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Investment Opportunities Efficient Frontiers Modelling

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Abstract

Paper presents some techniques for portfolios efficient set modelling in Excel. From the technical point of view one needs to solve a series of quadratic programming problems that are known as Markowitz portfolio selection problems. For an effective realisation of solving the process, the Excel solver specially created VBA procedures were used. The first part of the paper illustrates an application of the procedures for modelling efficient portfolio sets of selected countries on the base of one year performances of their markets in period from 1900 to 2000. In the second part the methodology of efficient frontier modelling in Excel environment is presented together with its illustration for modelling of efficient portfolios sets of selected capital market segments.

Keywords: *investment analysis, efficient portfolios sets, investor's risk attitude, VBA procedure*

JEL Classification: G11

1. Introduction

As investors enter the third millennium, they have more reason than ever to ask where the markets are heading, what returns can be expected from equities, bonds and bills in different markets around the world, what are the risk of stocks and bond market investments from the perspective of both domestic and international investors and what are the likely long-term rewards.

It is difficult to form a judgment about future prospects or about required or "fair" rates of return without making a comparison with the past. Recent market returns are widely published. But it has been difficult to get a reliable impression of what investors have achieved over the long term.

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Table 1

Real Returns on Equities, Bonds, and Bills in the Year 2000 with Ten and One-Hundred Year Comparisons

Country	Period	Annualised Real Returns, % per year				Best Asset
		Equities	Bonds	Bills	Inflation	
Australia	Year 2000	–0.7	7.1	0.0	5.8	Bonds
	1990 – 1999	9.0	9.0	4.9	2.3	Bonds
	1900 – 1999	7.6	1.0	0.4	4.0	Equities
Belgium	Year 2000	–6.0	4.4	1.5	2.7	Bonds
	1990 – 1999	9.6	7.1	4.1	2.1	Equities
	1900 – 1999	2.7	–0.5	–0.4	5.6	Equities
Canada	Year 2000	4.5	7.4	2.6	2.8	Bonds
	1990 – 1999	8.2	9.5	4.3	2.1	Bonds
	1900 – 1999	6.4	1.8	1.7	3.1	Equities
Denmark	Year 2000	15.6	6.5	2.5	2.4	Equities
	1990 – 1999	7.2	8.1	5.1	2.1	Bonds
	1900 – 1999	5.1	2.8	2.9	4.0	Equities
France	Year 2000	–1.0	5.4	2.6	1.6	Bonds
	1990 – 1999	12.6	7.4	4.6	1.8	Equities
	1900 – 1999	4.0	–1.1	–3.4	8.0	Equities
Germany	Year 2000	–11.8	5.0	2.0	2.2	Bonds
	1990 – 1999	9.6	5.1	3.0	2.4	Equities
	1900 – 1999	3.7	–2.3	–0.6	5.2	Equities
Ireland	Year 2000	10.1	4.0	–0.9	5.1	Equities
	1990 – 1999	11.9	8.3	5.5	2.3	Equities
	1900 – 1999	5.5	1.5	1.3	4.5	Equities
Italy	Year 2000	4.6	4.1	1.4	2.8	Equities
	1990 – 1999	6.4	8.7	1.7	4.1	Bonds
	1900 – 1999	2.7	–2.3	–4.1	9.2	Equities
Japan	Year 2000	–24.6	2.9	0.9	–0.6	Bonds
	1990 – 1999	–5.2	5.7	1.6	1.0	Bonds
	1900 – 1999	5.0	–1.6	–2.1	7.7	Equities
Netherlands	Year 2000	–4.9	4.4	1.3	2.9	Bonds
	1990 – 1999	17.7	5.7	3.2	2.5	Equities
	1900 – 1999	5.9	1.1	0.7	3.0	Equities
Spain	Year 2000	–16.1	2.9	0.1	4.1	Bonds
	1990 – 1999	12.1	8.1	4.7	4.1	Equities
	1900 – 1999	4.7	1.2	0.4	6.2	Equities
Sweeden	Year 2000	–12.4	10.2	2.6	1.4	Bonds
	1990 – 1999	15.2	11.1	5.1	3.0	Equities
	1900 – 1999	8.2	2.3	2.0	3.7	Equities
Switzerland	Year 2000	11.0	3.1	1.3	1.5	Equities
	1990 – 1999	13.9	5.0	1.9	2.1	Equities
	1900 – 1999	5.0	2.8	1.1	2.2	Equities
UK	Year 2000	–7.6	4.5	3.0	2.9	Bonds
	1990 – 1999	11.2	9.3	4.4	3.5	Equities
	1900 – 1999	5.9	1.3	1.0	4.1	Equities
USA	Year 2000	–13.8	10.2	2.4	3.4	Bonds
	1990 – 1999	14.2	6.4	2.0	2.9	Equities
	1900 – 1999	6.9	1.5	1.1	3.2	Equities

Source: The Table 1 was constructed on the base of data published in: Dimson, Marsh and Staunton (2001).

