GEOLOGICA CARPATHICA, FEBRUARY 2007, 58, 1, 97–102

Geophysical indirect effect on interpretation of gravity data in Slovakia

PETER VAJDA and JAROSLAVA PÁNISOVÁ

Geophysical Institute, Slovak Academy of Sciences, Dúbravská cesta 9, 845 28 Bratislava, Slovak Republic;
peter.vajda@savba.sk; jpanisova@yahoo.co.uk

(Manuscript received November 22, 2005; accepted in revised form June 22, 2006)

Abstract: This paper deals with the proper definition and correct compilation of the Bouguer gravity anomaly that differs from the standard definition repeatedly advocated in the geophysical literature. The difference between the new "correct" definition of the Bouguer anomaly and its "standard" definition has become known in geophysics as the "geophysical indirect effect". Here we investigate the magnitude and variation of the geophysical indirect effect as a systematic error in gravity data inversion or interpretation in Slovakia. It is found that in Slovakia this effect is of the order of 3 mGal with a spatial variability of about 0.5 mGal over some 100 km. The impact of the geophysical indirect effect on the determination of the depth of a density interface is also estimated. In Slovakia, the impact of this systematic error is of the order of about 300 meters for a density contrast of 300 kg/m^3.

Key words: gravity data, Bouguer anomaly, geophysical indirect effect.

Introduction

When interpreting gravity data, we seek to determine the density distribution in order to gather knowledge of subsurface geological structure. The inverse problem in gravimetry is typically formulated in terms of relating the anomalous density distribution to the parameters of the anomalous field derived from the disturbing potential. These parameters include the gravity anomaly, gravity disturbance, geoidal undulation, and various derivatives (horizontal, vertical, mixed) of the disturbing potential.

We focus in particular on the relation between anomalous masses and anomalous gravity. In geophysical practice, the Bouguer gravity anomaly has been used most extensively. Many geophysicists have already argued that correct interpretation of gravity data requires that the Bouguer anomaly is compiled using normal gravity evaluated at the station (which implies the use of ellipsoidal heights as opposed to heights above sea level in the formula for computing normal gravity), and using the reference ellipsoid (as opposed to sea level) as the lower boundary of topographic masses in evaluating the topographic correction (Chapman & Bodine 1979; Vogel 1982; Jung & Rabinowitz 1988; Meurers 1992; Talwani 1998; Hackney & Featherstone 2003; Vajda et al. 2006; and others). The failure to do so leads to a systematic error that has become known in gravimetry as the "geophysical indirect effect" (ibid).

Vajda et al. (2006) have recently demonstrated that if we look for a *rigorous match* to the "gravitational effect" (attraction) of anomalous density, we arrive at the quantity compiled from observed gravity named by them the "NETC gravity disturbance" (where NETC denotes 'No ellipsoidal topography of constant density', the definition and compilation of which is reviewed in section 2) to distinguish it from the commonly used "Bouguer gravity anomaly". In other words, in the quest to determine the anomalous density by means of gravity inversion or forward and inverse modeling, the gravity data to be modeled or to be inverted are strictly those gravity data defined as the "NETC gravity disturbance". The difference between the *NETC gravity disturbance* and the *Bouguer gravity anomaly*, which constitutes the geophysical indirect effect (cf. ibid), is relatively small, of the order of 10 mGal. The issue we shall examine here is whether or not this relatively small difference is significant to geophysical interpretations. Our aim is to numerically assess the geophysical indirect effect in Slovakia, as well as to estimate its impact on the interpreted anomalous density distribution, especially its impact on the interpreted depth of density interfaces.

Gravity inversion/interpretation formulated in terms of the attraction of anomalous masses

We shall use the following notation: the horizontal positions of points are represented by geographical coordinates, latitude ϕ and longitude λ, vertical position of a point by ellipsoidal (geodetic) height h, which is reckoned from the reference ellipsoid as a height datum, not by a height above sea level (in Slovakia: "normal height") H^N, which is reckoned from "sea level" (in Slovakia: "quasigeoid") as a vertical datum. For brevity, the horizontal position is often denoted by $\Omega \equiv (\phi, \lambda)$. Actual (measured) gravity is denoted by g, normal gravity by γ, and quasigeoidal height (the separation between "sea level" and the reference ellipsoid) by N. In our developments, the Newton volume integrals are evaluated using a spherical approximation (e.g. Vajda et al. 2004a) with a mean earth radius R. Thus, the reciprocal Euclidean distance between a computation point (h, Ω) and any integration point (h', Ω') is (ibid, Eqs. [19] and [22]):

$$L^{-1}(h, \Omega, h', \Omega') = \left[(R+h)^2 + (R+h')^2 - 2(R+h)(R+h')\cos\psi\right]^{-\frac{1}{2}}, \tag{1}$$

where $\cos\psi = \sin\phi \sin\phi' + \cos\phi \cos\phi' \cos(\lambda - \lambda')$, ψ being the angular (spherical) distance between Ω and Ω'.

