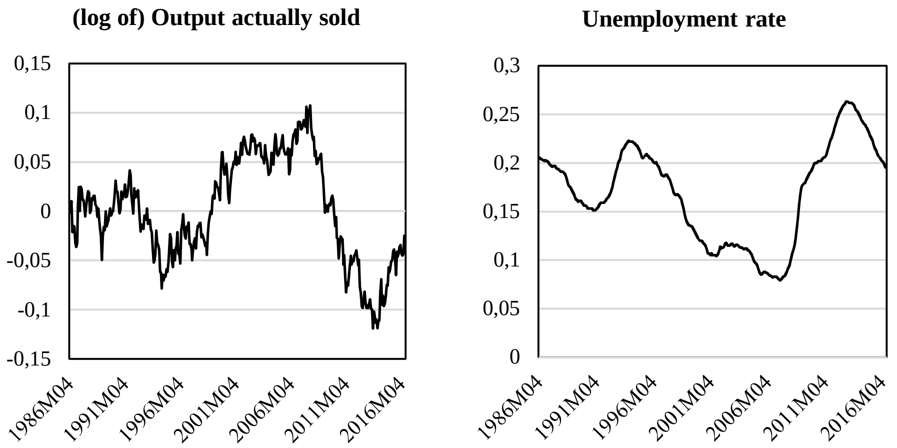
Parameter	Description	Density	Mean	Std. Dev.
γ	Weak demand	Beta	0.20	0.20

Source: Own calculations.

The choice of this prior mean is motivated by the fact that a value of 0.20 for γ together with the above mentioned prior means for ρ^p and σ_1 (0.8 and 0.01, respectively) generates data for $\log\left(p_t^s\right)$ with reasonable characteristics when the observed data for unemployment rate u_t is used in equation (12). The evolution of the variable $\log\left(p_t^s\right)$ (representing log of output actually sold) generated in this way is illustrated in Figure 1.

F i g u r e 1

The Evolution of Output Actually Sold (in logarithms) Generated by Equation (12) with $\gamma = 0.2$, $\rho^p = 0.8$ and $\sigma_1 = 0.01$ (using observed data for unemployment rate)



Source: Own calculations.

The WD model with above mentioned priors was econometrically estimated in Dynare. Initial values for variables serving as a starting point for finding a steady state had to be chosen wisely and standard algorithms implemented in Dynare had to be replaced with numerically demanding Markov Chain Monte Carlo (MCMC) methods in order to obtain estimates. The results of the estimation are summarized in the following table.

Table 4

Parameter Estimates of the Weak Demand Model

Parameter	Posterior mean	90% confidence interval
ρ^p	0.814	(0.811; 0.818)
ρ^m	0.976	(0.967; 0.985)
c_K	0.534	(0.528; 0.541)
c_W	0.1113	(0.1112; 0.1114)
ξ	0.669	(0.665; 0.673)
β	0.635	(0.629; 0.641)
η	0.623	(0.620; 0.625)
z	0.9837	(0.9829; 0.9844)
γ	0.268	(0.264; 0.272)
σ_1	0.0188	(0.0188; 0.0189)
σ_2	0.023	(0.019; 0.028)
σ_3	0.0023	(0.0022; 0.0024)
σ_4	0.099	(0.098; 0.099)

Source: Own calculations.

Posterior means will be used as parameter values in steady state analysis in the next chapter 3. This analysis will reveal that these parameter estimates imply a situation very close to the case of multiple equilibria. The important fact is that this result is robust to the choice of prior mean of γ. The result that the estimated WD model is close to the case of multiple equilibria was detected for all the values 0.1; 0.2; 0.3; 0.4 and 0.5 used as a prior mean of γ.

Another robust result also is that there were technical problems in Dynare when using posterior means as parameter values in stochastic simulations. The simulation was either not performed at all or the dynamics was explosive which happened due to approximations stemming from a linearization around a steady state.

