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# MODEL OF STABILIZING OF THE INTEREST RATE ON DEPOSITS BANKING SYSTEM USING BY MOMENT EQUATIONS

Irada Dzhalladova — Mária Michalková — Miroslava Růžičková

ABSTRACT. The paper deals with a system of difference equations, where coefficients depend on Markov chains. The functional equations for particular density and the moment equations for the system are derived and used in the investigation of solvability and stability. An application of the results is shown how to solve various economic problems.

## 1. Introduction

At the beginning of the 20th century French mathematician Louis Bachelier (1870–1946) evaluated stock options on the Paris market applying stochastic processes. A. Einstein in the same manner described Brownian motion of suspended particles in the liquid. Although the genesis of the theory of stochastic processes is in economics, after L. Bachelier it was mostly developed in physics.

In the twenties of the 20th century it was found that even in a sequence of equally distributed independent random variables could occur quite naturally marginal distributions other than the normal. Mechanism of creation majority of such regularities can be understood only using the theory of the Markov processes.

Process in a system is called a Markov process, if at any time the probability of any future state of the system depends only on the state of the system at the moment and does not depend on how the system has come into this state.

Markov models are widely used in controlling. They form the basis of modern arsenal of probabilistic methods in relation to the description of the state


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of the managed object and the transition from one state to another at time with an acceptable degree of accuracy and reliability. Basic properties of Markov processes are represented in [12].

In our opinion, based on the stochastic approach, we can study a number of aspects concerning the operation of the banking system. Our aim is to build models in the economic system, using elements of random processes. In this paper, we will present how this approach can be used to study the profits of banks, using a model based on the constructed moment equations. Focusing on the economic feasibility of management decision making at each stage of such project will not only save resources, but will also ensure the planned profit level.

Obviously, in developing solutions, management of complex systems always has to take into account the uncertainty and risk, while allowing some regularities of the probabilistic nature in accordance with the role of individual or mass of random phenomena. Uncertainty is especially characteristic of the decisions that have to be made in fast changing circumstances. Reduce uncertainty in two ways: either to try to obtain additional relevant information and reexamine the problem; or to act in strict accordance with the past experiences, thoughts or intuition and make an assumption about the probability of events when there is not enough time to collect additional information or it costs too high.

So, development of governance profit models in banks is possible within the framework of the stochastic approach.

Investigating stability of solutions of difference equations with random coefficients depending on Markov or non-Markov, in particular semi-Markov, process represents a current problem.

A dynamic system called a system with random parameters is considered in this paper. Some dynamic systems with random parameters are mentioned in the papers by V. V. Anisimov [1], I. Ya. Katz–N. N. Krasovskii [13], A. N. Kolmogorov [14], V. S. Korolyuk [15], [16], P. Levi [19] and others.

Investigating stability in mean and stability in mean square using traditional methods of Lyapunov functions is considered in [2], [3], [7], [8], [9], [11], [20], [22], [24], [27]. The class of systems investigated in this paper, i.e., systems with jumps, is considered in [12], [23] and others.

Considering a more accurate models with semi-Markov coefficients remains an open problem. It will be possible to obtain necessary and sufficient conditions for stability in mean square and conditions for $L_2$-stability of systems with semi--Markov coefficients.

## 2. Statement of the problem

Let us consider the initial value problem on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ formulated for stochastic dynamic system with random coefficients

$$x_{n+1} = A(n, \xi_n)x_n, \qquad n = 1, 2, \ldots, \tag{1}$$

$$x_0 = \varphi(\omega), \tag{2}$$

where $\varphi : \Omega \to \mathbb{R}^m$, $\varphi \in C(\Omega), A$ is $m \times m$ matrix with random elements, $\xi_n$ is the Markov chain of finite number of the states $\theta_1, \theta_2, \ldots, \theta_q$ with the probabilities $p_k(n) = P\{\xi_n = \theta_k\}$, $k = 1, 2, \ldots, q$, $n = 1, 2, \ldots$ that satisfy the system of difference equations

$$p_k(n+1) = \sum_{s=1}^{q} \pi_{ks} p_s(n), \qquad k = 1, 2, \ldots, q, \tag{3}$$

with transition matrix $\big(\pi_{ks}(t)\big)_{k,s=1}^{q}$.