The *NETC gravity disturbance*, at the observation point, is defined as (Vajda et al. 2006):

$$\delta g^{NETC}(h, \Omega) = g(h, \Omega) - \gamma(h, \Omega) - G\rho_0 \int_0^{h_T(\Omega')} \iint_{\Omega_0} J(h, \Omega, h', \Omega')(R+h')^2 \, dh' d\Omega' \tag{2}$$

The first term on the right hand side is the *actual* (observed) *gravity* at the observation point (station), the second term is the *normal gravity* at the observation point, and the third term is the *topographic correction to the gravity disturbance*, evaluated at the observation point, adopting the reference ellipsoid ($h = 0$) as the lower boundary of topographic masses, the topographic surface ($h = h_T(\Omega)$) as the upper boundary, and constant density ρ_0 for topographic masses. G is the Newton gravitational constant, Ω_0 stands for the solid angle (representing integration over the entire globe — full sphere), $d\Omega' = \cos\phi' \, d\phi' \, d\lambda'$ and the kernel of the volume integral reads:

$$J(h, \Omega, h', \Omega') \equiv \left[-\frac{\partial L^{-1}(h, \Omega, h', \Omega')}{\partial h}\right] = \left[(R+h) - (R+h')\cos\psi\right]\left(L^{-1}\right)^3. \tag{3}$$

This topographic correction should strictly be evaluated over the whole globe. It represents the negative attraction of masses of constant density between the reference ellipsoid and the topographic surface.

Let us now define (ibid) the *gravitational effect of anomalous masses* (δA), i.e. the *attraction* of anomalous density ($\delta\rho$) within the whole earth (below the topographic surface), again assuming a spherical approximation (cf. Vajda et al. 2004a):

$$\delta A(h, \Omega) = G \int_{-R}^{h_T(\Omega')} \iint_{\Omega_0} \delta\rho(h', \Omega') \, J(h, \Omega, h', \Omega')(R+h')^2 \, dh' d\Omega'. \tag{4}$$

Equation (4) represents the Newton volume integral (with the kernel J given by Eq. (3)) of the anomalous density within the entire volume of the earth, bound by the topographic surface. The background (reference) density model used to construct the anomalous density $\delta\rho$ from the real density is described in detail in (Vajda et al. 2004b, sec. 3.1; Vajda et al. 2006, sec. 3). It consists of a model 'normal' density distribution inside the reference ellipsoid that generates the normal gravitational field, and of the constant density distribution between the reference ellipsoid and the topographic surface.

Vajda et al. (2006) give a rigorous proof that:

$$\delta g^{NETC}(h, \Omega) = \delta A(h, \Omega), \tag{5}$$

which, in words, means that the *attraction of anomalous masses* inside the earth (below the topographic surface) is exactly equal to the anomalous gravity quantity defined as the *NETC gravity disturbance*. Put another way: if we are to solve for anomalous density using gravity data inversion/interpretation, we have to (rigorously) match synthetic gravity data, generated by our model of anomalous density (via the Newton volume integral), with "observed" NETC gravity disturbances (as opposed to "observed" Bouguer gravity anomalies). Here "observed" means "compiled from measured gravity". As we have already mentioned, the difference between the *Bouguer gravity anomaly* and the *NETC gravity disturbance* is subtle, and the aim here is to estimate the impact and significance of the difference on gravity data interpretation, with particular focus on Slovakia.

