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ON AN ESTIMATION OF INTERACTION FLOWS FROM TRANSPORTATION COSTS

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The spatial interaction model resulting from maximizing entropy subject to only marginal (row and column) flows is extremely simple since it does not contain any parameters requiring estimation. The deficiency of this model is, however, that it does not contain any variable representing the spatial separation of the origin zone *i* from the destination zone *j*. Such a variable is considered essential in spatial interaction modelling. The purpose of this paper is to design a model exhibiting the simplicity of this model and containing at the same time such a variable in the form of transportation cost, c_{ij}. The basic idea to achieve this lies in replacing T_{ij} by the product $T_{ij}\,c_{ij} = Z_{ij}$ and dealing with this new variable, Z_{ij}, as with T_{ij}; so the corresponding entropy objective function is defined and then maximized subject to only marginal (row and column) transportation costs. The resulting model exhibits the simplicity of the model mentioned above, but it estimates Z_{ij}, not T_{ij}. Therefore, in order to estimate T_{ij}, it is necessary to assume additionally that c_{ij} is known; then $T_{ij} = Z_{ij}/c_{ij}$. If this operation is admissible, and the author finds it crucial for the model, then the final form of the model designed bears the structure of a spatial interaction model of gravity type. The purpose of this model is only to give first, preliminary estimates of interaction flows when one wants to avoid the estimation of model parameters, a procedure which sometimes may not be computationally simple.

Key words: spatial interaction modelling, entropy maximization

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INTRODUCTION

The size of flows between zones or nodes is usually estimated on the basis of spatial interaction models (SIMs). One of the most preferred classes among them is probably the class of entropy maximizing models. The objective function used for their derivation is usually that given by Wilson (1967 and 1970), namely

$$W = \frac{T\,!}{\prod_i \prod_j T_{ij}\,!} \tag{1}$$

where T_{ij} is the size of interaction flow between the origin zone (node) i and the destination zone (node) j, T an overall interaction flow, that is

$$T = \sum_i \sum_j T_{ij}$$

and W the number of microstates.

If (1) is maximized subject to

$$\sum_j T_{ij} = O_i \qquad (i=1, 2, ..., m) \tag{2}$$

$$\sum_i T_{ij} = D_j \qquad (j=1, 2, ..., n) \tag{3}$$

$$\sum_i \sum_j T_{ij} c_{ij} = C \tag{4}$$

(where O_i is the size of flows generated by the zone (node) i, D_j the size of flows absorbed by the zone (node) j, c_{ij} transportation (movement) cost from i to j, and C an overall transportation cost) one obtains

$$T_{ij} = \exp(-\lambda_i)\exp(-\gamma_j)\exp(-\beta c_{ij})$$

which after the evaluation of terms containing the zonal (nodal) Lagrange multipliers λ_i and γ_j from (2) and (3) yields finally

$$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}) \tag{5}$$

$$A_i = \left[\sum_j B_j D_j \exp\left(-\beta c_{ij}\right) \right]^{-1} \tag{6}$$

$$B_j = \left[\sum_i A_i O_i \exp\left(-\beta c_{ij}\right) \right]^{-1} \tag{7}$$

Relations (5)-(7) represent the familiar doubly (production-attraction) constrained SIM of gravity type since its structure corresponds to the gravity model structure given either by Wilson (1974, p. 67) as

size of interaction = coefficient(s) × mass × mass × distance function (8)

or by Sen and Sööt (1981, p. 165) as

size of flow = origin factor × destination factor × separation factor. (9)

If constraint (4) is replaced by

$$\sum_i \sum_j T_{ij} \ln c_{ij} = C' \tag{10}$$

one obtains the doubly constrained SIM with power distance function, i. e.

$$T_{ij} = A_i B_j O_i D_j c_{ij}^{-\beta} \tag{11}$$

$$A_i = \left[\sum_j B_j D_j c_{ij}^{-\beta} \right]^{-1} \tag{12}$$

$$B_j = \left[\sum_i A_i O_i c_{ij}^{-\beta} \right]^{-1} \tag{13}$$

Note that while O_i, D_j and c_{ij} are variables which are given, A_i, B_j and β are parameters that have to be estimated.