**Definition 1** ([6]). The $m$-dimensional random vector $x_n$ is called a solution of the initial value problem (1), (2) if $x_n$ satisfies (1) and initial condition (2) in the sense of strong solution of the initial Cauchy problem.

To define stochastic value $x_n$ it needs: at first, to determine a discrete set of its values, it means a discrete phase space of states; next to determine the probability distribution on this set.

The space $X$ of solutions is often interpreted as a phase space of states of random space, of which measurable subsets represent set of observed states of the space. As a phase space of states we consider complete separable metric space, usually Euclidean space or finite set of $\sigma$-algebra of all subsets of $X$.

Our task is to obtain reliable and simple method for investigating stability of solutions of this class of systems, also its justification and application to solving different practical problems representing a continuation of the series of papers, for example [5], [10], [21], referring to this field of study.

In this article we present an appropriate method for investigating stability, it means the method of moment equations.

**Definition 2.** Let $x_n \in \mathbb{R}^m$ be the random variable depending on a random Markov chain $\xi_n$ with $q$ possible states $\theta_k$, $k = 1, 2, \ldots, q$. The matrices

$$E(n) = \sum_{k=1}^{q} E^{(k)}(n), \qquad D(n) = \sum_{k=1}^{q} D^{(k)}(n),$$

where

$$E^{(k)}(n) = \int_{\mathbb{E}_m} x f_k(n, x)\, dx, \quad D^{(k)}(n) = \int_{\mathbb{E}_m} x\, x^* f_k(n, x)\, dx, \quad k = 1, 2, \ldots, q,$$

are called moments of the first or the second order of the random variable $x_n$ respectively. The values $E^{(k)}(n)$ and $D^{(k)}(n)$, $k = 1, 2, \dots, q$, are called particular moments of the first or the second order respectively.

**Remark 1.** The moments of the random variable $x_n$ in a scalar case, $x_n \in \mathbb{R}$, are defined for any $s = 1, 2, \dots$, and are called moments of the $s$th order. The particular moments are defined by the formula

$$E_s^{(k)}(n) = \int_{-\infty}^{\infty} x^s f_k(n, x)\, dx, \quad s = 1, 2, \dots, \quad k = 1, 2, \dots, q. \tag{4}$$

Several different stability statements are possible. We recall here mean square stability definition.

**Definition 3.** The trivial solution of the system (1) is said to be mean square stable, if for any $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ such that the mathematical expectation

$$E\big(\|x_n\|^2\big) < \varepsilon \qquad \text{for all} \quad n = 1, 2, \dots$$

whenever the initial probability distribution $x_0$ satisfies $E\big(\|x_0\|^2\big) < \delta(\varepsilon)$.

## 3. Moment equations for the linear homogenous difference equations

At first, on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ we consider initial value problem (1), (2) when the system (1) is stochastic difference equation of the first order in the form

$$x_{n+1} = a(\xi_n) x_n, \tag{5}$$

where $\xi_n$ is random Markov chain which has only two possible states $\xi_n = \theta_1$, $\xi_n = \theta_2$ with probabilities

$$p_k(n) = P\{\xi_n = \theta_k\}, \qquad k = 1, 2.$$

In the proof of the next theorem we denote

$$a(\theta_1) = a_1,$$
$$a(\theta_2) = a_2.$$

We will suppose that for $0 \le \lambda \le 1$, $0 \le \nu \le 1$, the probabilities $p_k(n)$, $k = 1, 2$, satisfy the system of difference equations

$$\begin{aligned} p_1(n+1) &= (1-\lambda) p_1(n) + \nu p_2(n), \\ p_2(n+1) &= \lambda p_1(n) + (1-\nu) p_2(n). \end{aligned} \tag{6}$$

**Theorem 1.** *Moment equations of any order $s = 1, 2, 3, \dots$ for the equation (5) are of the form*

$$E_s^{(1)}(n+1) = (1-\lambda)a_1^s E_s^{(1)}(n) + \nu a_2^s E_s^{(2)}(n), \tag{7}$$

$$E_s^{(2)}(n+1) = \lambda a_1^s E_s^{(1)}(n) + (1-\nu)a_2^s E_s^{(2)}(n). \tag{8}$$

