Household Production with Inventories

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Abstract

An innovation in this paper is the introduction of inventories into the market sector of the Real Business Cycle model with household production. My main finding is that the controversy between procyclicity of hours spent to produce consumption good versus correlation between business and household investment, which has been prevailing in all business cycles models so far, is resolved.

Keywords: *home production, inventory, business cycle, investment*

JEL: C61, E22, E32

Introduction

The household sector is large.[1] According to Greenwood and Hercowitz (1991), household capital (defined as purchases of consumer durables and residential structures) exceeds market capital. So the inclusion of household sector into real business cycle models has been very important step in understanding many macroeconomic phenomena.

When comparing to real US data, statistical properties of home production models has improved in many directions. Just to name few (see Benhabib et al., 1991):

- increased volatility of output
- increased fluctuations of labour hours and consumption relative to output
- decreased fluctuations
- decreased correlation between productivity and output.

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Acknowledgement: I would like to thank professor Kydland for valuable comments.

[1] Studies show that a typical married couple spends 25 % of their discretionary time on unpaid work in the home, such as cooking, cleaning and childcare, compared to 33 % on work for pay in the market (Juster and Stafford, 1991).

However, there are still anomalies yet to be explained. One of them is the controversy between correlation of output and labour hours spent on producing consumption good (let's use acronym CorYHc) and correlation of market and home investment (CorImIn). Their values in US data are 0.76 and 0.41, respectively. This means that hours spent on producing consumption good are largely procyclical and market and home investments are fairly positively correlated.

In traditional models without home production, CorYHc is always large negative value while CorImIn is not defined in this one-sector environment.

One contribution of Benhabib-Rogerson-Wright model with home production (Benhabib, Rogerson and Wright, 1991) is that they managed to turn this large negative value to a small positive value 0.10. But CorImIn in their model is large negative value –0.75.

Gomme, Kydland and Rupert has tackled the anomaly of negative correlation of market and home investment. Their value is 0.41, which is exactly the value observed in US data. But their CorYHc is large negative number (–0.95).
The problem is that these two values tend to go in opposite directions.

The aim of this work is to try to reconcile these two facts. I have tackled it by introducing inventories in the Gomme-Kydland-Rupert model in the same fashion as it was done in (Kydland and Prescott, 1982), where inventories were implemented into the standard growth model.

The rest of work is organized as follows: In section (2) I describe the economic environment and specify the model. In section (3) calibration is discussed. In section (4) I simulate numerical solution of the model and compare its properties with other models and with the actual data. In section (5) I summarize and make conclusions.

1. The Economic Environment

1.1 . Household

The representative agent maximizes its expected lifetime utility as given by

$$E_0 \sum_{t=0}^{\infty} \beta U(c_{Mt}, c_{Nt}, h_{Mt}, h_{Nt}),\ 0 < \beta < 1 \tag{1.1.1}$$

where

E_0 – an expectation operator at time *0*,
β – the time discount factor,
c_{Mt} – market consumption,
c_{Nt} – non-market (home) production,
h_{mt} – market hours,
h_{Nt} – non-market hours.

The momentary utility function has the following form:

$$U(c_{Mt}, c_{Nt}, h_{Mt}, h_{Nt}) = \omega \log C(c_{Mt}, c_{Nt}) + (1-\omega)\log(1 - h_{Mt} - h_{Nt}) \quad (1.1.2)$$

where the consumption aggregator is

$$C(c_{Mt}, c_{Nt}) = c_{Mt}^{\psi} \,.\, c_{Nt}^{1-\psi} \quad (1.1.3)$$

Combining (2) and (3), the utility has the following form

$$U(c_{Mt}, c_{Nt}, h_{Mt}, h_{Nt}) = \omega\psi \log C(c_{Mt}) + \omega(1-\psi)\log(c_{Nt}) + (1-\omega)\log(1 - h_{Mt} - h_{Nt}) \quad (1.1.4)$$

The agent has one unit of labour time to divide between leisure, market labour and household labour.