All these problems demonstrate that multiple equilibria models cannot be estimated by standard algorithms implemented in Dynare as these algorithms are based on linearization around a uniquely determined steady state. For this reason, a simplified backward-looking version of the weak demand model will be formulated and analyzed later in this paper in chapter 4.

3. Analysis of Steady States

Steady state analysis is performed for the WD model as well as for the baseline HM model in the first two subchapters. The third section interprets the multiplicity of unemployment rates from an economic point of view.

3.1. The Weak Demand Model

The following steady state relation follows immediately from equation (12)

$$p^s(u) = \exp\left(-\frac{\gamma}{1-\rho}\cdot(u-\bar{u})\right) \tag{15}$$

where

$p^s(u)$ represents sold output as a function of steady state unemployment rate *u*,

$\bar{u}$ – an arithmetic mean of unemployment rate.

Therefore, flow cost $c(u)$ is a function of unemployment rate

$$c(u) = c_K \cdot p^s(u) + c_W \cdot \left(p^s(u)\right)^{\xi} \tag{16}$$

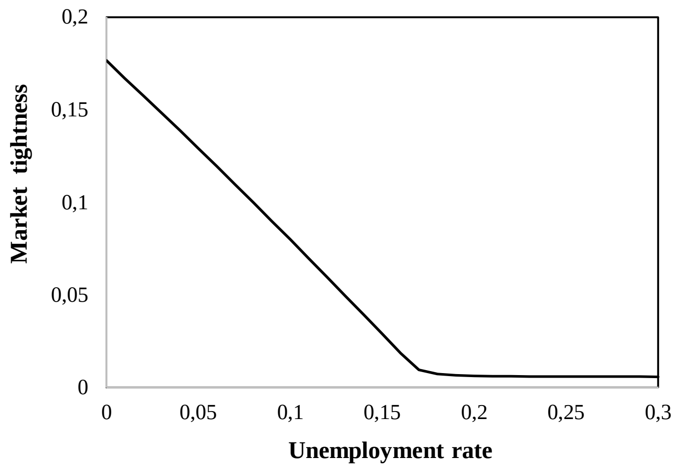
Steady state values of market tightness $\theta(u)$ is then implicitly defined by relation (13) as follows:

$$\frac{c(u)}{\delta \cdot q(\theta(u))} = E\left[(1-\beta)\cdot\left(p^s(u)-z\right) - c(u)\cdot\beta\cdot\theta(u) + \frac{(1-s)\cdot c(u)}{q(\theta(u))}\right] \tag{17}$$

Market tightness $\theta(u)$ is a decreasing function of unemployment. This can be illustrated by using posterior means of the parameters in this relation which yields the following function $\theta(u)$.

F i g u r e 2

Reaction Function $\theta(u)$ when Posterior Means are Used as Parameter Values



Source: Own calculations.

The lower bound $\underline{\theta} = 0.006 \geq 0$ was assumed for the function $\theta(u)$, where the value of 0.006 is the minimum observed in the dataset.

From an economic point of view, the mechanisms behind the decreasing function $\theta(u)$ can be summarized as follows:

$$\boxed{\begin{array}{l}\uparrow \text{ unemployment } \rightarrow \downarrow \text{ purchasing power } \rightarrow \downarrow \text{ demand for final goods} \\ \downarrow \text{ labour demand } \rightarrow \downarrow \text{ vacancies } \rightarrow \downarrow \text{ market tightness}\end{array}} \tag{18}$$

High unemployment rate u leads to low aggregate purchasing power. Firms do not demand labour because of weak demand for their products. For this reason, only few vacancies are posted by companies and market tightness $\theta_t = v_t / u_t$ is low.[8]

The dependence of market tightness on unemployment rate implies that a job finding probability is also a function of unemployment rate

$$f(u) = 0.287 \cdot \theta(u)^{1-0.623} \tag{19}$$

where the value of 0.623 is the posterior mean of η and $m0 = 0.287$ was calculated according to $m_0 = mean\left(f_t / \theta_t^{1-0.623}\right)$.