P r o o f. We consider the possible states of the random variable $\xi_n$ (see in [26]). The random variable $\xi_n$ can be in the state $\xi_n = \theta_1$. The particular density function corresponding to the $x_n$ is $f_1(n, x)$. The random variable $\xi_{n+1}$ can also be in the state $\xi_{n+1} = \theta_1$ with probability $(1-\lambda)$ and the particular density function corresponding to the $x_{n+1}$ is $f_1\bigl(n, \frac{x}{a_1}\bigr)\frac{1}{a_1}\cdot$ If the random variable $\xi_n$ is in the state $\xi_n = \theta_2$ then the particular density function corresponding to the $x_n$ is $f_2(n, x)$. The transition probability to the state $\xi_{n+1} = \theta_1$ of the random variable $\xi_{n+1}$ is $\nu$ and the density function corresponding to the random variable $x_{n+1}$ is $f_2\bigl(n, \frac{x}{a_2}\bigr)\frac{1}{a_2}\cdot$ Now, in accordance with the formula for total probability, we obtain the first relationship for the particular density functions:

$$f_1(n+1, x) = \frac{1-\lambda}{a_1} f_1\left(n, \frac{x}{a_1}\right) + \frac{\nu}{a_2} f_2\left(n, \frac{x}{a_2}\right)\cdot \tag{9}$$

In the same way the second relationship can be obtained

$$f_2(n+1, x) = \frac{\lambda}{a_1} f_1\left(n, \frac{x}{a_1}\right) + \frac{1-\nu}{a_2} f_2\left(n, \frac{x}{a_2}\right) \tag{10}$$

for the particular density functions.

By some modification of the equation (9) we obtain the first moment, i.e., the equation (7). Specifically, we multiply the left side of equation (9) and (10) by the $x$ and we integrate them from $-\infty$ to $\infty$. So, we get

$$\int_{-\infty}^{\infty} x\, f_1(n+1,\, x)\, dx = E^{(1)}(n+1),$$

$$\int_{-\infty}^{\infty} x\, f_2(n+1,\, x)\, dx = E^{(2)}(n+1),$$

or, by using linear change of integrating variables, we obtain

$$\int_{-\infty}^{\infty} x\, f_1\left(n, \frac{x}{a_1}\right)\frac{dx}{a_1} = a_1 E^{(1)}(n),$$

$$\int_{-\infty}^{\infty} x\, f_2\left(n, \frac{x}{a_2}\right)\frac{dx}{a_2} = a_2 E^{(2)}(n).$$

Taking this into account, the equation (9) can be rewritten in the form

$$E^{(1)}(n+1) = (1-\lambda)a_1E^{(1)}(n) + \nu a_2E^{(2)}(n). \tag{11}$$

Let us explain meaning of the equation (11). If $\xi_n = \theta_1$ then the random variable $\xi_{n+1}$ is in the same state $\theta_1$ with probability $(1-\lambda)$. Using the linear transformation $x_{n+1} = a_1x_n$ the first particular moment $E^{(1)}(n+1)$ of random variable $x_{n+1}$ can be derived from the first particular moment $E^{(1)}(n)$ of random variable $x_n$ multiplied by $a_1$.

If $\xi_n = \theta_2$, $\xi_{n+1} = \theta_1$, using the linear transformation $x_{n+1} = a_2x_n$, the first particular moment $E^{(1)}(n+1)$ of random variable $x_{n+1}$ can be derived from the first particular moment $E^{(2)}(n)$ of random variable $x_n$ multiplied by $a_2$. Therefore, according to the formula of mathematical expectation we obtain the equation (11).

By the similar considerations that have been used above, it can be also found the equation

$$E^{(2)}(n+1) = \lambda a_1E^{(1)}(n) + (1-\nu)a_2E^{(2)}(n). \tag{12}$$

The system of difference equations (11), (12) describes behavior of the first particular moments of random variables $x_n$, $x_{n+1}$.