By reducing the set of constraints (2)-(4) or replacing them by others it is possible to derive a whole set of entropy maximizing SIMs (e.g. Wilson 1971). The prime objective of this paper is to enlarge that set, namely by deriving an entropy maximizing SIM enabling the estimation of interaction flows from purely transportation costs and being at the same time extremely simple in terms that it does not contain any parameters requiring estimation.

DERIVING A MODEL

Derivation on the basis of Boltzmann's entropy

Instead of going "in medias res" we start, to create a framework, by deriving an entropy maximizing (SIM) subject to only (2) and (3). By applying this procedure one obtains

$$T_{ij} = \exp(-\lambda_i) \exp(-\gamma_j) \tag{14}$$

If $\exp(-\lambda_i) = A_i$ and $\exp(-\gamma_j) = B_j$ then

$$T_{ij} = A_i B_j \tag{15}$$

Evaluation of A_i and B_j yields

$$\sum_j T_{ij} = A_i \sum_j B_j = O_i \Rightarrow A_i = \frac{O_i}{\sum_j B_j} \tag{16}$$

$$\sum_i T_{ij} = B_j \sum_i A_i = D_j \Rightarrow B_j = \frac{D_j}{\sum_i A_i} \tag{17}$$

Note, since according to (15), that

$$\sum_i \sum_j T_{ij} = \sum_i \sum_j A_i B_j = T \tag{18}$$

It follows now from (16), (17) and (18) that

$$T_{ij} = \frac{O_i}{\sum_j B_j} \frac{D_j}{\sum_i A_i} = \frac{O_i D_j}{\sum_i \sum_j A_i B_j} = \frac{O_i D_j}{T} = \frac{1}{T} O_i D_j = K O_i D_j$$

where

$$K = \frac{1}{T} \tag{19}$$

Model (19) is, as can be seen, extremely simple; it does not contain any parameters requiring estimation and all variables are clearly defined. However, it does contain a serious deficiency, namely that it does not contain any variable representing the spatial separation *i* from *j* (e.g. in the form of transportation cost or physical distance), being considered essential in spatial interaction modelling. This situation initiates, or even provokes, a question whether it would not be possible to eliminate somehow this deficiency and to keep the simplicity of the model at the same time. Let us try to do it!

Try first to maximize (1), or, better, its natural logarithm, subject to the following constraints

$$\sum_j T_{ij} c_{ij} = X_i \qquad (i = 1, 2, ..., m) \tag{20}$$

$$\sum_i T_{ij} c_{ij} = Y_j \qquad (j = 1, 2, ..., n) \tag{21}$$

The corresponding Lagrangian, as the first step in this procedure, is

$$L = \ln W + \sum_i \lambda_i (X_i - \sum_j T_{ij} c_{ij}) + \sum_j \gamma_j (Y_j - \sum_i T_{ij} c_{ij})$$

Differentiation of L with respect to T_{ij} yields

$$\frac{\partial L}{\partial T_{ij}} = -\ln T_{ij} - \lambda_i c_{ij} - \gamma_j c_{ij}$$

from which, if $\frac{\partial L}{\partial T_{ij}} = 0$

it follows that

$$T_{ij} = \exp(-\lambda_i c_{ij}) \exp(-\gamma_j c_{ij}) \tag{22}$$

To arrive at the final form of the model it is necessary to evaluate terms constraining the zonal Lagrange multipliers λ_i and γ_j from constraints (20) and (21). Inspection of (22) shows, however, that it is not possible; objective function (1) is simply not compatible with constraints (11) and (12) in this respect.

