Regional Input-Output Model: Implications for the Slovak Republic

*Jaroslav HUSÁR**

Abstract

Facing new challenges of the Slovak economy with regard to European Union, lagging regions will need ideas from regional science. An input-output model is considered a good way for strategy making. In this paper we deal with the two-regional input-output model. We consider that systems of national accounts and input-output models are more than just the source of data. They are means of specifying the categories under which data are to be organized. For policy purposes we decided to use and modify a common input-output table for the two-regional model construction. There are two distinct regions In Slovakia.

Keywords: *two-regional model, input-output table, economic analysis, final demand, intermediate output, systematic analysis of the economic impacts*

JEL Classification: C67

Introduction

Economic policy and the systematic analysis of the economic impacts of final demand and of the investment projects in particular regions requires regional economic models. The input-output tables mainly applied so far are pertaining to national economy. Exports and imports are all lumped in one sector (one columns, one raw) regardless of their destinations and origins, and production sectors are distinguished only by their products regardless of their geographical locations. This is valid for the Slovak Republic too. There are two regions of Slovakia – Western Slovakia and Eastern Slovakia. It can be said that west part of Slovakia is developed and east part is underdeveloped. We consider that economic policymakers can use input-output model as strong instrument for predicting the economic development. It is known that a national input-output table (I/O) is much more useful in empirical application than the systems of national accounts

* prof. Ing. Mgr. Jaroslav HUSÁR, PhD., Ekonomická univerzita v Bratislave, Fakulta hospodárskej informatiky, Katedra operačného výskumu a ekonometrie, Dolnozemská cesta 1/b, 852 35 Bratislava 5; e-mail: Husar@euba.sk

by virtue of its finer breakdown of the economy by industries. Nevertheless, for certain problems, a further breakdown of industries by geographical region is also necessary. Facing new challenges of the Slovak economy with regard to the European Union, lagging regions will need ideas from regional science.

1. Input Output Framework of the Systematic Analysis

Systematic analysis of economic impacts must account for the interindustry relationships within regions because these relationships largely determine how regional economies are likely to respond to projects and programs changes. European system of accounts provides a data base for economic analysis and economic policy. As it is known, this system is compulsory. Analytical interests and policy problems vary from country to country. Having two distinct regions in Slovakia, what is needed is a systematic and coherent framework which can accommodate the main bodies of data, such as intermediate product flow, final demand, flow of funds and so on by regions, needed for a statistical description of the functioning of the Slovak economy. But there is no attempt made to find room for information which no-one collects but the policymaker needs them. We think that a strategy is required in order to extract the best possible quantitative description of the regional economy in Slovakia from the available data of input-output table. This table is constructed in Slovakia but not by regions. The input-output framework applied consists of symmetric input-output tables. This symmetric input-output table is an industry by industry matrix describing the domestic production processes and the transactions in products of the national economy in great detail. Up to these days the regional aspects of these processes are not involved. It requires an enormous work to be done. The filling in of regional input-output framework reveals endless statistical difficulties which come to light only gradually and cannot be resolved immediately. Most statistical information that can be obtained from producer units indicates what type of products they have sold and, usually less detailed information on regions. To find a way we decided to use regional input-output model based on the special type matrices. The latter table we use in calculating the cumulated coefficients, i.e. the Leontiev inverse. So the input-output framework is an excellent way of economic policy making by regional input-output model.

2. A Two-Regional Model

From the literature and practical applications we can learn that several multiregional I/O tables in which input and output are identified by region as well as by industry were constructed. We in Slovak Republic (SR) we do not have a real history in this field. For policymaking we consider the input-output table to be the proper tool. For this reason we shall focus our discussion on regional input-output model construction. The Statistical Office of the Slovak Republic is now in the problem formulation phase. For this reason we shall also focus our discussion on regional input-output analysis that may serve as the basis of our theoretical approach. The main problem is that we do not construct the regional input-output tables. But there are several alternatives for simplifying the data collection and statistical problems of regional input-output models (W. Isard, R. Miller and P. Drennan).