Notice that using Eq. (5) the gravimetric inverse problem can be formulated, for instance, at the topographic surface ($h = h_T$). This means that in the case of forward and inverse modeling, the matching of observed and synthetic data can be done at the observation points (stations) and elsewhere, also on the topographic surface. The gravimetric inverse problem is solved in three steps, as follows:

1 — Compile the NETC gravity disturbance (δg^{NETC}) from *observed gravity* using Eq. (2). For more details regarding the evaluation of normal gravity at the observation point (with a focus on Slovakia), refer to Vajda & Pánisová (2005). For more details regarding the topographic correction to gravity disturbance, see Vajda et al. (2004a) and references therein;

2 — Match the "observed" NETC gravity disturbance with the gravitational effect of anomalous masses (cf. Eqs. (4) and (5)) at the observation points (say on the topography) as follows:

$$\delta g^{NETC}(h, \Omega) = G \int_{-R}^{h_T(\Omega')} \iint_{\Omega_0} \delta\rho(h', \Omega') J(h, \Omega, h', \Omega')(R+h')^2 \, dh' d\Omega'; \tag{6}$$

3 — Solve for anomalous density from the functional on the right hand side of Eq. (6) using either direct inversion methods or forward modeling (trial and error iterative) methods (e.g. Blakely 1995).

As we have already stressed, in practice the role of the left hand side in Eq. (6) is typically and commonly played by the *Bouguer gravity anomaly*, be it planar or spherical, complete or incomplete etc. (e.g. Hayford & Bowie 1912; Bullard 1936; Heiskanen & Moritz 1967; Vaníček & Krakiwsky 1986; LaFehr 1991; Chapin 1996; Talwani 1998; Arafin 2004; Vaníček et al. 2004). For our purposes we consider an "ideal" *spherical complete Bouguer anomaly* (SCBA). "Ideal" means that the topographic correction needed to construct the SCBA is considered as ideally evaluated over the whole globe. Of course, we acknowledge that in practice the topographic correction for the Bouguer anomaly is generally computed only to the Hayford-Bowie radius ($1^\circ 29' 58''$ or 167 km). The effect of the truncation error in computing the topographic correction to the Bouguer anomaly has been discussed in detail most recently by Mikuška et al. (2006), and previously by Talwani (1998), LaFehr (1991), Kotake & Hagiwara (1987), and Pick et al. (1973). This systematic error may become significant when interpreting gravity data on a regional or global scale and on a local scale in mountainous areas (Mikuška et al. 2006). This effect is, however, outside the scope of our investigations. We shall focus on the "ideal" SCBA. Let us at last define the SCBA (cf. Vaníček et al. 1999, Eq. [38]; Vaníček et al. 2004):

$$\Delta g^{SCB}(h,\Omega) = g(h,\Omega) - \gamma(h=H^N,\Omega) - G\rho_0 \int_{N(\Omega')}^{h_T(\Omega')} \iint_{\Omega_0} J(h,\Omega,h',\Omega')(R+h')^2 dh' d\Omega'. \quad (7)$$

Notice that compared to the NETC gravity disturbance (Eq. (2)), when the SCBA is compiled (using Eq. (7)) from observed gravity, normal gravity is not evaluated at the station. This is because normal height (height above sea level) is used in the formula for normal gravity computation instead of the ellipsoidal (geodetic) height. Further, sea level (geoid or quasigeoid) is used as the lower boundary of topographic masses in the topographic correction needed to compile the SCBA. This contrasts to the case in which the ellipsoid is used as the lower topographic boundary in the topographic correction for computing the NETC gravity disturbance.

Geophysical indirect effect

The Geophysical Indirect Effect (GIE) is defined, in its most general form, as the systematic deviation of the ("ideal") SCBA from the gravitational effect of the subsurface anomalous masses:

$$\Delta^{GIE}(h,\Omega) \equiv \delta A(h,\Omega) - \Delta g^{SCB}(h,\Omega). \quad (8)$$

Realizing that the gravitational effect of anomalous masses is rigorously equal to the NETC gravity disturbance (Vajda et al. 2006), we get:

$$\Delta^{GIE}(h,\Omega) = \delta g^{NETC}(h,\Omega) - \Delta g^{SCB}(h,\Omega). \quad (9)$$

By substituting for the two anomalous gravity quantities from Eqs. (2) and (7), we get:

$$\Delta^{GIE}(h,\Omega) = \left[\gamma(h=H^N,\Omega) - \gamma(h,\Omega)\right] - G\rho_0 \int_{0}^{N(\Omega')} \iint_{\Omega_0} J(h,\Omega,h',\Omega')(R+h')^2 dh' d\Omega'. \quad (10)$$