In this connection the very interesting and useful book by *E. Dimson, P. Marsh,* and *M. Staunton* (2001) provides investors with a comprehensive record of past investment returns around the world and helps them understand the historical record, so they can make informed judgment about the future. It does this by providing a comprehensive record of the returns from equities, bond and bills, as well as inflation rates and currency movements in fifteen different counties over the whole of the last 101 years, from the 1900 to 2000.

This part of the paper presents some selected facts from the mentioned publication that were used for constructions and analysis of the efficient portfolio sets of individual countries on the basis of modern portfolio theory applications (Mlynarovič, 2001). Some detailed results can be found at (Mlynarovič and Kolárik, 2002).

Table 1 shows the real returns on equities, bonds and bills, plus the inflation rate for selected countries. For each country, the first row records the returns during 2000, while for comparative purposes, the next two rows show the annualised returns over the previous decade and the whole of the 20th century. The final column shows which asset category – equities, bonds or bills – performed best over each period. It is clear that the returns during 2000 contrast starkly with the excellent returns over the previous decade. The 1990s were a golden age. While the events in the capital markets during 2000 are endlessly fascinating, it is clear that they were very unusual, and do not form a basis for generalising about capital market returns. But it is also seen that going back in time by a further decade does little to clarify the picture. Golden ages, by definition, recur quite rarely.

To form a balanced judgement about the returns achieved both during 2000, and also over the last decade, we need to look at much longer periods of history. An insight into the perspective this can bring is provided in the final row for each country in Table 1. This shows the real returns for each asset class over the one hundred years. Clearly, these 100-year returns are much less favourable than the corresponding returns for 1990s, but they are generally much more favourable than the disappointing returns for the year 2000. The most striking feature of these 100-years returns, however, is shown in the final column of the table. Without exception, equities were the best performing asset class in all fifteen countries.

In order to understand risk and return in the capital market – which is the principal objective for the all investors – one therefore needs to examine much longer periods of history than either a single year, or even a decade. This is because stock markets are volatile, with considerable variation in year to year returns and we therefore need long time series before we can make inferences. The examined period also needs to be long enough to incorporate the bad times as well as the good.

Table 2

Characteristics of Capital Markets for Period 1900 – 2000

		Arithmetic Mean (in %)	Standard Deviation (in %)	Covariance of Returns Bonds	Covariance of Returns Bills	Covariance of Excess Returns Bonds	Standard Dev. of Excess Returns (in %)
Australia	Equities	8.65	13.67	0.005321	0.000644	0.003841	13.71
	Bonds	5.20	8.56		0.002223		6.54
	Bills	2.46	3.72				
Belgium	Equities	6.62	17.19	0.006435	0.001352	0.004716	17.00
	Bonds	3.36	7.15		0.002406		4.83
	Bills	1.81	4.52				
Canada	Equities	7.72	11.27	0.003624	–0.0006	0.004050	12.14
	Bonds	5.90	9.51		0.000994		8.88
	Bills	2.96	2.86				
Denmark	Equities	7.30	16.44	0.0076	–0.0005	0.007808	17.17
	Bonds	5.80	9.67		0.001747		8.55
	Bills	3.93	3.82				
France	Equities	8.95	18.09	0.005859	0.002217	0.003151	18.06
	Bonds	3.41	9.15		0.004817		5.54
	Bills	0.65	6.58				
Germany	Equities	6.70	16.46	0.00568	0.001233	0.003532	16.25
	Bonds	1.79	9.03		0.002698		6.73
	Bills	1.25	4.22				
Ireland	Equities	10.15	18.74	0.017166	0.000507	0.016366	18.92
	Bonds	5.17	11.22		0.001964		10.17
	Bills	3.12	4.09				
Italy	Equities	6.13	19.70	0.007304	0.003192	0.003102	19.36
	Bonds	3.47	10.87		0.006059		6.89
	Bills	–0.94	7.11				
Japan	Equities	1.01	21.87	0.006096	0.004675	0.000889	21.26
	Bonds	2.52	10.65		0.007247		5.98
	Bills	0.15	8.20				
Netherlands	Equities	11.54	15.66	0.003969	0.000115	0.003638	15.91
	Bonds	3.49	6.77		0.001242		5.59
	Bills	1.88	3.20				
Spain	Equities	8.63	20.13	0.012228	0.001501	0.010258	19.74
	Bonds	4.65	8.36		0.001911		6.79
	Bills	2.35	3.80				
Sweden	Equities	12.29	22.14	0.005447	0.000822	0.003934	22.07
	Bonds	7.10	10.91		0.002035		9.58
	Bills	3.48	3.67				
Switzerland	Equities	10.14	17.06	0.006973	0.000378	0.006590	17.05
	Bonds	3.96	6.11		0.000738		5.46
	Bills	1.50	2.71				
United Kingdom	Equities	8.13	11.32	0.011964	0.001816	0.006680	10.88
	Bonds	5.53	10.94		0.001095		9.71
	Bills	2.64	3.31				
United states	Equities	9.73	12.77	0.006039	0.001061	0.004571	12.14
	Bonds	4.50	9.12		0.000952		8.34
	Bills	1.59	2.33				