The system of difference equations

$$D^{(1)}(n+1) = (1-\lambda)\, a_1^2\, D^{(1)}(n) + \nu\, a_2^2\, D^{(2)}(n),$$

$$D^{(2)}(n+1) = \lambda\, a_1^2\, D^{(1)}(n) + (1-\nu)a_2^2\, D^{(2)}(n)$$

for the second particular moments $D^{(k)}(n) = E_2^{(k)}(n)$, $k = 1, 2$ can be obtained by the same way as for the first particular moments.

Finally, the system of linear difference equations of the type (7), (8) for moments $E_s^{(k)}(n)$, $k = 1, 2$, of any order $s = 1, 2, 3, \dots$ can be derived by the same way, in accordance with the formula (4). □

**Remark 2.** The system of equations (7), (8) can be rewritten in simpler form

$$\begin{aligned} u_{n+1} &= (1-\lambda)a_1^s u_n + \nu a_2^s \upsilon_n, \\ \upsilon_{n+1} &= \lambda a_1^s u_n + (1-\nu)a_2^s \upsilon_n, \qquad s = 1, 2, 3, \dots, \end{aligned} \tag{13}$$

where $u_n = E^{(1)}(n)$, $\upsilon_n = E^{(2)}(n)$, $u_{n+1} = E^{(1)}(n+1)$, $\upsilon_{n+1} = E^{(2)}(n+1)$.

**Remark 3.** For the probability density functions

$$f(n,x) = f^{(1)}(n,x) + f^{(2)}(n,x)$$

it is impossible to create a simple system of ordinary difference equations, that would reflect the relationship among values of $f(n,x)$ for different values of $n$. It is possible only for particular values of the probability density functions $f^{(1)}(n,x)$, $f^{(2)}(n,x)$.

Example 1. Let us find condition for mean stability of solutions of linear equations (5) such that probabilities $p_k(n) = P\{\xi_n = \theta_k\}$, $k = 1, 2$ satisfy the system of difference equations (6). The mean stability of solutions of the difference equation (5) is equivalent to the stability of solutions of the system (11), (12) or, in view to the Remark 2, is equivalent to its simpler form (13) with $s = 1$.

The characteristic equation, under assumption $\nu = \lambda$, is

$$\begin{vmatrix} z-(1-\lambda)a_1 & -\lambda a_2 \\ -\lambda a_1 & z-(1-\lambda)a_2 \end{vmatrix} = z^2 - z(1-\lambda)(a_1+a_2) + (1-2\lambda)\, a_1 a_2 = 0. \quad (14)$$

Both roots of the equation (14) are real. For the maximum absolute value of this roots the condition

$$\begin{aligned} z_{max} &= \max\{|z_1|, |z_2|\} \\ &= \frac{(1-\lambda)(a_1+a_2)}{2} + \sqrt{(1-\lambda)^2\frac{(a_1-a_2)^2}{4} + \lambda^2 a_1 a_2} < 1, \end{aligned}$$

is valid, if the inequality below holds

$$(a_1+a_2)(1-\lambda) < 1 + a_1 a_2 (1-2\lambda). \quad (15)$$

The condition (15) specifies the domain of mean stability of solutions of difference equation (5).

By the same way it can be derived the condition

$$(a_1^2+a_2^2)(1-\lambda) < 1 + a_1^2 a_2^2 (1-2\lambda),$$

which defines the domain of mean square stability of solutions of equations (5) and the condition

$$(a_1^s+a_2^s)(1-\lambda) < 1 + a_1^s a_2^s (1-2\lambda)$$

defines the domain of stability in the $s$th mean of the solutions.

Let us discuss some values of $a_1$, $a_2$. It is simple to prove the inequality

$$a_1 \le z_{max} \le a_2$$

under the assumption $0 < a_1 < a_2$. Moreover, if

$$0 < a_k < 1, \qquad k = 1, 2, \quad \text{then} \quad z_{max} < 1$$

and the zero solutions of the system (13) are asymptotically stable. In the other side, if

$$a_k > 1, \qquad k = 1, 2,$$

then the zero solution oscillates. The most interesting situation is when

$$0 < a_1 < 1 < a_2.$$

The considered equation (5) becomes deterministic if $\lambda = 1$ and takes the form of equation

$$x_{n+2} = a_1\, a_2\, x_n.$$

The domain of stability in the case of deterministic equation is once shaded in the Figure 1. The domain of the mean stability of solutions of equation (5) in the case $\lambda \neq 1$ obtained from (15) is twice shaded in the Figure 1.