Equation (10) represents the most general form of the *geophysical indirect effect* (Vajda et al. 2006). Notice that the GIE is a spatial rather than a surface quantity, that is, its value varies not only with horizontal position, but also with the altitude of the observation station. The GIE consists of two terms, $\Delta^{GIE}(h,\Omega) = \Delta_1^{GIE}(h,\Omega) + \Delta_2^{GIE}(h,\Omega)$. The first term:

$$\Delta_1^{GIE}(h,\Omega) \equiv \gamma(h=H^N,\Omega) - \gamma(h,\Omega) \quad (11)$$

accounts for the *change of normal gravity* along the *vertical displacement* (between the telluroid and the topographic surface, if the observation point lies on the topographic surface (ibid)). It can be approximated (Vajda et al. 2006, sec. 9; Vajda & Pánisová 2005, sec. 5) as:

$$\Delta_1^{GIE}(\Omega) \cong \gamma_0(\phi) - \gamma(N(\Omega),\Omega) \cong -\frac{\partial\gamma_0(\phi)}{\partial h} N(\Omega) \cong 0.3086\,[mGal/m]\;N(\Omega) \quad (12)$$

where $\gamma_0(\phi)$ is the normal gravity on the reference ellipsoid, the vertical gradient of normal gravity is given in [*mGal/m*], and the geoidal height is given in [*m*]. In Slovakia, this approximation is good to 90 μGal, while the

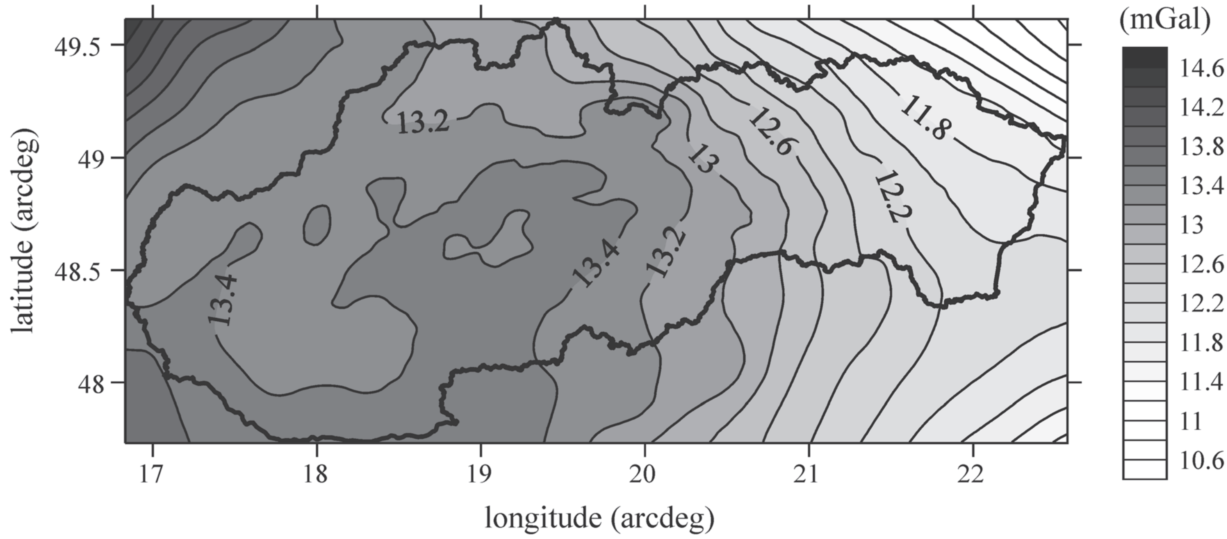

Fig. 1. The $\Delta_1^{GIE}(\Omega)$ term, Eq. (11), in Slovakia [mGal].

effect itself is of the order of 10 mGal (Vajda & Pánisová 2005, sec. 5). Notice that with this approximation, this term becomes a surface quantity, i.e. it only varies with horizontal location and no longer varies with the altitude of the observation station. This effect computed for Slovakia is displayed in Fig. 1.

The second term:

$$\Delta_2^{GIE}(h,\Omega) \equiv -G\rho_0 \int_0^{N(\Omega')} \iint_{\Omega_0} J(h,\Omega,h',\Omega')(R+h')^2 dh' d\Omega' \tag{13}$$

is the *gravitational effect of masses of constant density between the reference ellipsoid and the sea level* (*geoid* or *quasigeoid*). It is computed by numerical integration of the volume integral, while the integration must be carried out over the whole globe. Alternatively, it can be approximated by the gravitational effect of the Bouguer shell (Blakely 1995, Secs 3.2.1 and 3.2.2) with a thickness equal to the (quasi-)geoidal height at the horizontal position of the observation point:

$$\Delta_2^{GIE}(h,\Omega) \cong -\frac{4}{3}\pi \mathrm{G}\rho_0 \frac{[R+N(\Omega)]^3 - R^3}{(R+h)^2}\,. \tag{14}$$