Households face the following budget constraint:

$$c_{Mt} + i_{Mt} + i_{Nt} + i_{Vt} = (1-\tau_K)r_t k_{Mt} + (1-\tau_H)w_t h_{Mt} + q_t v_t + \delta_M \tau_K k_{Mt} + \tau_t \quad (1.1.5)$$

where

k_{Mt} – household's stock of market capital,

r_t – rental price of capital,

w_t – real wage,

i_{Mt} – market investment,

i_{Nt} – household investment,

i_V – inventory investment,

v_t – stock of inventories,

q_t – rental price of inventories.

Capital income is taxed at the rate τ_K, labour income at the rate τ_H.

The term $\delta_M \tau_K k_{Mt}$ represents the depreciation allowance, τ_t is lump-sum transfer to household from the government.

Time to build technology, as in Kydland and Prescott (1982) is used. This means that it takes J periods until the capital investment i_M becomes productive capital. Let s_{jt} be resources which are j periods from completion at period t and φ_j be the fraction of the resources allocated to the investment project in the jth stage from the last. Then total market investment is given by

$$i_{Mt} = \sum_{j=1}^{J} \varphi_j \,.\, s_{jt} \quad (1.1.6)$$

where s_{jt} evolve according to

$$s_{j-1,t+1} = s_{jt} \; j = 2,\ldots J \quad (1.1.7)$$

Then the laws of motion for market capital and household capital respectively are

$$k_{M,t+1} = (1-\delta_M)k_{Mt} + s_{1t} \tag{1.1.8}$$

$$k_{N,t+1} = (1-\delta_N)k_{Nt} + i_{Nt} \tag{1.1.9}$$

where δ_M and δ_N are depreciation rates of market and household capital, respectively.

The law of motion for inventories is

$$v_{t+1} = v_t + i_{Vt} \tag{1.1.10}$$

Home production is described by

$$c_{Nt} = e^{z_N} k_{Nt}^{\eta} h_{Nt}^{1-\eta} \tag{1.1.11}$$

where home productivity shocks evolve as

$$z_{N,t+1} = \rho \,.\, z_{Nt} + \varepsilon_{N,t+1} \tag{1.1.12}$$

1.2. Firms

Market firms behave competitively and seek to maximize their profit

$$\Pi_t = F(K_{Mt}, H_{Mt}, V_t, z_{Mt}) - r_t K_t - w_t H_{Mt} - q_t V_t \tag{1.2.1}$$

where

$$F(K_M, H_M, V, z_M) = e^{z_M} [(1-\sigma)K_M^{-\nu} + \sigma \,.\, V^{-\nu}]^{-\frac{\theta}{\nu}} H_M^{1-\theta} \tag{1.2.2}$$

Market productivity shocks are described in the next section.

1.3. Government

In this model the government raises revenue through capital and income taxes and rebate the household.

$$\tau_t = \tau_K r_t K_{Mt} + \tau_H w_t H_{Mt} - \delta_M \tau_K K_{Mt} \tag{1.3.1}$$

1.4. Market shocks

Productivity shocks in the market sector have more complicated structure than those in the home sector. I assume (following Kydland and Prescott, 1982) that the technology parameter is subject to a stochastic process with components of different persistence. The productivity parameter is not observed but consumer observes an indicator of this parameter at the beginning of the period. On the basis of the indicator, decision o f how much new investment projects to initiate and of how much of the time is allocated to market and home production are made. Subsequently to observing aggregate output, the consumption level is chosen and decision of inventory investment is taken.

Specifically, the technology shocks, z_{Mt} is the sum of a permanent component z_{M1t} and a transitory component z_{M2t}:

$$z_{Mt} = z_{M1t} + z_{M2t} \tag{1.4.1}$$

The permanent component is highly persistent so

$$z_{M1,t+1} = \rho z_{M1t} + \zeta_{1t} \tag{1.4.2}$$

where

ρ – less than but close to 1,

ζ_{1t} – a permanent shock.