The equation (7) implies that a change in unemployment is given by[9]

$$u_{t+1} - u_t = s - (s + f_t) \cdot u_t \tag{20}$$

Stationary unemployment rate $u_t = u$ satisfies $u_{t+1} - u_t = 0$, which yields

$$u = \frac{s}{s + f(u)} \tag{21}$$

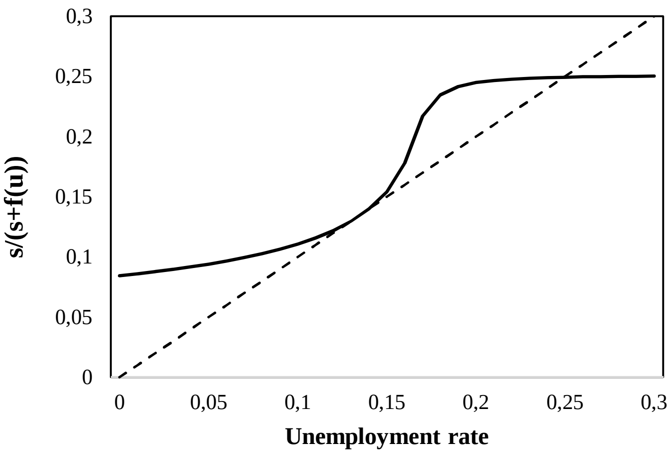
The function $s / (s + f(u))$ together with a 45°line representing variable u on the left-hand side of the equation (21) is depicted at the following Figure 3.

There are two equilibrium unemployment rates $u^1 = 0.14$ and $u^2 = 0.25$. The equilibrium point u^2 is stable, while the equilibrium u^1 is semi stable. These results on stability are best seen from equation (20). We know that the right-hand side of this equation corresponds to change in unemployment rate Δu. Also observe that $\Delta u > 0 \Leftrightarrow s / (s + f(u)) > u$. Therefore, unemployment rate u is rising whenever $s / (s + f(u))$ is above the 45° line.

[8] Market tightness $\theta(u)$ is not a decreasing function of unemployment rate u just because the variable u is in the denominator in the definition of market tightness $\theta \equiv v / u$.

[9] The random error is set to zero in this calculation of steady states.

Figure 3
Equilibrium Unemployment Rates for the Weak Demand Model



Source: Own calculations.

3.2. The Baseline HM Model

In the baseline HM model, a rise in unemployment rate u causes proportional rise in number of vacancies v so that market tightness $\theta \equiv v/u$ is constant and does not depend on unemployment rate u. The basic idea of this mechanism can be summarized as follows

$$\boxed{\begin{array}{l}\uparrow \text{ unemployment } \rightarrow \uparrow \text{ availability of labour } \rightarrow \\ \uparrow \text{ probability of filling a vacancy } \rightarrow \uparrow \text{ vacancies}\end{array}} \qquad (22)$$

Comparing the mechanisms (18) and (22) reveals that a supply (availability of labour) plays a crucial role in the baseline HM model while a demand is essential in the WD model.

The independence of θ on u in the baseline HM model is easily seen by noting that the equilibrium value of the productivity is $p = 1$ and that the condition (17) boils down in this case to

$$\frac{c}{\delta \cdot q(\theta)} = E\left[(1-\beta)\cdot(1-z) - c\cdot\beta\cdot\theta + \frac{(1-s)\cdot c}{q(\theta)}\right] \qquad (23)$$

where

$c = c_K + c_W$.

The value of the market tightness θ is determined by solving equation (23) and this value of θ is independent of unemployment rate *u*. In the HM model, the variable $\theta \equiv v/u$ is thus constant and independent of unemployment rate *u* despite the fact that unemployment rate *u* is in the denominator in the definition of the market tightness $\theta \equiv v/u$.

Therefore, the equilibrium condition (21) reduces to $u = s/(s+f)$ in the baseline HM model. The function $s/(s+f)$ is a horizontal straight line crossing the 45° line in only one point. For this reason, the equilibrium unemployment rate is unique.