Notice that the second term of the GIE, as well as its Bouguer shell approximation, is a spatial quantity, i.e. it varies not only with horizontal location, but also with altitude. This effect, the second term of the GIE, computed for Slovakia using the Bouguer shell approximation for stations on the topographic surface, is displayed in Fig. 2. The accuracy of the Bouguer shell approximation of Δ_2^{GIE} will be subject to future investigation. The variability of Δ_2^{GIE}, as computed using the Bouguer shell approximation, with the altitude of the station is (for altitudes up to 8 km) three orders of magnitude smaller than the effect itself. Hence, Δ_2^{GIE} with the Bouguer shell approximation can be further approximated as:

$$\Delta_2^{GIE}(\Omega) \approx -4\pi G\rho_0 N(\Omega), \tag{15}$$

which becomes a surface quantity. Assuming that the density of masses between sea level and the reference ellipsoid is constant ($\rho_0 = 2670\ \mathrm{kg/m^3}$), this leads to the following approximation for the whole geophysical indirect effect as a surface quantity (cf. Eqs. (12) and (15)):

$$\Delta^{GIE}(\Omega) \approx [0.3086 - 4\pi G\rho_0]\, N(\Omega) = 0.0863\,[mGal/m]\, N(\Omega)\,[m]. \tag{16}$$

This formula gives a simple estimate of the GIE. A separation between sea level and the ellipsoid of 100 m produces a GIE of about 8 mGal.

If the second term of the GIE is neglected, then the GIE is referred to as the “Free-Air GIE”. If the second term of the GIE is approximated by the gravitational effect of the Bouguer plate/shell, then the GIE is referred to as the “Bouguer GIE”. The GIE for Slovakia, computed using the Bouguer shell approximation and evaluated on the topographic surface, is displayed in Fig. 3.

The geophysical indirect effect in Slovakia has an amplitude of about 3 mGals. Spatially, it varies on a regional scale by a few tenths of a mGal. It appears that for local studies in Slovakia, the GIE can be neglected as a trend of no interest. However, in regional studies of larger extent, this may not be the case.

Let us give a rough estimate of the impact of the GIE on the determination of the depth to a density interface. The error Δd in the interpreted depth d of a density interface (with a density contrast $\Delta\rho$) stemming from neglecting the GIE can be estimated using the gravitational effect of a Bouguer plate (e.g. Blakely 1995, sec. 10.3.1) as:

$$\Delta d(\Omega) \approx \frac{\Delta^{GIE}(\Omega)}{2\pi G \Delta\rho}\,. \tag{17}$$