The transitory component equals the transitory shock

$$z_{M2,t+1} = \zeta_{2t} \tag{1.4.3}$$

The indicator of productivity χ_t is the sum of actual productivity z_{Mt} and a third shock ζ_{3t}

$$\chi_t = z_{Mt} + \zeta_{3t} = z_{M1t} + z_{M2t} + \zeta_{3t} \tag{1.4.4}$$

1.5. Equilibrium

As there are externalities in the model – distortionary taxes, it is not possible to find equilibrium allocations by solving a planning problem. Instead it is necessary to solve for equilibrium allocations directly by solving a fixed-point problem.

The maximization problem (1) with laws of motion (2.1.7) – (2.1.10) and budget constraints (2.1.5), (2.3.1) can be expressed as a dynamic programming problem of the following form:

$$v(z, S, s) = \max_{d}\{U(z, S.s, D, d) + \beta . E[v(z', S', s')/z]\} \tag{2.5.1}$$

where $v(z, S, s)$ is a value function, z, S, D is defined by

$$z = (z_M, z_N),\ S = (K_M, K_N, S_1, S_2, S_3),\ D = (S_4, I_N, I_V, H_M, H_N)$$

where S, D are per capita aggregate state and decision variables, respectively, and s, d, are their house-specific counterparts. (z', S', s') evolves according to the laws of motion (2.1.7) – (2.1.10) and prime indicates future values of the state vector. Relationships (2.1.5), (2.1.11) and (2.3.1) were used for reducing number of decision variables.

This standard optimization problem has to be slightly modified to accommodate the information structure defined in section 2.4.

Let $D = (D_1, D_2)$ where $D_1 = (S_4, I_N, I_V, H_M)$, $D_2 = I_V$ and d is partitioned similarly.

The first set of decisions d_{1t}, D_{1t} is contingent upon $(z_{M1t}, z_{Nt}), s_t, S_t, I_{Vt}, I_{Vt}$.

The second set of decisions d_{2t}, D_{2t} is contingent upon $(z_{M2t}, 0), s_t, S_t, d_{1t}, D_{1t}$.

In order to calculate decision rules, steady state is found and utility function U is approximated by a quadratic function around it. This allows us to drop the expectation operator E in (2.5.1) and solve a certainty case instead. For computing the value function v, the method of successive approximations, which is fully described in Hansen and Prescott (1995), is used.

2. Calibration

As I did not deal with real data in this work, I took parameter values used in Gomme-Kydland and Rupert (1999) and in Kydland and Prescott (1995). These parameters are calibrated so as to match certain long run averages observed in postwar US economy. I just list these values.

Preferences		
β	0.9855	Discount factor
ω	0.6755	Consumption-leisure weight
ψ	0.5583	Market-home consumption weight
Household Production		
η	0.3526	Capital-labour weight
δ_N	0.027	Depreciation rate
Market Production		
J	4	Number of project periods
φ	0.25	Fraction of resources used at stage j
α	0.3267	Capital share
δ_M	0.0295	Depreciation rate
σ	0.0000028	Market capital-inventory share
ν	4.0	$1/(1+\nu)$ is capital-inventory elasticity of substitution
Government		
τ_H	0.25	Tax rate on labour income
τ_K	0.70	Tax rate on capital income
Shocks		
ρ_M	0.95	Market shock autocorrelation
ρ_N	0.95	Home shock autocorrelation
$\sigma_{\zeta 1}, \sigma_{\zeta 3}$	0.009	Standard deviation of ζ_1, ζ_3 market shocks
$\sigma_{\zeta 2}$	0.0018	Standard deviation of ζ_2 market shock
σ_N	0.00763	Standard deviation of home shock
σ_{MN}	0.6667	Correlation of z_{M1} and z_N

3. Results

In this section I compare statistics from simulations of the model with real data and with data of other models. All series are filtered using the Hoderick-Prescott filter and statistics listed in table 1 are averages over 50 simulations.

I have focused on two statistics in this work. Correlation between output and hours spent on producing consumption good in market sector (CorYHc) and correlation between market and home investment (CorImIn) and their mutual interaction. Just recall that in US data these values are – CorYHc = 0.76 and CorImIn = 0.41 (the first figure varies depending on the measurement) but it can be said that it is large positive number.