3.3. Interpretation of Equilibrium Multiplicity

The line $s/(s+f(u))$ is upward-sloping in the WD model because of the endogenisation of the job finding probability $f(u)$ [10] which makes it possible to cross the 45° line in more than one point.

The reason for the existence of multiple equilibriums in the WD model is that the labour market is "less effective" when unemployment is high. This is modelled by making job finding probability $f(u)$ endogenous in a simple and novel way which is the main contribution of this paper. The probability $f(u)$ is low (high) when unemployment rate *u* is high (low). But low (high) value of $f(u)$ keeps unemployment at high (low) levels because it is hard (easy) to find a job. Similar results were obtained by other authors (Diamond, 1982; Kaplan and Menzio, 2016).

From an economic point of view, the mechanisms behind the decreasing function $f(u)$ closely corresponds to those behind the decreasing function $\theta(u)$ which were already summarized by the transition mechanism (18). High unemployment rate *u* leads to low market tightness by the mechanisms described by (18). Consequently, low market tightness leads to low job finding probability which is a direct implication of the matching function.

4. Backward-looking Version of the WD Model

There were technical problems associated with econometric estimation as well as stochastic simulation of the weak demand model in Dynare. For this reason, a simplified backward-looking version of the WD model is formulated, estimated and analysed in this chapter.

[10] Separation rate was not made endogenous as there is empirical evidence that it is nearly acyclic (Shimer, 2012).

4.1. Formulation

The formulation of the backward-looking version of the WD model is based on the following approximation of the function $\theta(u)$

$$\theta(u) = max\{a - b \cdot u,\ \underline{\theta}\} \tag{24}$$

where $a,\ b > 0$ and the lower bound $\underline{\theta} = 0.006 \geq 0$ is the minimum observed in the dataset.

The complete backward-looking version of the WD model can be summarized as follows:

$$\theta_t = max\left\{a - b \cdot u_t + \varepsilon_t^1,\ \underline{\theta}\right\} \tag{25}$$

$$f_t = m_0 \cdot \theta_t^{1-\eta} \cdot \exp\left(\varepsilon_t^2\right) \tag{26}$$

$$n_{t+1} = (1 - s) \cdot n_t + f_t \cdot u_t \tag{27}$$

The random errors turned out to be autocorrelated. The first-order autocorrelation was therefore assumed:

$$\varepsilon_t^j = \rho^j \cdot \varepsilon_t^{j-1} + \tilde{\varepsilon}_t^j \tag{28}$$

where $\tilde{\varepsilon}_t^j \sim N\left(0,\ \sigma_j^2\right)$ are i.i.d. random shocks, $j = 1,\ 2$.

The presented backward-looking formulation of the WD model is only viewed as an approximation and has many disadvantages compared to the baseline HM model. Firstly, it is not microfounded. Secondly, the coefficients *a* and *b* are not deep structural parameters. Thirdly, only the main labour market indicators such as market tightness, job finding probability and unemployment rate are modelled while other (potentially important) variables are neglected.

On the other hand, there are no technical problems with econometric estimation or stochastic simulation despite the fact that the backward-looking WD model has multiple equilibrium unemployment rates. The important advantage of this model also is that it outperforms the baseline HM model from an empirical point of view which is discussed in detail later in chapter 5.

4.2. Econometric Estimation

The equations (25) – (28) were estimated in Eviews by nonlinear least squares algorithm in order to take autocorrelation of random errors ε_t^j into account. Estimation sample was $t \in \{1986\text{ M4}, ..., 2005\text{ M4}\}$ for the two observable variables θ_t and u_t. The results are summarized in the following table.

T a b l e 5

Parameter Estimates of the Backward-looking WD Model

Parameter	a	b	ρ^1	log (m_0)	η	ρ^2
Value	0.12	0.57	0.92	–1.31	0.65	0.70
Std. error	0.03	0.19	0.02	0.12	0.03	0.05

Source: Own calculations.