Source: Own computations.

2. Historical Efficient Frontiers Modelling

The goal of this part of the paper is a construction of historical efficient sets of capital markets of the selected countries for the period from 1900 to 2000 that are aggregated into three classes of assets, namely equities, bonds and bills. The sets contain portfolios that are optimal from viewpoints of individual investors. An individual investor then selects his optimal portfolio on the base of her (his) attitude to risk. It means that investors with high level of risk aversion select portfolios from this part of the efficient set where the risk (measured by standard deviation of yearly returns) is low and investor with a tolerance to risk select portfolios from this part of the efficient set where the risk is higher.

Modern portfolio theory applications enable to construct step by step frontier of investment opportunities with different assumptions concerning short sales possibilities and risk of some assets. At the first it was assumed that short sales are allowed and this assumption was applied for two following situations: a) all assets are risky; b) bills are risk free assets.

The latter of the assumption permitted to approximate capital market line. Finally, frontiers of investment opportunities were constructed on the assumption that short sales are not allowed.

As it was mentioned above, all data were drawn from *E. Dimson, P. Marsh,* and *M. Staunton* (2001). The goal was to construct investment opportunities frontiers on the base of yearly data. But for the period from 1900 to 1990 only the data for decades are published. For these periods the corresponding yearly data were computed using the geometric means. Resulting real characteristics of capital markets are shown in Table 2. Of course, these characteristics differ from these published in *Millennium Book* II (2001), which are derived from full year time series for the period 1900 – 2000. The result is that presented frontiers are some approximations of the actual ones. Figure 1 and Table 3 present results of analysis in the case when short sales are allowed and all assets are assumed as risky ones. Figure 1, and following figures as well, consists of two parts owing to higher lucidity. Note that curves for Australia are always in the both of them to help to see mutual positions of all of them.

In the following part of the analysis it was assumed that there are only two risky assets, equities and bonds, and returns of bills are assumed as risk free ones. In Table 4 corresponding market portfolios and their characteristics are described and Figure 2 shows capital market lines for individual countries. Ireland was excluded from this analysis. The reason is that in this type of the analysis the coefficients of the covariance matrix measure covariance of excess returns, i.e. covariance of the equities and bonds returns less risk free return of the bill. In the case of Ireland the return that corresponds to the global minimum variance portfolio (2.93%) in this situation is less than risk free return (3.12%) and it generates a negative slope of the CML what is out of a reasonable economic interpretations.