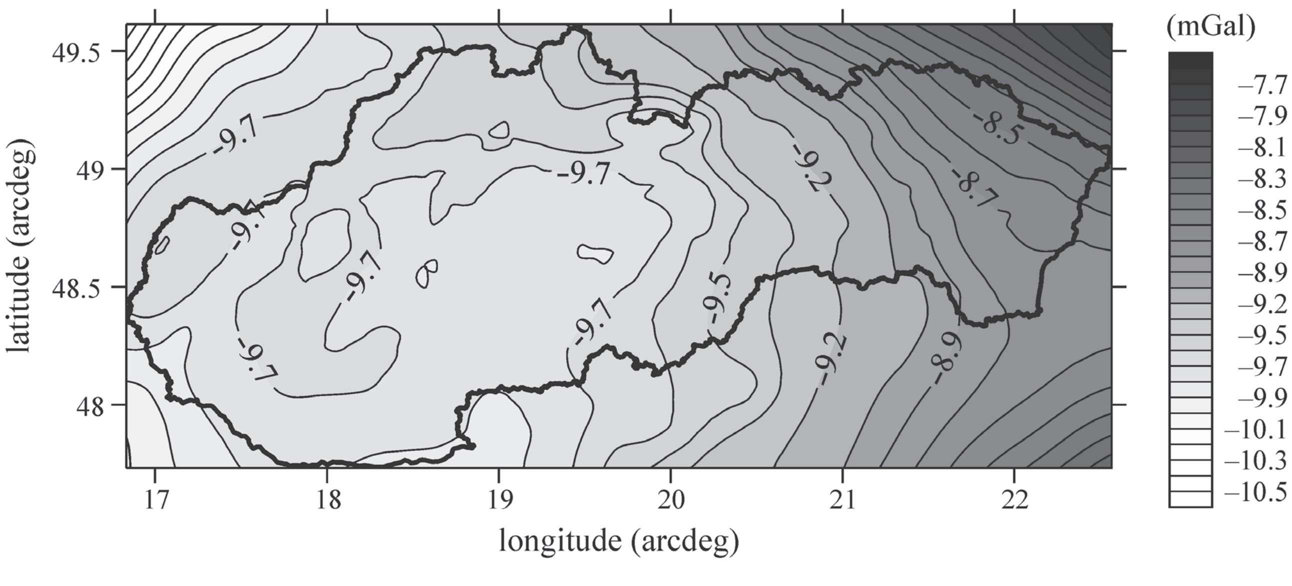

Fig. 2. The $\Delta_2^{GIE}(h = h_T, \Omega)$ term in Slovakia [mGal], evaluated with the Bouguer shell approximation for stations on the topographic surface, Eq. (14).

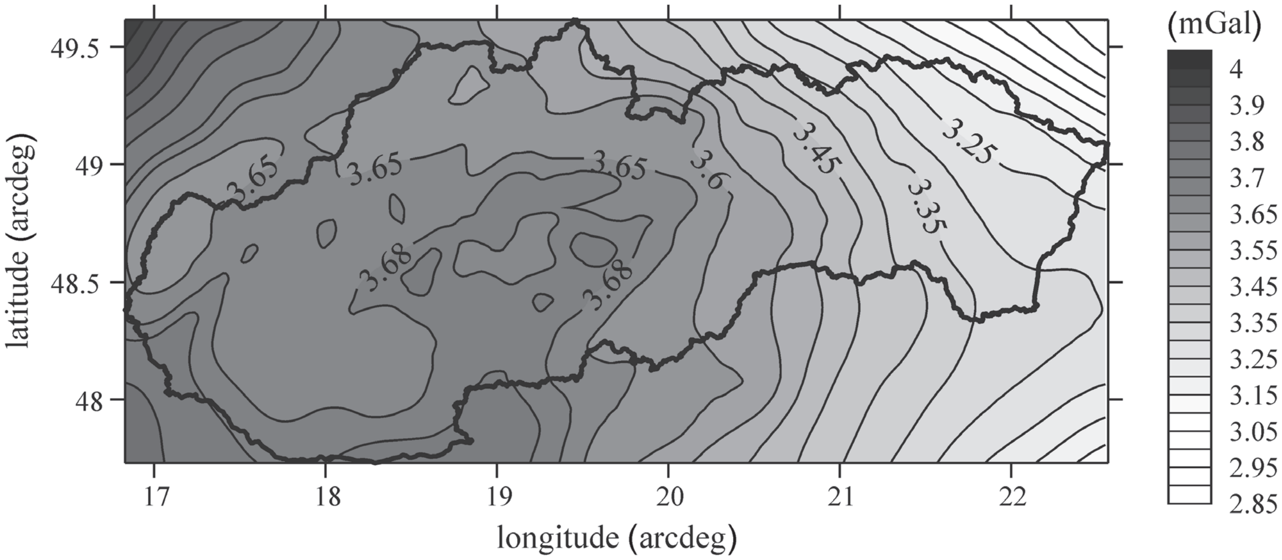

Fig. 3. The geophysical indirect effect $\Delta^{GIE}(h=h_T, \Omega)$ in Slovakia [mGal], evaluated on the topographic surface using the Bouguer shell approximation, Eqs. (11) and (14).