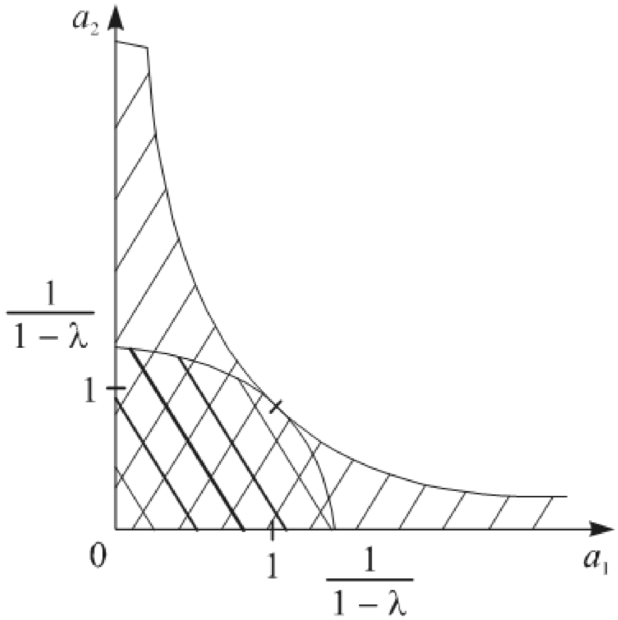


FIGURE 1. Domains of stability for equation (5).

## 4. Model problem

We apply the results on the stability of solutions of the system (6) established above in economical model problem. We consider the following situation: the bank randomly changes the rate interest of deposits while choosing from two values $p$ or $q$. Our aim is to find the mean value of the rate for sufficiently large $n$. Suppose that the change of the rate is described by difference equations (5), where

$$p_k(n) = P\{\xi_n = \theta_k\}, \qquad k = 1, 2, \quad n = 1, 2, \ldots$$

Let $x_n$ be random value of the rate through $n$ time intervals. Then changes of the rate can be described by difference equation

$$x_{n+1} = a(\xi_n)x_n.$$

The coefficient of equation (5) when the random variable is in the state $\theta_1$ is

$$a_1 = a(\theta_1) = 1 + \frac{p}{100}.$$

If the random variable is in the state $\theta_2$ then the coefficient is

$$a_2 = a(\theta_2) = 1 + \frac{q}{100}.$$

The first moment

$$E(n) = E^{(1)}(n) + E^{(2)}(n)$$

of random solution is described by the system of difference equations (11), (12), where values of coefficients established here are used.

Growth of solutions is defined by $z_{max}$

$$z_{max} = \frac{(1-\lambda)a_1 + (1-\nu)a_2}{2} + \sqrt{\left(\frac{(1-\lambda)a_1 - (1-\nu)a_2}{2}\right)^2 + \lambda\nu a_1 a_2}\,. \tag{16}$$

If the rate is not changing, it means $a_1 = a_2$, then from the formula (16) we obtain

$$z_{max} = a_1.$$

It is interesting to calculate mean increase of the rate for various values of $\lambda$ and $\nu$. The results are in Table 1. The average time of remain in the first state is $\lambda^{-1}$, in the second state $\nu^{-1}$.

As we can see in Table 1, increase of the rate interest is equal to mean value of rates $p$ and $q$ if $\lambda = \nu = 0,5$.