The estimate $\hat{\eta}=0.65$ is in line with the results of other empirical studies (Petrongolo and Pissarides, 2001), according to which this parameter ranges from 0.2 to 0.8. All estimated parameters have expected signs and are statistically significant even at 1% level. Coefficient of determination in the first and the second equation is $R^2 = 0.96$ and $R^2 = 0.89$ respectively.

4.3. Calculation of Equilibria

Equilibrium unemployment rates are again derived from relation (21). The function $f(u)$ in this relation is again obtained from the function $\theta(u)$

$$\theta(u) = max\{0.12 - 0.57 \cdot u;\ \underline{\theta}\} \tag{29}$$

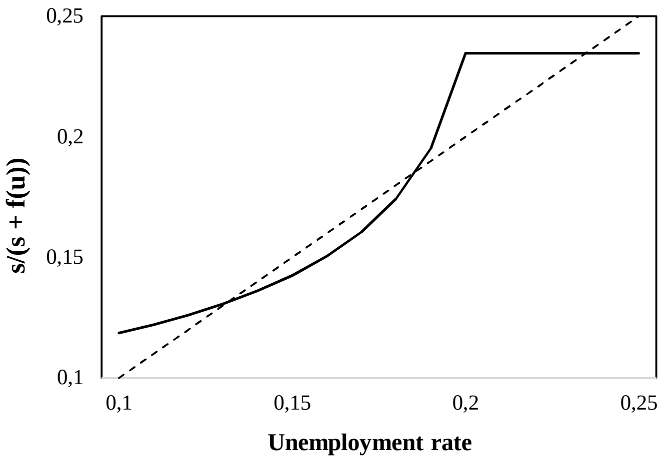
The probability of finding a job is given by

$$f(u) = exp(-1.31) \cdot \theta(u)^{1-0.65} \tag{30}$$

The function $s/(s+f(u))$ together with 45° line representing variable u on the left-hand side of the equation (21) is depicted at the following figure.

F i g u r e 4

Equilibrium Unemployment Rates for the Backward-looking WD Model



Source: Own calculations.

There are three stationary unemployment rates $u^1 = 0.13$, $u^2 = 0.18$ and $u^3 = 0.23$. The equilibrium points u^1, u^3 are stable, while the equilibrium u^2 is unstable for the same reason discussed already in chapter 0.

5. Comparison of the Models

This chapter compares empirical performance of the backward-looking version of the weak demand model with the baseline forward looking Hagedorn--Manovskii model. The attention is given to comparing moments of variables and to forecasting performance.

5.1. Comparison of Moments

The key question is how good these models are at generating data with similar properties as real observed data. Selected moments of the main labour market variables – market tightness, job finding probability and unemployment rate – are compared in the following table in order to answer this question.

Table 6

Comparison of Moments Calculated from Observed Data, from Data Generated by the Baseline HM Model and the Backward-looking Version of the WD Model

		Market tightness	Job finding probability	Unemployment rate
Mean	Observed data	0.037	0.077	0.169
	HM model	0.037	0.085	0.150
	WD model	0.030	0.072	0.174
Standard deviation	Observed data	0.027	0.036	0.051
	HM model	0.018	0.034	0.044
	WD model	0.025	0.028	0.048

		$corr(\theta, f)$	$corr(\theta, u)$	$corr(f, u)$
Correlation	Observed data	0.754	–0.715	–0.863
	HM model	0.898	–0.715	–0.815
	WD model	0.893	–0.868	–0.835