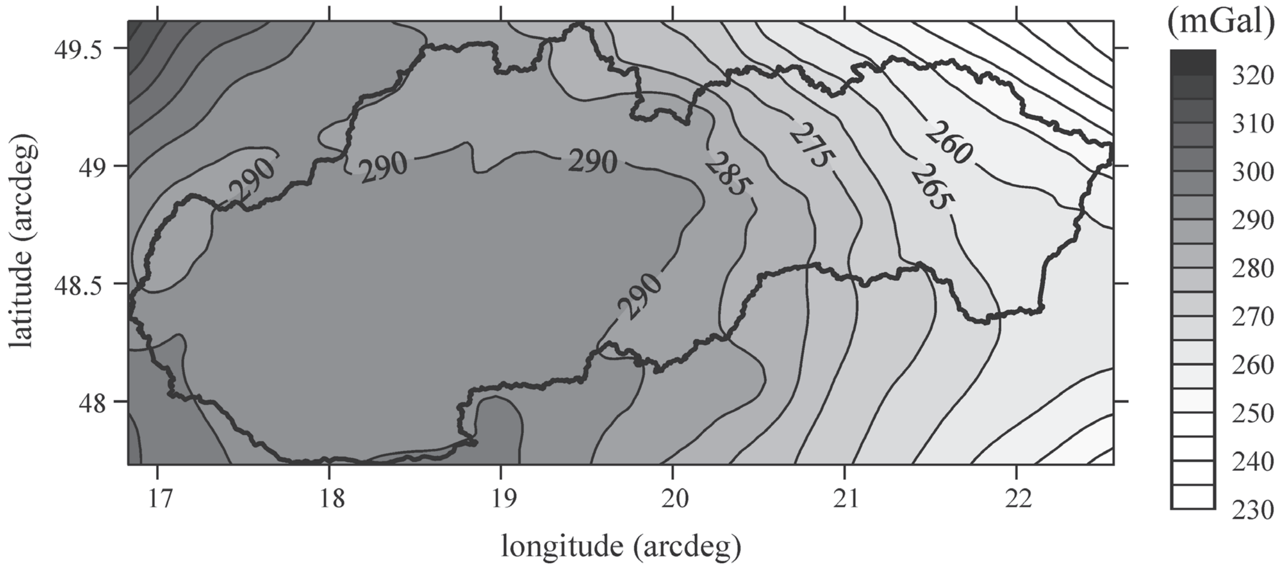

Fig. 4. The impact of the GIE on gravimetric interpretation in Slovakia. This map shows a coarse estimate of the error [m], due to neglecting the GIE, in the interpreted depth of a density interface with a contrast of 300 kg/m^3 (chosen as an example), cf. Eq. (17).

Naturally, the smaller the density contrast across the interface, the greater the impact of the GIE on the interpreted depth of the interface. The estimated error in the interpreted depth of a density interface of, for example, $\Delta\rho = 300$ kg/m^3 due to neglecting the GIE (as displayed in Fig. 3) is portrayed in Fig. 4. In Slovakia, the magnitude of this error is about 300 m.

Using the approximation of Eq. (16) we can estimate this impact as:

$$\Delta d(\Omega) \approx \frac{2\,073\,[kg/m^3]}{\Delta\rho\,[kg/m^3]} N(\Omega). \tag{18}$$

The above simple estimate tells us that if the GIE is neglected, for each meter of separation between sea level and the reference ellipsoid there is an error of about 7 meters in the interpreted depth of a density interface with a density contrast of 300 kg/m^3. The above estimate based on the gravitational effect of a Bouguer plate is coarse. Better insight would be gained by adding the GIE to the input Bouguer anomalies in a specific gravity interpretation, and then examining the change imposed on the interpreted model.

Historically, the lack of ellipsoidal (geodetic) heights at observation points meant that the separation between sea level (geoid or quasigeoid) and the reference ellipsoid was ignored in gravity data inversion or interpretation. Nowadays, in the era of GPS positioning and with the widespread availability of geoid/quasigeoid models, ellipsoidal heights are readily available. This means that it is now possible to compute the GIE and to add it to the Bouguer anomaly, thereby leading to improved gravimetric interpretation and inversion.

Discussion and conclusions

The geophysical indirect effect, as a systematic error in gravity data inversion or interpretation, originates in: 1) the failure to use ellipsoidal heights instead of heights above sea level in evaluating the normal gravity used for compiling the Bouguer gravity anomaly; and 2) the failure to compute the topographic correction using the reference ellipsoid instead of sea level as the lower boundary of the topographic masses. The issue we have pursued here was the size and shape of the geophysical indirect effect as the source of a systematic error, and its impact on gravity data interpretation.

In Slovakia, the GIE is small (Fig. 3). It has a magnitude of 3 mGal and varies by about 0.5 mGal/100 km. We also give a simple estimate of the impact of this systematic error in interpretation of density distribution, namely in terms of an error in the computed depth of a density interface (Eq. (18)). In Fig. 4, we portray the error of the determination of the depth of a density interface with a density contrast of 300 kg/m^3 (taken as an example) caused by the GIE in Slovakia. This error has a magnitude of about 300 meters. We conclude that in most applications in Slovakia, the GIE as a systematic error may be considered insignificant and neglected. However, in other areas with higher mountains, or on larger regional scales, this may not be the case. On the other hand, since this effect is a systematic error, and since it is possible to evaluate it numerically, it can be added to Bouguer gravity anomalies for the sake of an improved gravimetric interpretation. The addition of the GIE to the Bouguer gravity anomaly transforms the Bouguer gravity anomaly, rigorously speaking, to the NETC gravity disturbance (Vajda et al. 2006).