For all kinds of regional analysis, ideally, a national table should be further decomposed into a multiregional table like that in Table 1 (for illustrative purposes 3 regions, 3 industries):

Table 1

Multiregional I/O Table

		Region I			Region II			Region III		
		1	2	3	1	2	3	1	2	3
Region I	1 2 3									
Region II	1 2 3									
Region III	1 2 3									

Inputs of the same commodity from different regions are considered distinct inputs. We feel immediately that the preparation of such table is extremely complicated for the sources and destinations of inputs and outputs must be identified by region as well as by industry.

As the reader may know, several alternatives for simplifying the data collection and statistical problems of regional I/O tables had been applied in the past (Z. Hirsh, H. Chenery, W. Isard...) (see Hirsch, 1959).

To get the regional input-output table, we propose the technique of regional I/O model (analysis) construction using the so called „supply coefficient“. In order to explain it, we consider a 2 region, 3 industry case. One region will be the west part of SR and the second region will be the east part of SR. First of all, assume that there is no international trade so that exports of one region automatically reflect imports of the other region. The principal fact is that we have one technical I/O matrix for the two regions, the input-output table for Slovakia, that we indicate by **B**:

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \tag{1}$$

Using matrix **B**, we need the matrix indicating the fractions of various demands in region i supplied by region j. We do not have it. To solve the problem, we propose the matrices of ***supply coefficients*** $\mathbf{R}_{11}$, $\mathbf{R}_{21}$, $\mathbf{R}_{21}$ and $\mathbf{R}_{22}$ for our two regions – West Slovakia and East Slovakia – as follows:

$$\mathbf{R}_{11} = \begin{pmatrix} r_{1,11} & 0 & 0 \\ 0 & r_{2,11} & 0 \\ 0 & 0 & r_{3,11} \end{pmatrix}, \quad \mathbf{R}_{12} = \begin{pmatrix} r_{1,12} & 0 & 0 \\ 0 & r_{2,12} & 0 \\ 0 & 0 & r_{3,12} \end{pmatrix}$$

$$\mathbf{R}_{21} = \begin{pmatrix} r_{1,21} & 0 & 0 \\ 0 & r_{2,21} & 0 \\ 0 & 0 & r_{3,21} \end{pmatrix}, \quad \mathbf{R}_{22} = \begin{pmatrix} r_{1,22} & 0 & 0 \\ 0 & r_{2,22} & 0 \\ 0 & 0 & r_{3,22} \end{pmatrix}$$

where

$\mathbf{R}_{ij}$ is the matrix indicating the fractions of various demands in region i supplied by region j;

$r_{i,jk}$ is the fraction of the commodity i demanded in region j and supplied by region k (the first subscript indicates commodity, the second the source demanded and the third the source of supply).

These matrices will help us to get detailed data on the input-output flows in our two regions. Suppose that import of certain commodity is distributed to all users in proportion to their total purchases (domestic plus imported goods) of that commodity. Then we can construct a 6 x 6 matrix to identify inputs by supplying region as well as by commodity. Premultiplying matrix **B** by $\mathbf{R}_{11}$, we obtain the matrix *indicating the portion of input requirements per unit of output* in region 1 supplied by *domestic* production:

$$\mathbf{R}_{11}\mathbf{B} = \begin{pmatrix} r_{1,11} & 0 & 0 \\ 0 & r_{2,11} & 0 \\ 0 & 0 & r_{2,11} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} b_{11}r_{1,11} & b_{12}r_{1,11} & b_{13}r_{1,11} \\ b_{21}r_{2,11} & b_{22}r_{2,11} & b_{23}r_{2,11} \\ b_{31}r_{3,11} & b_{32}r_{3,11} & b_{33}r_{3,11} \end{pmatrix} \tag{2}$$

Similarly, the product matrix, $\mathbf{G}_{12}\mathbf{B}$ indicates the portion of input requirements per unit of output in region 1 supplied by products of region 2:

$$\mathbf{R}_{12}\mathbf{B} = \begin{pmatrix} b_{11}r_{1,12} & b_{12}r_{1,12} & b_{13}r_{1,12} \\ b_{21}r_{2,12} & b_{22}r_{2,12} & b_{23}r_{2,12} \\ b_{31}r_{3,12} & b_{32}r_{3,12} & b_{33}r_{3,12} \end{pmatrix}$$