Figure 1a
Efficient Frontiers with Short Sales Allowed

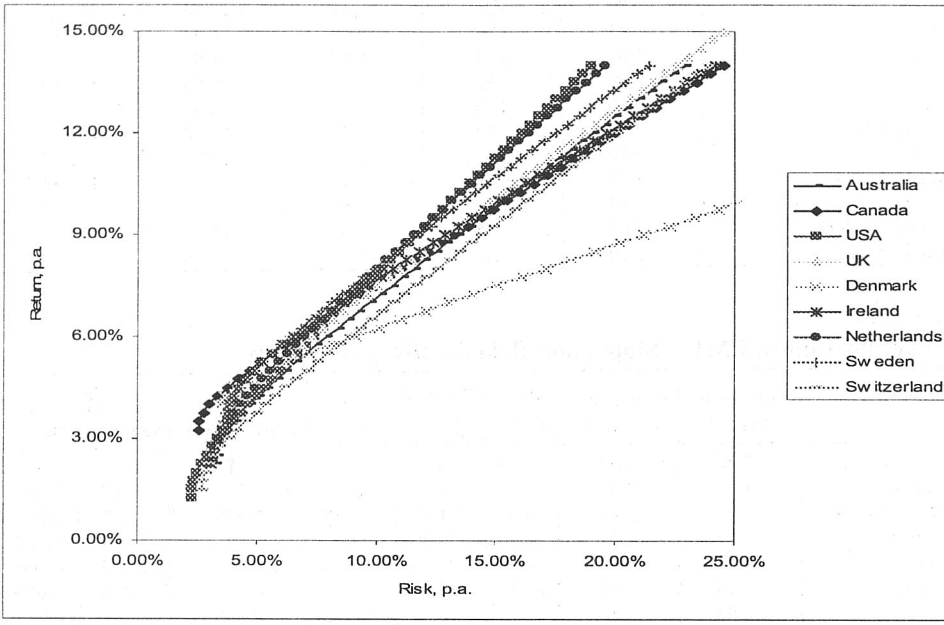


Figure 1b
Efficient Frontiers with Short Sales Allowed

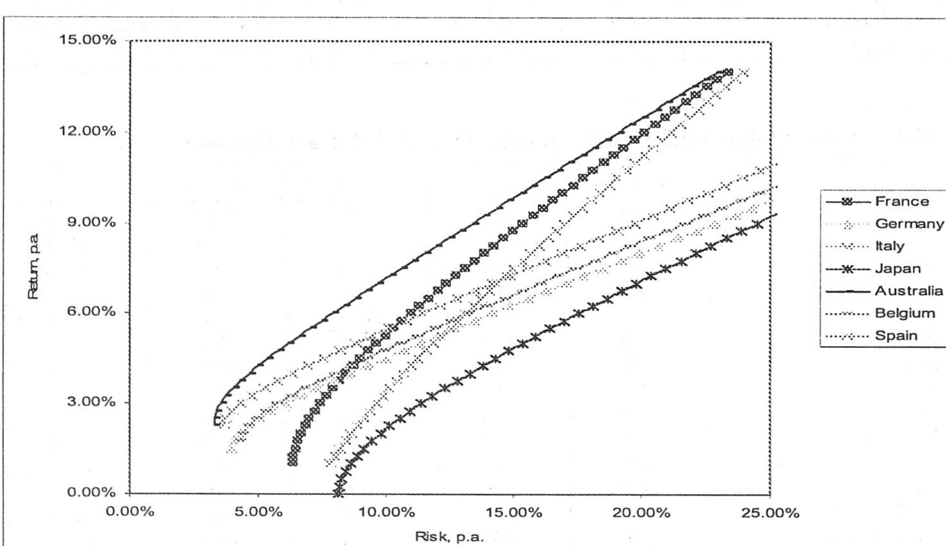


Table 3
Portfolios with Global Minimum Variance – Short Sales Allowed (in %)

	Return, p.a.	Risk, p.a.	Equities	Bonds	Bills
Australia	2.28	3.28	9.74	–28.32	118.58
Belgium	1.68	4.33	7.38	–30.66	123.28
Canada	3.28	2.52	11.93	–8.33	96.40
Denmark	4.00	3.47	10.72	–15.42	104.70
France	0.70	6.34	8.89	–25.16	116.27
Germany	1.41	3.91	5.33	–24.33	118.99
Ireland	3.69	3.73	16.50	–28.97	112.47
Italy	–1.59	6.82	7.10	–25.95	118.84
Japan	–0.19	8.08	4.83	–16.07	111.24
Netherlands	2.20	3.08	5.52	–13.34	107.82
Spain	2.19	3.65	6.10	–23.74	117.64
Sweden	3.33	3.58	1.74	–8.28	106.54
Switzerland	1.58	2.69	2.51	–5.72	103.20
United Kingdom	2.69	3.15	7.93	–13.27	105.34
United states	1.29	2.27	–2.12	–4.45	106.57