I compare the following models:

- Greenwood-Hercowitz model (GH)
- Benhabib-Rogerson-Wright model (BRW)
- Gomme-Kydland-Rupert model (GKR)
- Model 1 that is basically the GKR model enhanced by introduction of inventory stock in market sector
- Model 2 is identical to the Model 1 except from shock structure.

In the benchmark GH model with elasticity of substitution between home capital and home hours equal to 1 (Cob-Douglas production function), CorImIn is negative. The intuition behind it goes as follows (Greenwood and Hercowitz, 1991): if positive shock hits the economy, the optimal levels of business and household capital increase. Because capital goods are produced in business sector only, the induced scarcity of market goods reduces the shadow price of home goods. This mechanism implies a ten-dency for business capital to build first. This induces negative co movements of business and home investment. The authors argue that any specification with a Cob-Douglas home technology tends to make these two investments negatively correlated. This problem is overcome by increasing complementarity of time and capital in home sector by decreasing elasticity of substitution from value 1 (Cob-Douglas) to negative values. Hence they got desirable value for CorImIn. But the effect of decreasing elasticity causes CorYHc to become large negative.

BRW model focuses on procyclicity of macroeconomic variables with output and ignores their mutual correlation. To achieve positive correlation of Hc (among others), the Cob-Douglas production function is used. This inevitably drives (as reasoned above) CorImIn to negative values (in their model the CorImIn equals –0.75).

In GKP model, the effect of turning CorImIn from negative to positive values is achieved by introducing time-to-build technology in the market sector. The scarcity of market goods that decreases the shadow price of home produced good is mitigated by spreading market investment into J periods instead of 1. Their value of CorImIn actually coincides with real data value but this is achieved at the expense of CorYHc, which becomes large negative –0.95.

I have tried to resolve the problem between CorYHc and CorImIn by introducing inventory stock in market sector. The intuition behind it goes as follows: As a positive shock hits the economy, it raises the investment in market sector and hence increases hours spent on producing investment good. As the total amount of hours worked in market sector is fairly constant, hours spent on producing consumption good are reduced. If however, part of the market investment goes to inventory stock, then the mechanism described above can be altered. Why? Inventory investment is highly procyclical and a large part of it is consumption good. So if inventory investment is divided between consumption and investment goods, say in the same proportions as are their average fractions of GNP (about three quarters and one quarter, respectively) then it might turn the countercyclical pattern of hours to procyclical one.

When I considered the usual shock structure, that means shocks are observed at the beginning of each period, almost nothing happened to any statistics (see Table 1). The reason is that there are almost no incentives for household to invest in inventory stock, as the weight of it in the home production function is very small. The situation changes though, when shock timing is as following. At the beginning of each period agent observe only an indicator of technology shock but not their actual level. Decisions about fundamental variables – capital investment and hours worked are made and after this the true level of technology is observed. Then inventory decision is taken.

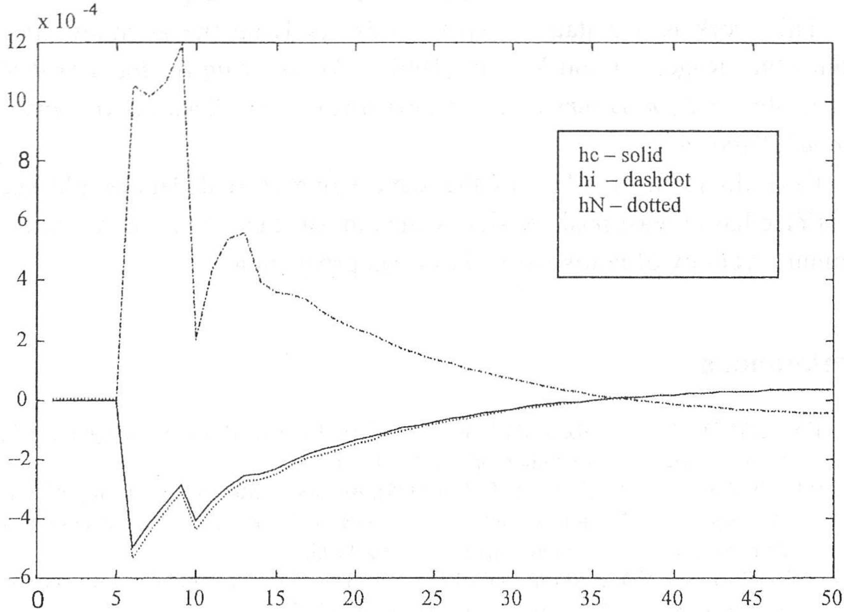
This scenario is quite a realistic one. At the beginning of a period when decisions have to be taken, agents just estimate the technology level for that period. Towards the end of the period as the technology level estimates are fine tuned, there is no chance to alter decisions already been taken. The only adjustment that can be made is in inventory stock.