Acknowledgments: This work was carried out with partial support from the Science and Technology Assistance Agency of the Slovak Republic under contract No. APVT-51-002804, and the VEGA Grant Agency Projects No. 2/3057/23 and 2/3004/23. We are grateful for the comments of the reviewers Bruno Meurers and Miroslav Bielik and especially for the detailed comments by Ron Hackney.

References

Arafin S. 2004: Relative Bouguer anomaly. *The Leading Edge* 23, 850–851.

Blakely R.J. 1995: Potential theory in gravity and magnetic applications. *Cambridge University Press,* New York, 1–441.

Bullard E.C. 1936: Gravity measurements in East Africa. *Phil. Trans. Roy. Soc. London, Ser. A* 235, 445–534.

Chapin D.A. 1996: The theory of the Bouguer gravity anomaly: A tutorial. *The Leading Edge* 15, 361–363.

Chapman M.E. & Bodine J.H. 1979: Considerations of the indirect effect in marine gravity modeling. *J. Geophys. Res.* 84, 3889–3892.

Hackney R.I. & Featherstone W.E. 2003: Geodetic versus geophysical perspectives of the ‘Gravity Anomaly’. *Geophys. J. Int.* 154, 35–43.

Hayford J.F. & Bowie W. 1912: The effect of topography and isostatic compensation upon the intensity of gravity. *U.S. Coast and Geodetic Survey, Spec. Publ.* 10, 1–132.

Heiskanen W.A. & Moritz H. 1967: Physical geodesy. *Freeman,* San Francisco, 1–364.

Jung W.Y. & Rabinowitz P.D. 1988: Application of the indirect effect on regional gravity fields in the North Atlantic Ocean. *Mar. Geodesy* 12, 127–133.

Kotake Y. & Hagiwara Y. 1987: Truncation error estimates for the spherical terrain correction of gravity anomaly (in Japanese). *J. Geodetic Soc. Japan* 33, 321–328.

LaFehr T.R. 1991: Standardization in gravity reduction. *Geophysics* 56, 1170–1178.

Meurers B. 1992: Untersuchungen zur Bestimmung und Analyse des Schwerefeldes im Hochgebirge am Beispiel der Ostalpen. *Österr. Beitr. Met. Geoph.* 6, 1–146.

Mikuška J., Pašteka R. & Marušiak I. 2006: Estimation of distant relief effect in gravimetry. *Geophysics* 71, 6, 59–69.

Pick M., Pícha J. & Vyskočil V. 1973: Theory of the Earth’s gravity field. *Academia,* Prague, 1–513 (in Czech).

Talwani M. 1998: Errors in the total Bouguer reduction. *Geophysics* 63, 1125–1130.

Vajda P., Vaníček P., Novák P. & Meurers B. 2004a: On evaluation of Newton integrals in geodetic coordinates: Exact formulation and spherical approximation. *Contr. Geophys. Geod.* 34, 4, 289–314.

Vajda P., Vaníček P. & Meurers B. 2004b: On the removal of the effect of topography on gravity disturbance in gravity data inversion or interpretation. *Contr. Geophys. Geod.* 34, 4, 339–369.

Vajda P., Vaníček P. & Meurers B. 2006: A new physical foundation for anomalous gravity. *Stud. Geophys. Geod.* 50, 189–216.

Vajda P. & Pánisová J. 2005: Practical comparison of formulae for computing normal gravity at the observation point with emphasis on the territory of Slovakia. *Contr. Geophys. Geod.* 35, 2, 173–188.

Vaníček P. & Krakiwsky E.J. 1986: Geodesy: The Concepts, 2nd rev. edition, North-Holland P.C. *Elsevier Science Publishers,* Amsterdam, 1–697.

Vaníček P., Huang J., Novák P., Pagiatakis S., Véronneau M., Martinec Z. & Featherstone W.E. 1999: Determination of the boundary values for the Stokes-Helmert problem. *J. Geod.* 73, 180–192.

Vaníček P., Tenzer R., Sjöberg L.E., Martinec Z. & Featherstone W.E. 2004: New views of the spherical Bouguer gravity anomaly. *Geoph. J. Int.* 159, 2, 460–472.

Vogel A. 1982: Synthesis instead of reductions — new approaches to gravity interpretations. Earth evolution sciences. 2 Vieweg, Braunschweig, 117–120.