The solution to this problem is to reformulate objective function (replacing T and T_{ij} by other variables) and relabelling $T_{ij} c_{ij}$ as Z_{ij}, that is $T_{ij} c_{ij} = Z_{ij}$. Then the constraints can be expressed as follows

$$\sum_j Z_{ij} = X_i \qquad (i = 1, 2, ..., m) \tag{23}$$

$$\sum_i Z_{ij} = Y_j \qquad (j = 1, 2, ..., n) \tag{24}$$

The corresponding objective function compatible with (23) and (24) is now

$$W = \frac{Z!}{\prod_i \prod_j Z_{ij}!} \tag{25}$$

where

$$\sum_i \sum_j Z_{ij} = C = Z$$

Maximizing natural logarithm of (25) (assuming that Z_{ij} can be expressed as a natural number) subject to (23) and (24) the corresponding Lagrangian will be

$$L = \ln W + \sum_i \lambda_i (X_i - \sum_j Z_{ij}) + \sum_j \gamma_j (Y_j - \sum_i Z_{ij})$$

from which, after differentiation with respect to Z_{ij}, one obtains

$$\frac{\partial L}{\partial Z_{ij}} = -\ln Z_{ij} - \lambda_i - \gamma_j$$

If $\frac{\partial L}{\partial Z_{ij}} = 0$, then

$$Z_{ij} = \exp\left(-\lambda_i\right) \exp\left(-\gamma_j\right) \tag{26}$$

After relabelling exp(-λ_i) as A_i and exp(-γ_j) as B_j one can write

$$Z_{ij} = A_i B_j \tag{27}$$

Evaluation of A_i and B_j yields

$$\sum_j Z_{ij} = A_i \sum_j B_j = X_i \Rightarrow A_i = \frac{X_i}{\sum_j B_j} \tag{28}$$

$$\sum_i Z_{ij} = B_j \sum_i A_i = Y_j \Rightarrow B_j = \frac{Y_j}{\sum_i A_i} \tag{29}$$

It follows from (28) and (29), since

$$\sum_i \sum_j Z_{ij} = \sum_i \sum_j A_i B_j = Z$$

that, according to (27),

$$Z_{ij} = \frac{X_i}{\sum_j B_j} \frac{Y_j}{\sum_i A_i} = \frac{X_i Y_j}{\sum_i \sum_j A_i B_j} = \frac{X_i Y_j}{Z} = \frac{1}{Z} X_i Y_j = K X_i Y_j$$

where

$$K = \frac{1}{Z} \tag{30}$$

Relation (30) is as simple as relation (19), but it does estimate Z_{ij}, not T_{ij}. Remember, however, that $Z_{ij} = T_{ij}\, c_{ij}$. If one now assumes that c_{ij} is known, though Z_{ij} as a whole is an endogenous variable, then

$$T_{ij} = \frac{Z_{ij}}{c_{ij}} = K \frac{X_i Y_j}{c_{ij}} \tag{31}$$

where

$$K = \frac{1}{Z}$$

Note that relation (31) is an interaction model of gravity type since its structure fits the gravity model structure given either by (8) or (9). The model does not contain any parameters requiring estimation and T_{ij} is estimated only on the basis of transportation costs as declared in the title of the paper. Note also and once more, and this has to be emphasized, that T_{ij} has not been estimated directly as an output of the derivation (this relates only to Z_{ij}); T_{ij} has been estimated assuming additionally that c_{ij} is known.

Derivation on the basis of Shannon's entropy

Note that objective function (1) has a combinatorial nature. However, by replacing T by Z and T_{ij} by Z_{ij} one could argue against using (25) as an objective function since Z and Z_{ij} may no longer be natural numbers. Taking this objection into account we now present a derivation of the model on the basis of Shannon's entropy as an objective function which is free of any combinatorial associations.

Let us introduce the following variables:

$$P_{ij} = \frac{Z_{ij}}{Z} \quad Q_i = \frac{X_i}{Z} \quad R_j = \frac{Y_j}{Z}$$

where P_{ij}, O_i and R_j are interpreted as probabilities. The constraints, corresponding to (23) and (24), are now as follows

$$\sum_j P_{ij} = Q_i \qquad (i = 1, 2, ..., m) \tag{32}$$

$$\sum_i P_{ij} = R_j \qquad (j = 1, 2, ..., n) \tag{33}$$

Maximizing Shannon's entropy as an objective function, that is

$$H = -\sum_i \sum_j P_{ij} \ln P_{ij} \tag{34}$$

subject to (32) and (33) leads to the following Langrangian

$$L = H + \sum_i \lambda_i (Q_i - \sum_j P_{ij}) + \sum_j \gamma_j (R_j - \sum_i P_{ij})$$