Table 4
Market Portfolios, CML – Slopes and Betas of Risky Assets (in %)

	Risk free Return (in %)	Excess Return (in %)	Risk (in %)	Equities (in %)	Bonds (in %)	CML – Slope	*Betas*	
							Equities	Bonds
Australia	2.46	4.00	7.75	36.46	63.54	0.516	1.547	0.686
Belgium	1.81	2.04	5.93	14.79	85.21	0.344	2.360	0.764
Canada	2.96	3.88	8.83	51.67	48.33	0.439	1.227	0.757
Denmark	3.93	2.26	9.48	25.96	74.04	0.238	1.494	0.827
France	0.65	3.91	6.61	20.76	79.24	0.591	2.122	0.706
Germany	1.25	6.80	20.18	127.33	–27.33	0.337	0.803	0.080
Italy	–0.94	4.74	6.98	12.15	87.85	0.679	1.494	0.932
Japan	0.15	2.36	5.94	0.89	99.11	0.398	0.364	1.006
Netherlands	1.88	8.13	13.35	80.96	19.04	0.609	1.188	0.199
Spain	2.35	3.00	8.53	17.41	82.59	0.351	2.096	0.769
Sweden	3.48	5.28	10.46	31.98	68.02	0.505	1.669	0.685
Switzerland	1.50	4.91	9.39	39.65	60.35	0.523	1.759	0.501
United Kingdom	2.64	5.70	11.29	108.27	–8.27	0.505	0.962	0.507
United states	1.59	7.54	11.21	88.45	11.55	0.673	1.080	0.386

Table 5
Portfolios with Global Minimum Variance – Short Sales not Allowed (in %)

	Return, p.a.	Risk, p.a.	Equities	Bonds	Bills
Australia	2.70	3.68	3.95	0.00	96.05
Belgium	1.92	4.50	2.38	0.00	97.62
Canada	3.42	2.61	9.64	0.00	90.36
Denmark	4.06	3.67	3.95	0.00	96.05
France	1.19	6.47	6.46	0.00	93.54
Germany	1.36	4.21	2.08	0.00	97.92
Ireland	3.35	4.04	3.25	0.00	96.75
Italy	–0.59	7.04	4.95	0.00	95.05
Japan	0.19	8.14	4.52	0.00	95.48
Netherlands	2.23	3.15	3.60	0.00	96.40
Spain	2.70	3.68	3.95	0.00	96.05
Sweden	3.57	3.66	1.07	0.00	98.93
Switzerland	1.61	2.70	1.22	0.00	98.78
United Kingdom	2.67	3.31	0.44	0.00	99.56
United states	1.59	2.33	0.00	0.00	100.00