TABLE 1. Table of mean increase of the rate.

| $\lambda = \nu$ | % | $\lambda = \nu$ | % | $\lambda$ | $\nu$ | % |
|---|---|---|---|---|---|---|
| $0,000$ | $5,000$ | $0,2$ | $4,014$ | $0,1$ | $0,2$ | $4,353$ |
| $0,001$ | $4,900$ | $0,3$ | $4,006$ | $0,1$ | $0,3$ | $4,511$ |
| $0,010$ | $4,395$ | $0,4$ | $4,002$ | $0,1$ | $0,4$ | $4,606$ |
| $0,100$ | $4,038$ | $0,5$ | $4,000$ | $0,1$ | $0,5$ | $4,670$ |

## 5. Moment equations for the non-homogenous linear difference equations

Next, on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ we consider the initial value problem (1), (2) when the system (1) is stochastic nonhomogenous linear difference equation of the first order in the form

$$x_{n+1} = a(\xi_n)x_n + b(\xi_n), \tag{17}$$

where $\xi_n$ is a random Markov chain which has $q$ states

$$\xi_n = \theta_k, \qquad k = 1, 2, \dots, q$$

with probabilities

$$p_k(n) = P\{\xi_n = \theta_k\}, \qquad k = 1, 2, \dots, q$$

that satisfy equation (3).

In the next theorem we also denote

$$a_k = a(\theta_k), \quad b_k = b(\theta_k), \quad a_k \neq 0, \quad k = 1, 2, \dots, q.$$

**Theorem 2.** *Moment equations of any order $s = 1, 2, \dots$ for the equations* (17) *are of the form*

$$E_s^{(k)}(n+1) = \sum_{r=1}^{q} \pi_{kr} \sum_{j=0}^{s} \binom{s}{j} a_r^j \, b_r^{s-j} \, E_j^{(r)}(n), \qquad k = 1, 2, \dots, q. \tag{18}$$

P r o o f. For particular probability density function we obtain system of functional equations

$$f_k(n+1, x) = \sum_{s=1}^{q} \pi_{ks} f_s\left(n, \frac{x - b_s}{a_s}\right) \cdot \frac{1}{a_s}, \qquad k = 1, 2, \dots, q. \tag{19}$$

The system (18) of linear difference equations for particular $s$th moments, $s = 1, 2, \dots$ can be obtained multiplying particular probability density functions by the $x^s$ and integrating them from $-\infty$, to $\infty$. $\square$

Let us recall that the total density function is a sum of particular density functions

$$f(n, x) = \sum_{k=1}^{q} f_k(n, x).$$

The particular moment of any order is defined by the formula (4). For $s = 0$ we have

$$E_0^{(k)}(n) \equiv p_k(n), \qquad k = 1, 2, \dots, q.$$

In general, to find particular moment of order $s$ it supposes to know all the particular moments of order $0, 1, \ldots, s-1$. Step by step in accordance to the formula (18) we get

$$\begin{aligned}
E_0^{(k)}(n+1) &= \sum_{r=1}^{q} \pi_{kr} E_0^{(r)}(n), \\
E_1^{(k)}(n+1) &= \sum_{r=1}^{q} \pi_{kr} \Big( a_r E_1^{(r)}(n) + b_r E_0^{(r)}(n) \Big), \\
E_2^{(k)}(n+1) &= \sum_{r=1}^{q} \pi_{kr} \Big( a_r^2 E_2^{(r)}(n) + 2a_r b_r E_1^{(r)}(n) + b_r^2 E_0^{(r)}(n) \Big), \\
&\vdots
\end{aligned}$$

The system of difference equations (18) has simpler form

$$E_s^{(k)}(n+1) = \sum_{r=1}^{q} \pi_{kr} a_r^s E_s^{(r)}(n), \qquad k = 1, 2, \ldots, q$$

in the case when $b(\xi) \equiv 0$ in the system of equations (19).

## 6. Moment equations for the non-homogenous system of linear difference equations

In the last section we consider the system of linear difference equations

$$x_{n+1} = A(\xi_n) x_n + B(\xi_n), \tag{20}$$

on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ together with the initial condition (2). In the considered system $\xi_n$ is Markov chain which has $q$ different states $\theta_1, \theta_2, \ldots, \theta_q$. We suppose that probabilities $p_k(n) = P\{\xi_n = \theta_k\}$ satisfy the system of linear difference equations

$$p_k(n+1) = \sum_{k=1}^{q} f_k(n, x) \delta(\xi - \theta_k),$$

where $\delta(\xi)$ is a Dirac function.