from which, after differentiation with respect to P_{ij}, one obtains

$$\frac{\partial L}{\partial P_{ij}} = -\ln P_{ij} - \lambda_i - \gamma_j$$

If $\frac{\partial L}{\partial P_{ij}} = 0$, then

$$P_{ij} = \exp(-\lambda_i)\exp(-\gamma_j) \tag{35}$$

Relabelling $\exp(-\lambda_i)$ as A_i and $\exp(-\gamma_j)$ as B_j means that P_{ij} can be rewritten as

$$P_{ij} = A_i B_j \tag{36}$$

Evaluation of A_i and B_j yields

$$\sum_j P_{ij} = A_i \sum_j B_j = Q_i \Rightarrow A_i = \frac{Q_i}{\sum_j B_j} \tag{37}$$

$$\sum_i P_{ij} = B_j \sum_i A_i = R_j \Rightarrow B_j = \frac{R_j}{\sum_i A_i} \tag{38}$$

Since

$$\sum_i \sum_j P_{ij} = \sum_i \sum_j A_i B_j = 1$$

one can write

$$P_{ij} = \frac{Q_i}{\sum_j B_j} \frac{R_j}{\sum_i A_i} = \frac{Q_i R_j}{\sum_i \sum_j A_i B_j} = \frac{Q_i R_j}{1} = Q_i R_j \qquad (39)$$

Relation (39) presents an important feature of maximizing entropy procedure, namely independence of events.

The question is now whether maximization on the basis of Boltzmann's entropy leads to the identical model as maximization on the basis of Shannon's entropy. Therefore, let us rewrite the probabilities in relation (39). This yields

$$\frac{Z_{ij}}{Z} = \frac{X_i}{Z} \frac{Y_j}{Z} = \frac{X_i Y_j}{Z^2} \Rightarrow Z_{ij} = \frac{X_i Y_j}{Z} = \frac{1}{Z} X_i Y_j = K X_i Y_j \qquad (40)$$

where

$$K = \frac{1}{Z}$$

It is clear from relation (40) that the resulting models are really identical.

ADDITIONAL CONSIDERATION

So far c_{ij}, as a measure of separation of i from j, has been considered in its simplest form. It is however, possible to consider also its functional modifications, e.g. $\ln c_{ij}$, $c_{ij}^{1/n}$ and, of course, others. Take, for instance, $\ln c_{ij}$. Then Z_{ij} as $T_{ij} c_{ij}$ must, of course, be replaced by another variable, e.g. $S_{ij} = T_{ij} \ln c_{ij}$. The corresponding constraints will now be

$$\sum_j S_{ij} = U_i \quad (i = 1, 2, ..., m) \qquad (41)$$

$$\sum_i S_{ij} = V_j \quad (j = 1, 2, ..., n) \qquad (42)$$

and the corresponding objective function

$$W = \frac{S!}{\prod \prod S_{ij}!} \qquad (43)$$

where

$$\sum_i \sum_j S_{ij} = S$$

It is now possible to write down directly the final form of the model resulting from maximizing (43) subject to (41) and (42) since its derivation is identical to that presented above. This final form will be

$$S_{ij} = KU_iV_j \tag{44}$$

where

$$K = \frac{1}{S}$$

and since, $S_{ij} = T_{ij}\ln c_{ij}$, assuming again that c_{ij} is known (and therefore also $\ln c_{ij}$), will be

$$T_{ij} = \frac{S_{ij}}{\ln c_{ij}} = K\frac{U_iV_j}{\ln c_{ij}} \tag{45}$$

where

$$K = \frac{1}{S}$$

An analogical procedure could, of course, be applied if some other modifications of c_{ij} are used.