For the input requirements per unit of output in region 2, the portion supplied by imports from region 1 is indicated by the product matrix, $\mathbf{R}_{21}\mathbf{B}$ and the portion supplied by its own production is $\mathbf{R}_{22}\mathbf{B}$:

$$\mathbf{R}_{21}\mathbf{B} = \begin{pmatrix} b_{11}r_{1,21} & b_{12}r_{1,21} & b_{13}r_{1,21} \\ b_{21}r_{2,21} & b_{22}r_{2,21} & b_{23}r_{2,21} \\ b_{31}r_{3,21} & b_{32}r_{3,21} & b_{33}r_{3,21} \end{pmatrix}$$

$$\mathbf{R}_{22}\mathbf{B} = \begin{pmatrix} b_{11}r_{1,22} & b_{12}r_{1,22} & b_{13}r_{1,22} \\ b_{21}r_{2,22} & b_{22}r_{2,22} & b_{23}r_{2,22} \\ b_{31}r_{3,22} & b_{32}r_{3,22} & b_{33}r_{3,22} \end{pmatrix}$$

Each of these matrices contains important economic information. We need to understand them. Now we are in a position to create a new matrix, that is the core of our idea, by combining above four matrices into an *aggregate* matrix $\mathbf{B}^*$, that is:

$$\mathbf{B}^* = \begin{pmatrix} R_{11}B & R_{21}B \\ R_{12}B & R_{22}B \end{pmatrix}$$

In the real economy this may be a large matrix. Country may have many industries and many regions. For the model we consider in our discussion we will get a 6 x 6 $\mathbf{B}^*$ matrix as shown in Table 2. The imports are distinguished by region as well as by industry.

Table 2

Two-Regional Technical Matrix

		Region I			Region II		
		Industry			Industry		
		1	2	3	1	2	3
Region I	1	$b_{11}r_{1,11}$	$b_{12}r_{1,11}$	$b_{13}r_{1,11}$	$b_{11}r_{1,21}$	$b_{11}r_{1,21}$	$b_{13}r_{1,21}$
	2	$b_{21}r_{2,11}$	$b_{22}r_{2,11}$	$b_{23}r_{2,11}$	$b_{21}r_{2,21}$	$b_{21}r_{2,21}$	$b_{23}r_{2,21}$
	3	$b_{31}r_{3,11}$	$b_{32}r_{3,11}$	$b_{33}r_{3,11}$	$b_{31}r_{3,21}$	$b_{31}r_{3,21}$	$b_{33}r_{3,21}$
Region II	1	$b_{11}r_{1,12}$	$b_{12}r_{1,12}$	$b_{13}r_{1,12}$	$b_{11}r_{1,22}$	$b_{12}r_{1,22}$	$b_{13}r_{1,22}$
	2	$b_{21}r_{2,12}$	$b_{22}r_{2,12}$	$b_{23}r_{2,12}$	$b_{21}r_{2,22}$	$b_{22}r_{2,22}$	$b_{23}r_{2,22}$
	3	$b_{31}r_{3,12}$	$b_{32}r_{3,12}$	$b_{33}r_{3,12}$	$b_{31}r_{3,22}$	$b_{32}r_{3,22}$	$b_{33}r_{3,22}$

The reader should try to understand the Table 2. The northwest quadrant of Table 2 indicates the distribution of domestic intermediate goods to various domestic industries in region 1.

The northeast quadrant shows the distribution of exports by region 1 to various industries in region 2. Similarly, the southwest quadrant indicates exports by region 2 and the southeast quadrant the distribution of domestic goods to various industries in region 2.

To get the real data for this table is an enormous statistical task. In our numerical example we got them on the base of our assumptions. From this table we can get regional input-output multipliers within regions. They are useful tools for regional economic impact analysis, a region's industrial structure and trading patterns. The change in output delivered for final users can be analyzed more deeply.

As we know, from I/O model, when the supply coefficients are sufficiently stable, matrix $\mathbf{B}^*$ serves as the basis for interregional impact analysis. But identical commodities produced by different regions are considered to be distinct commodities. We can then, as in I/O model, describe the calculation of total output requirements for given set of final demand. To show the complexity of the analysis, first we will give the notation of coefficients and variables. First of all:

$y_{i,r}$ is gross output of industry i in region r,
$F_{i,\,rh}$ is the final product for product i in region r met by the production of region h.