We denote

$$A(\theta_k) = A_k, \quad B(\theta_k) = B_k$$

and we suppose

$$\det A_k \neq 0.$$

**Theorem 3.** *Moment equations of first and second order for the system of equations* (20) *are of the form*

$$
\begin{aligned}
E^{(k)}(n+1) &= \sum_{s=1}^{q} \pi_{ks} \Big( A_s E^{(s)}(n) + B_s p_s(n) \Big), \\
D^{(k)}(n+1) &= \sum_{s=1}^{q} \pi_{ks} \Big( A_s D^{(s)}(n) A_s^* + A_s E^{(s)}(n) B_s^* \\
&\qquad + B_s \big( E^{(s)}(n) \big)^* A_s^* + B_s B_s^* p_s(n) \Big), \qquad k = 1, 2, \dots, q. \qquad (21)
\end{aligned}
$$

P r o o f. The probability density functions $f_k(n, x)$, $k = 1, 2, \dots, q$ satisfy the system of linear functional equations

$$
f_k(n+1,\, x) = \sum_{s=1}^{q} \pi_{ks} f_s\Big(n, A_s^{-1}(x - B_s)\Big) \det\, A_s^{-1}, \\
k = 1, 2, \dots, q, \quad n = 1, 2, \dots \qquad (22)
$$

The first equation of (21), in accordance with Definition 2, we obtain by the multiplying equation (22) by the column vector $x$, subsequently by integrating the product on $m$-dimensional phase space $E_m$. By the same way we get the second equation of (21). □

**Corollary 1.** *For the homogenous system of difference equations with random Markov coefficients, i.e., for the system*

$$
x_{n+1} = A(\xi_n) x_n, \qquad n = 1, 2, \dots, \qquad (23)
$$

*the system of the moment equations* (21), *is in the following form*

$$
\begin{aligned}
E^{(k)}(n+1) &= \sum_{s=1}^{q} \pi_{ks} A_s E^{(s)}(n), \\
D^{(k)}(n+1) &= \sum_{s=1}^{q} \pi_{ks} A_s D^{(s)}(n) A_s^*, \qquad k = 1, 2, \dots, q.
\end{aligned}
$$

Example 2. Let us construct system of moment equations for the system of difference equations (23), where

$$
A_1 = A(\theta_1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_2 = A(\theta_2) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},
$$

$$
\pi_{11} = 1 - \lambda, \quad \pi_{12} = \nu, \quad \pi_{21} = \lambda, \quad \pi_{22} = 1 - \nu.
$$

Then the system of moment equations of the first order is in the form

$$E^{(1)}(n+1) = (1-\lambda)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}E^{(1)}(n) + \nu\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}E^{(2)}(n),$$

$$E^{(2)}(n+1) = \lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}E^{(1)}(n) + (1-\nu)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}E^{(2)}(n).$$

The system of moment equations of the second order is of the form

$$\begin{aligned} D^{(1)}(n+1) &= (1-\lambda)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}D^{(1)}(n)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ &\quad + \nu\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}D^{(2)}(n)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix},\end{aligned}$$

$$\begin{aligned} D^{(2)}(n+1) &= \lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}D^{(1)}(n)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ &\quad + (1-\nu)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}D^{(2)}(n)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}.\end{aligned}$$

As a conclusion let us remark that considered difference equations describe the evolution of a state of dynamical systems. Development of mathematical modeling of natural phenomena requires new enhanced methods. The established moments equations can be used also for solving problems such as:

- finding a model of dynamics of populations that do not interfere in biology;
- solving a problem of bank operations—models of saturation of citizens saving;
- solving economics problem—to find a model of market, where there are delays of products selling and others.

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*Irada A. Dzhalladova*
*ISTF Advanced Mathematics Department*
*Kyiv National Economic*
*Vadym Getman University*
*54/1 Prospect Peremogy*
*03680 Kyiv*
*UKRAINE*
*E-mail*: irada–05@mail.ru

*Mária Michalková*
*Miroslava Růžičková*
*Department of Mathematics*
*Faculty of Humanities*
*University of Žilina*
*Univerzitn 8215/1*
*SK–010-26 Žilina*
*SLOVAKIA*
*E-mail*: maria.michalkova@fhv.uniza.sk
miroslava.ruzickova@fpv.uniza.sk