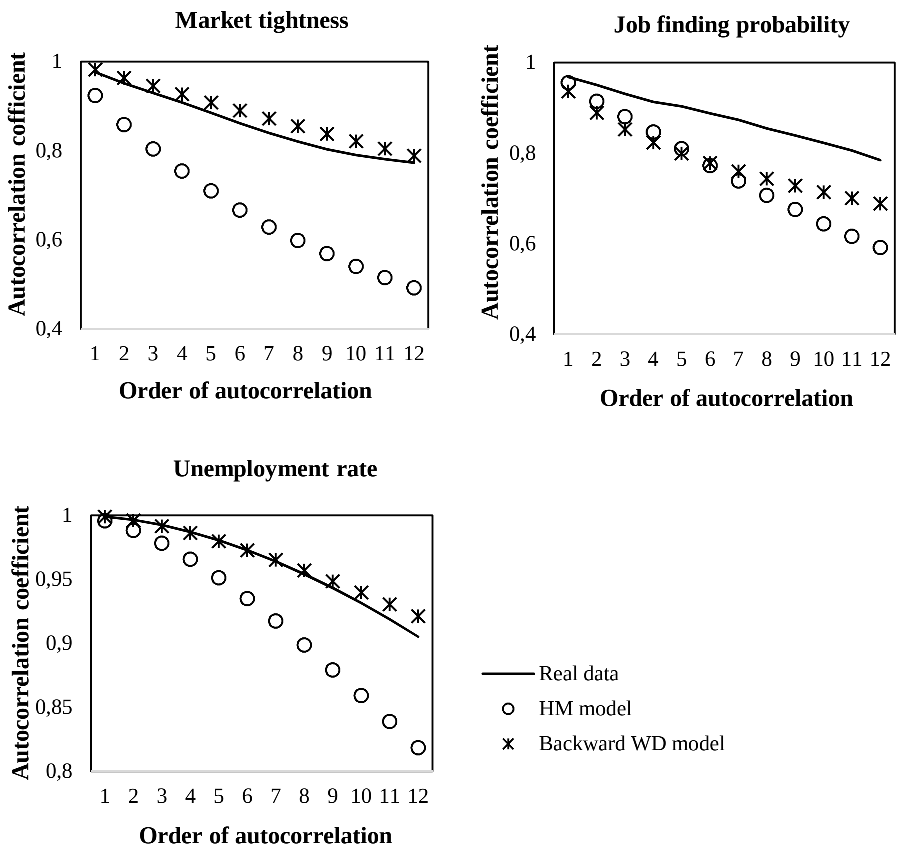
Source: Own calculations.

Both models are able to reproduce selected features of the observed data. Nonetheless, the WD model outperforms the baseline HM model in matching autocorrelation functions which is documented in the following Figure 5.

The figure illustrates that both models are able to match first-order autocorrelation coefficient. However, the baseline HM model fails to match higher-order autocorrelation coefficients and systematically underestimates them. Possible explanation of this result might be that search and matching models treats deviations from a steady state unemployment rate only as a short-run phenomenon. Nonetheless, unemployment in Spain can be very high for quite a long time. For this reason, higher-order autocorrelation coefficients calculated from the observed data are systematically higher than these coefficients calculated from the data generated by the baseline HM model.

F i g u r e 5

Comparison of Autocorrelation Functions



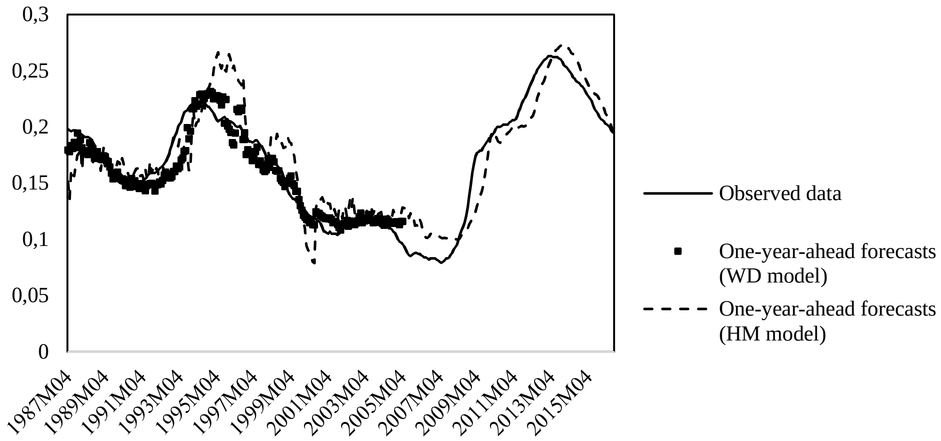
Source: Own calculations.