Note additionally that from relation, $Z_{ij} = T_{ij}c_{ij}$, or $S_{ij} = T_{ij}\ln c_{ij}$, also c_{ij} can be estimated if, of course, T_{ij} is assumed to be known. If $Z_{ij} = T_{ij}c_{ij}$ is considered, then c_{ij} will be

$$c_{ij} = \frac{Z_{ij}}{T_{ij}} = K\frac{X_iY_j}{T_{ij}} \tag{46}$$

where

$$K = \frac{1}{Z}$$

If $S_{ij} = T_{ij}\ln c_{ij}$ is considered then $\ln c_{ij}$ will be

$$\ln c_{ij} = \frac{S_{ij}}{T_{ij}} = K\frac{U_iV_j}{T_{ij}} \tag{47}$$

where

$$K = \frac{1}{S}$$

from which follows that c_{ij} will be

$$c_{ij} = \exp\left(K \frac{U_i V_j}{T_{ij}} \right) \tag{48}$$

Such a situation that c_{ij} instead of T_{ij} is to be estimated, can, though rarely, happen.

CONCLUSION

In this paper an attempt has been made to design an entropy maximizing SIM of gravity type exhibiting the simplicity of (19) and containing, at the same time, a variable representing the separation of i from j. This model estimates T_{ij} only from transportation costs. The open question in this derivation remains, however, whether c_{ij} in $Z_{ij} = T_{ij} c_{ij}$ can be considered as known since Z_{ij} as a whole is an endogenous variable in the model, or, in other words, whether the operation

$$T_{ij} = \frac{Z_{ij}}{c_{ij}}$$

is admissible under such a condition.

Note that the model presented above resembles Tobler's additive SIM (Dorigo and Tobler 1983, Tobler 1983 and 1985) having the form

$$M_{ij} = \frac{R_i + E_j}{d_{ij}}, \quad i \neq j \tag{49}$$

where M_{ij} corresponds to T_{ij}, R_i and E_j are the push and pull factors respectively and d_{ij} is a distance between i and j as a measure of separation of i from j. Though this model does not contain any distance function parameter requiring estimation it is worth noticing that R_i and E_j are not simple exogenous variables since they have to be computed before, namely from the production and the attraction constraints, that is from

$$\sum_j M_{ij} = O_i$$

and

$$\sum_i M_{ij} = D_j$$

R_i and E_j exhibit thus analogical features as balancing factors, A_i and B_j, in doubly constrained entropy maximizing SIM (5)-(7). It means that the model presented in this paper is even simpler than Tobler's one. However it is primarily intended to give only the first, preliminary and rough estimate of interaction flows when one wants to avoid the estimation of model parameters, a procedure which sometimes may not be computationally simple.

APPENDIX

In order to asses how the model designed fits the observed data a test has been performed. To have a comparison a whole set of SIMs has been tested, viz. (5)-(7), (11)-(13), (19), (31) and (45).

All data used refer to migration flows between highest order administrative units of former Czecho-Slovakia ("kraje") in 1982. The spatial separation between them is given in physical distances (kilometres), not in transportation costs.

As goodness-of-fit statistics a coefficient of correlation (*R*) and standardized root mean square error (*SRMSE*)[1)] were used (Knudsen and Fotheringham 1986). As expected, a clearly better fit can be observed in the case of models (5)-(7) and (11)-(13). Comparison of (19), (31) and (45) shows, and the author finds it decisive, better fit in the case of models (31) and (45), that is in the case of models designed by the author, though generally the fit in the case of models (19), (31) and (45) is comparatively low (Tab. 1).

Tab. 1. Results of a test: goodness-of-fit statictics

Model	R	SRMSE
$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij})$	0.7494	1,083
$T_{ij} = A_i B_j O_i D_j c_{ij}^{-\beta}$	0.7832	1,033
$T_{ij} = \frac{O_i D_j}{T}$	0.1777	1,621
$T_{ij} = \frac{Z_{ij}}{c_{ij}} = \frac{1}{Z}\frac{X_i Y_j}{c_{ij}}$	0.6720	1,306
$T_{ij} = \frac{S_{ij}}{\ln c_{ij}} = \frac{1}{S}\frac{U_i V_j}{\ln c_{ij}}$	0.3416	1,545