So, what are the distribution equations in this our two-regional model? Based on logical consideration we can say that the following identities hold (more in Husár, Mokrášová and Goga, 2000):

$$y_{i,1} = \underbrace{\sum_{j=1}^{3} b_{ij} r_{i,11} y_{j,1} + F_{i,11}}_{\text{(domestic uses)}} + \underbrace{\sum_{j=1}^{3} b_{ij} r_{i,21}\, y_{j,2} + F_{i,21}}_{\text{(exports)}}$$

$$y_{i,2} = \underbrace{\sum_{j=1}^{3} b_{ij} r_{i,12} y_{j,1} + F_{i,12}}_{\text{(exports)}} + \underbrace{\sum_{j=1}^{3} b_{ij} r_{i,22}\, y_{j,2} + F_{i,22}}_{\text{(domestic uses)}} \tag{3}$$

The critical variable is the final demand in each region. Given the value of final demand in each region, system (3) has 6 equations and 6 unknowns ($y_{i,r}$). In matrix notation, the output vector, $\mathbf{Y}^*$ and the final demand vector, $\mathbf{F}^*$ each has 6 components:

$$\mathbf{Y}^* = \begin{pmatrix} y_{1,1} \\ y_{2,1} \\ y_{3,1} \\ y_{1,2} \\ y_{2,2} \\ y_{3,2} \end{pmatrix} \qquad \mathbf{F}^* = \begin{pmatrix} F_{1,11} + F_{1,21} \\ F_{2,11} + F_{2,21} \\ F_{3,11} + F_{3,21} \\ F_{1,12} + F_{1,22} \\ F_{2,12} + F_{2,22} \\ F_{3,12} + F_{3,22} \end{pmatrix}$$

Everybody can grasp the content of these two vectors. So the system (3) can be written as:

$$\mathbf{Y}^* = \mathbf{B}^* \mathbf{Y}^* + \mathbf{F}^*$$

or

$$(\mathbf{I} - \mathbf{B}^*)\mathbf{Y}^* = \mathbf{F}^*$$

and finally

$$\mathbf{Y}^* = (\mathbf{I} - \mathbf{B}^*)^{-1}\mathbf{F}^* \tag{3a}$$

From (3a) we see that given the final demand in each region, the equilibrium output levels in each region can be calculated using the inverse matrix, $\mathbf{Y}^* = (\mathbf{I} - \mathbf{B}^*)^{-1}$. The technique is well known.

For illustration, suppose that the **B** matrix obtained for three sector economy is:

$$\mathbf{B} = \begin{pmatrix} 0{,}2 & 0{,}2 & 0 \\ 0{,}2 & 0{,}1 & 0{,}1 \\ 0 & 0{,}2 & 0{,}1 \end{pmatrix}$$

As we know, we need the matrices of supply coefficients. Say that they are:

$$\mathbf{R}_{11} = \begin{pmatrix} 0{,}35 & 0 & 0 \\ 0 & 0{,}4 & 0 \\ 0 & 0 & 0{,}3 \end{pmatrix}, \quad \mathbf{R}_{12} = \begin{pmatrix} 0{,}2 & 0 & 0 \\ 0 & 0{,}25 & 0 \\ 0 & 0 & 0{,}3 \end{pmatrix}$$

$$\mathbf{R}_{21} = \begin{pmatrix} 0{,}1 & 0 & 0 \\ 0 & 0{,}15 & 0 \\ 0 & 0 & 0{,}25 \end{pmatrix}, \quad \mathbf{R}_{22} = \begin{pmatrix} 0{,}4 & 0 & 0 \\ 0 & 0{,}35 & 0 \\ 0 & 0 & 0{,}45 \end{pmatrix}$$

Next what we need are $\mathbf{R}_{11}\mathbf{B}$, $\mathbf{R}_{21}\mathbf{B}$, $\mathbf{R}_{21}\mathbf{B}$ and $\mathbf{R}_{22}\mathbf{B}$ matrices. The reader can get them very easily.[1] Finally we need the combined matrix $\mathbf{B}^*$.