When I applied this shock structure (model 2) CorImIn remained plausibly unchanged (as well as all other statistics), and CorYHc has changed from a large negative value to a small positive one.

Table 1

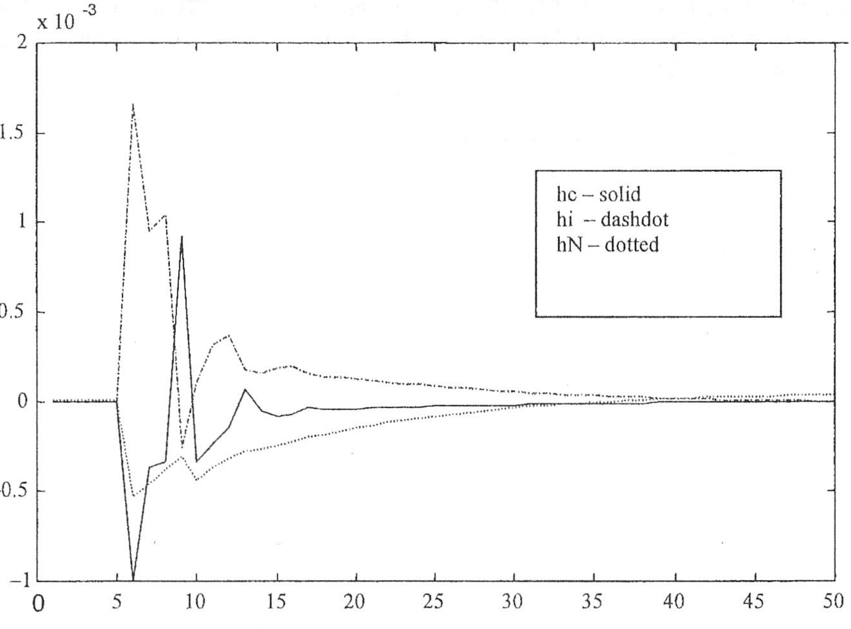
Cor(x ,GNP) x =	c_M	i_T	h_M	h_N	h_C	h_i	Cor(i_M , i_N)
Us Data	0.80	0.80	0.88		0.76		0.41
GH	0.92	0.99	0.95				0.37
BRW	0.69	0.94	0.94	–0.76	0.10		–0.75
GKP	0.97	0.99	–0.98	0.98	0.95	0.78	0.41
Model 1	0.96	0.99	0.98	0.98	–0.97	0.79	0.29
Model 2	0.61	0.80	0.76	–0.76	0.17	0.35	0.43

Figure 1
Positive Shock to Technology in the Model without Inventory Stock



In the Figure 1 h_i is highly procyclical while both h_c and h_N are highly countercyclical.

Figure 2
Positive Shock to Technology in the Model with Inventory Stock



In the Figure 2 the pattern is more complex but h_c and h_i tend to be procyclical and h_N remains highly countercyclical.

Conclusion

This work is a tentative step towards resolving the anomaly articulated in Benhabib, Rogerson and Wright (1991): *Constructing a Model that Simultaneously Account for Hours and the Two Investments Remains a Topic for Additional Research.*

CorImIn in my model has the same value as real data would suggest and CorYHc has at least positive sign, which means that I have managed to turn high countercyclicity of hours worked to weak procyclicity.

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