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Ján P a u l o v

O ODHADE INTERAKČNÝCH TOKOV Z DOPRAVNÝCH NÁKLADOV

Veľkosť tokov (napr. osôb, tovarov a pod.) medzi zónami (oblasťami) sa spravidla odhaduje prostredníctvom interakčných modelov. Jednou z najpreferovanejších tried takýchto modelov je trieda entropiu maximalizujúcich modelov. Ich charakter závisí od použitej cieľovej funkcie a od druhu a počtu ohraničení, ktoré sa na ňu „naložia" pri ich odvodení. Interakčný, entropiu maximalizujúci model, ktorý rezultuje z maximalizácie cieľovej funkcie

$$W = \frac{T!}{\prod_i \prod_j T_{ij}!}$$

iba pri „naložení" dvoch ohraničení, a to

$$\sum_j T_{ij} = O_i \text{ a } \sum_i T_{ij} = D_j$$

[kde T_{ij} je veľkosť interakčného toku medzi východiskovou zónou (oblasťou) i a cieľovou zónou (oblasťou) j,

$$T = \sum_i \sum_j T_{ij}$$

pričom O_i je veľkosť tokov generovaných východiskovou zónou (oblasťou) i a D_j, veľkosť tokov absorbovaných cieľovou zónou (oblasťou) j] je extrémne jednoduchý, pretože neobsahuje žiadne parametre vyžadujúce odhad a exogénne premenné sú jasne definované. Jeho tvar je

$$T_{ij} = \frac{O_i D_j}{T}$$

Tento model však trpí jedným vážnym nedostatkom, a to tým, že neobsahuje žiadnu premennú, prostredníctvom ktorej sa vyjadruje separácia medzi i a j (napríklad v podobe vzdialenosti, dopravných nákladov a pod.), ktorá sa v interakčnom modelovaní považuje za podstatnú. Takáto situácia iniciuje hľadať model, ktorý by vykazoval jednoduchosť vyššie spomenutého modelu a zároveň by obsahoval takúto premennú. Riešenie problému vidí autor v zavedení novej premennej, a to $Z_{ij} = T_{ij} c_{ij}$, kde c_{ij} sú dopravné náklady z i do j, v naformulovanom ohraničení

$$\sum_j Z_{ij} = X_i \quad \text{a} \quad \sum_i Z_{ij} = Y_j$$

a v naformulovaní cieľovej funkcie

$$W = \frac{Z!}{\prod_i \prod_j Z_{ij}!} \text{, kde} \quad Z = \sum_i \sum_j Z_{ij}$$

Ak je táto cieľová funkcia maximalizovaná vzhľadom na uvedené dve ohraničenia, potom výsledný model nadobudne tvar

$$Z_{ij} = \frac{X_i Y_j}{Z}$$

Tento model však odhaduje Z_{ij}, nie T_{ij}; odhad T_{ij} je konečným cieľom. Keďže však $Z_{ij} = T_{ij} c_{ij}$, potom, ak predpokladáme, že c_{ij} je známe, hoci Z_{ij} ako celok je endogénnou premennou, pre T_{ij} bude platiť

$$T_{ij} = \frac{Z_{ij}}{c_{ij}}$$

Po dosadení dostaneme

$$T_{ij} = \frac{X_i Y_j}{Z} \Big/ c_{ij} = \frac{1}{Z} \frac{X_i Y_j}{c_{ij}} = K \frac{X_i Y_j}{c_{ij}} \text{, kde} \quad K = \frac{1}{Z}$$

Tento model spĺňa vyššie vyslovenú požiadavku: je extrémne jednoduchý, t. j. neobsahuje žiadne parametre vyžadujúce odhad, exogénne premenné sú jasne definované, pričom model zároveň obsahuje premennú c_{ij}, prostredníctvom ktorej sa vyjadruje separácia medzi i a j. Ide o model gravitačného typu, pretože vykazuje štruktúru gravitačného modelu. Jediná otvorená otázka, ktorú autor vidí v odvodení tohto modelu, je, či operácia

$$Z_{ij} = T_{ij} c_{ij} \Rightarrow T_{ij} = \frac{Z_{ij}}{c_{ij}}$$

je prípustná vzhľadom na to, že Z_{ij} ako celok je endogénna premenná. V ďalšej časti príspevku sa sledujú ďalšie vlastnosti tohto modelu, pričom v dodatku sú obsiahnuté výsledky skúmania, ako model reprodukuje pozorované dáta, t. j. reálne toky T_{ij}.