The backward-looking version of the WD model is able to match higher-order autocorrelations much better than the baseline HM model. This suggests that a multiple equilibria model in which unemployment rate might fluctuate around ineffective equilibrium point for a long time might indeed be more appropriate for the Spanish labour market.

5.2. Comparison of Forecasts

Forecasting performance of the models are compared by calculating ex post dynamic one-year-ahead forecasts of unemployment rate. In the case of the baseline HM model, the information about u_t, p_t and ε_t^2 is used together with the knowledge of the policy function coefficients to make a dynamic forecast 12 months ahead $u_{t+12|t}^{HM}$ which is then compared to the observed value u_{t+12}. This exercise is repeated for $t = 1986\text{ M4}, ..., 2015\text{ M8}$. Smoothed value of the variable ε_t^2 was utilized as it is not directly observable. In the case of the backward-looking WD model, the values of the variables u_t, θ_t and f_t were used to make a dynamic forecasts 12 months ahead $u_{t+12|t}^{WD}$ which is also compared to the observed value u_{t+12}. These forecasts were calculated only for $t = 1986\text{ M4}, ..., 2004\text{ M4}$ because data for market tightness is not available after 2005 M4.

Figure 6
Comparison of Dynamic One-year-ahead Forecasts of Unemployment Rate



Source: Own calculations.

This figure illustrates that the WD model outperforms the baseline HM model. The mean squared error (MSE) of the HM forecasts is 5.1738e-04 while for the WD forecasts it is only 1.5019e-04.

Conclusion

This paper contributes to the existing literature by introducing the concept of weak demand into the basic search-matching framework of unemployment in both a simple and novel way. The significant finding is that incorporating this principle gives rise to a multiplicity of equilibrium unemployment rates which makes it impossible to econometrically estimate such a model by standard algorithms implemented in Dynare. For this reason, the backward-looking version of the forward-looking weak demand model was formulated and analysed in this paper.

Empirical performance of the backward-looking version of the weak demand model with multiple equilibria was compared to the baseline search-matching model formulated by Hagedorn and Manovskii (2008) which is characterized by a unique steady state. Firstly, the weak demand model outperforms the baseline model in matching autocorrelation functions. Higher order autocorrelation coefficients of unemployment rate were systematically underestimated by the baseline search-matching mode. This result suggests that the baseline model has difficulties in capturing highly persistent behaviour of unemployment in Spain which is characterized by huge swings ranging from 8% to 27%. This persistence is explained by aggregate demand transition mechanism according to which a decline in aggregate demand causes unemployment rate to converge to the higher equilibrium. Secondly, forecasting performance of the weak demand model is improved compared to the baseline search and matching model. These results of empirical comparison thus suggest that a multiple equilibria model in which unemployment rate might fluctuate around ineffective equilibrium for quite a long time is more appropriate for the Spanish labour market. High unemployment rate is thus treated as a long-run phenomenon which is in sharp contrast to the standard search and matching theory.

Another interesting finding is that a calibration strategy suggested by Hagedorn and Manovskii (2008) is supported by the results of econometric estimation. These authors showed that calibrating value of non-market activity close to 1 and bargaining power of workers close to 0 generates volatilities of unemployment rate and market tightness that are close to that observed in U.S. data. Econometric estimation of the baseline search-matching model thus supports the view that their proposed calibration strategy might be appropriate not only for U.S. data.

An empirical labour market analysis performed in this paper could be extended in several dimensions. Unemployment could be disaggregated for different groups of workers – high and low skilled (Hagedorn, Manovskii and Stetsenko, 2016), young and old (Hahn, 2009; Janíčko, 2012), long-term and short-term unemployed (Hynninen, 2009). Spatial econometric analysis of unemployment could also be performed (Di Addario, 2011; Formánek and Hušek, 2015). Empirical performance of the weak demand model could be compared to the baseline search and matching model not only for Spain but also for other economies. Comparison of the formulated model with DMP-DSGE approach is also an interesting topic for a future research.

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