[1] Similar approach is applied in the Regional Multipliers (1997).

$$\mathbf{B}^* = \begin{pmatrix} 0,07 & 0,07 & 0 & 0,04 & 0,04 & 0 \\ 0,08 & 0,04 & 0,04 & 0,05 & 0,025 & 0,025 \\ 0 & 0,06 & 0,03 & 0 & 0,06 & 0,03 \\ 0,02 & 0,02 & 0 & 0,08 & 0,08 & 0 \\ 0,03 & 0,015 & 0,015 & 0,07 & 0,035 & 0,035 \\ 0 & 0,05 & 0,025 & 0 & 0,09 & 0,045 \end{pmatrix}$$

We need yet the vector $\mathbf{F}^*$. Let it be:

$$\mathbf{F}^* = \begin{pmatrix} 80 \\ 300 \\ 100 \\ 150 \\ 260 \\ 140 \end{pmatrix}$$

Thus given the final demand in each region, equilibrium output levels in each region can be calculated from (3a). Using an Excel, we got these values:

$$\mathbf{Y}^* = \begin{pmatrix} 133,\ 703 \\ 352,\ 740 \\ 149,\ 716 \\ 189,\ 033 \\ 302,\ 271 \\ 197,\ 470 \end{pmatrix}$$

In principle the $\mathbf{F}^*$ vector is the GDP and its elements are regional components of it. They can serve as good insides into the regions' economy. Based on the above assumptions, the vector of final demand by products and industries has produced the vector of gross output of the economy $\mathbf{Y}^*$. This is the consistent output pertaining to given final demand. The first component is $y_{1,1}$, that is gross output of industry 1 in region 1 is 133,703 bill. SKK.

Similar interpretation can be given to other components. By changing the components of the vector $\mathbf{F}^*$ we will get other components of gross output and we ca make an analysis of consequences. Now we can envisage the field of application – final-demand multipliers for output, for earnings, and for employment. Facing new challenges of the Slovak economy with regard to European Union economy we showed some new ideas from regional input-output model on how to change the economic policy making.

Conclusion

We tackled an important macroeconomic problem of economic system functioning as we consider the regional problems. Using the technical input-output table (matrix) of a given economy, we showed how to construct the two-regional technical matrix that can serve as the bases for the input-output model application. The results of interregional reactions have some economic policy significance. The policymaker is able to project the economic development of the region based on the functioning of the economic system. He can make a systematic analysis of the economic impacts of the investment projects and final demand on affected regions.

We consider that systems of national accounts and input-output models are more than just a source of data. We can use them together with the matrices developed in this paper to adjust the national I-O table in order to reflect a region's industrial structure and trading patterns. We can understand the lagging region more deeply. They are means of specifying the categories under which data are to be (by regions) organized. For policy purposes we decided to use common input-output table for the two-regional model construction.

Literature

[1] ACOCELLA, N. (1998): The Foundations of Economic Policy. Cambridge: CUP.

[2] FOX, K. A. (1953): A Spatial Equilibrium Model of the Livestock-Feed Economy in the U.S. Econometrica, *21*, No. 4, October.

[3] HIRSCH, W. Z. (1959): Interindustry Relations of a Metropolitan Area. Review of Economics and Statistics, *41*, August.

[4] HUSÁR, J. – MOKRÁŠOVÁ, V. – GOGA, M. (2000): Input-output analýza a systém národných účtov. Bratislava: Vydavateľstvo EKONÓM.

[5] ISARD, W. et al. (1998): Methods of Interregional and Regional Analysis. Sydney: Ashgate.

[6] KUZNETS, S. (1955): Economic Growth and Income Inequality. American Economic Review, *45*, March.

[7] LEONTIEF, W. (1986): Input-Output Analysis. New York: Oxford University Press.

[8] MOORE, F. T. (1955): Regional Economic Reaction Paths. American Economic Review, *65*, May, pp. 133 – 148.

[9] REGAN W. M. – PRISLOE, M. (2003): Estimating the Impact of Public Policy and Investment Decissions. Economic Digest, *8*, No. 5, May.

[10] Regional Multipliers (1997): A User Handbook for the Regional I/O Modeling Systems. Washington, DC 20402: U.S. Government Printing Office, 1997.

[11] STONE, R. (1970): Mathematical Models of the Economy and Other Essays. London: Chapman and Hall.

[12] TREYZ, F. – TREYZ, G. (1997): The REMI Multi-regional U.S. Policy Analysis model. Washington: Annual North American Meeting of the Regional Science Association.