Figure 2a

Capital Market Lines

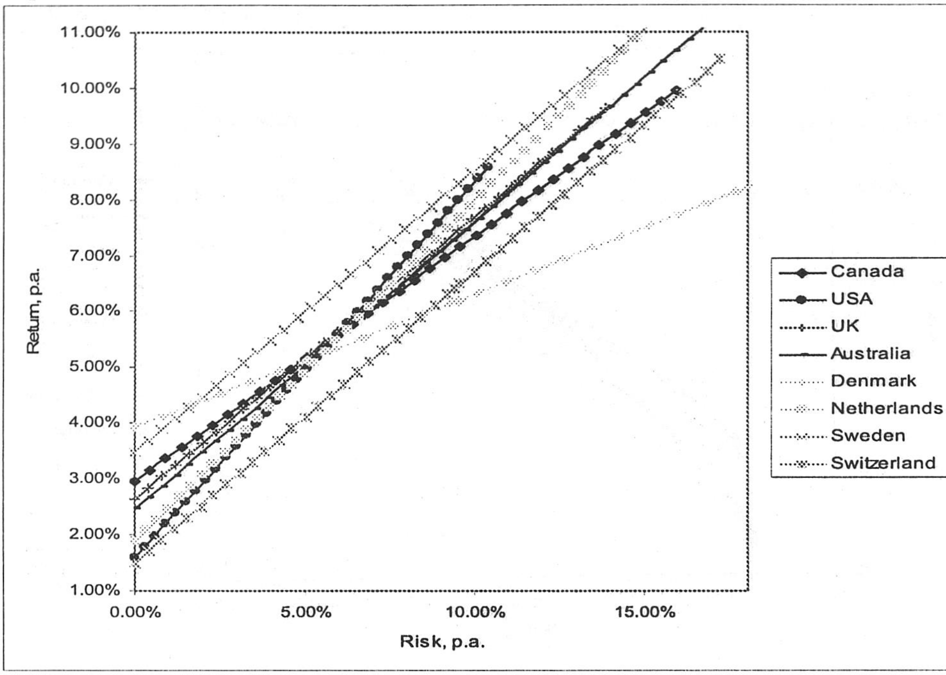


Figure 2b

Capital Market Lines

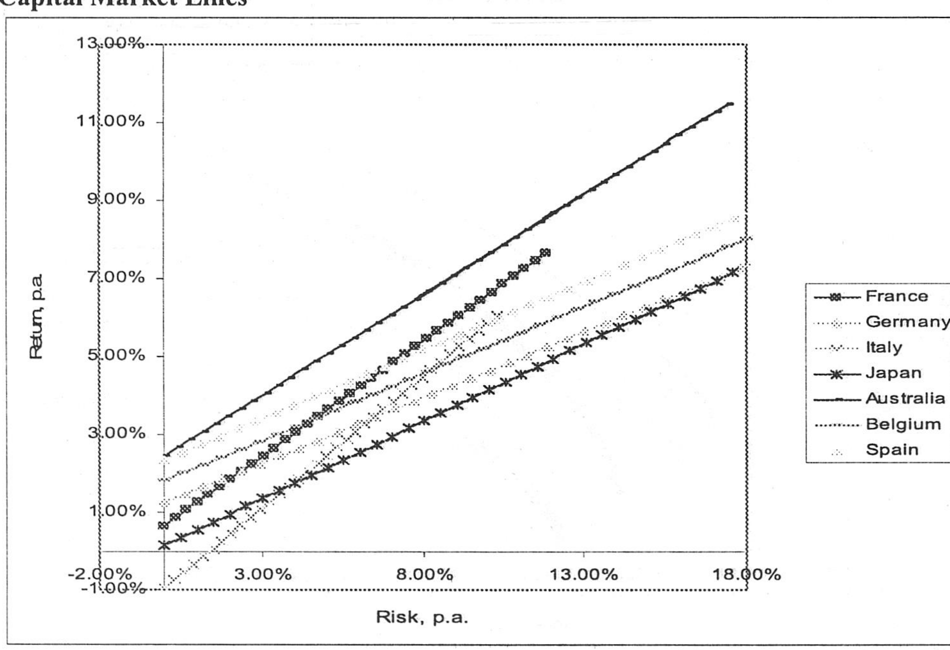


Figure 3a
Efficient Frontiers with Short Sales not Allowed

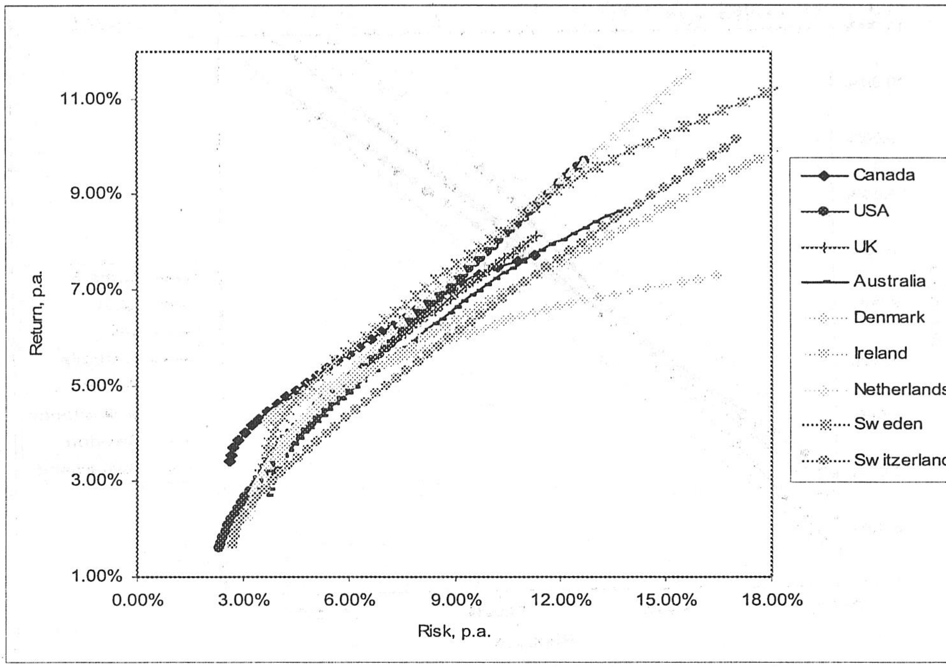
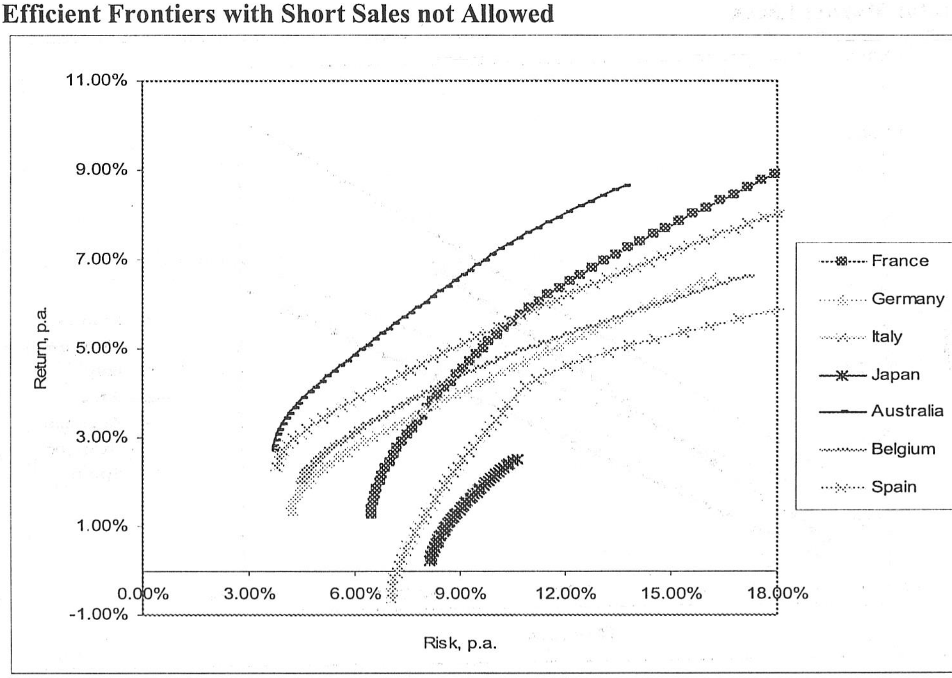


Figure 3b
Efficient Frontiers with Short Sales not Allowed



The historical analysis of the investment opportunity frontiers was completed on the assumption that short sales are not allowed. Corresponding frontiers are shown in Figure 3 and global minimum variance portfolios are described in Table 5.

3. Modelling Efficient Frontiers (from a Viewpoint of Slovak Investors)

Relying on historical discussion (Kroll, Levy, and Markowitz, 1984; Markowitz, 1991) mean – variance ($E - V$) frontier is assumed to be a suitable and elegant technique to obtain a description of efficient portfolios in modern portfolio theory. Formally it means that presented analyses are based on solving quadratic programming problems that look for such portfolios where defined expected returns E_P are achieved with as low risk as possible, where the risk is measured by the variance of portfolio returns.

The problem can be formally written in the form:

$$\min \quad V_P = \mathbf{w}^T \mathbf{C} \mathbf{w}$$

subject to

$$\mathbf{E}^T \mathbf{w} = E_P$$
$$\mathbf{e}^T \mathbf{w} = 1$$
$$\mathbf{w}^l \leq \mathbf{w} \leq \mathbf{w}^u$$

where

- $\mathbf{C}$ – covariance matrix $n \times n$, where n is the number of assets,
- $\mathbf{w}$ – vector of assets weights,
- $\mathbf{E}$ – vector of assets expected returns,
- E_P – portfolio expected return,
- $\mathbf{w}^l$ – vector of lower bounds on weights,
- $\mathbf{w}^u$ – vector of upper bounds on weights,
- $\mathbf{e}$ – vector whose elements equal 1.

Two small modifications of the above problem provide so called global minimum variance portfolio, with characteristics $(V_P, E_P) = (V_*, E_*)$, and portfolio with maximum expected return with characteristics $(V_P, E_P) = (V^*, E^*)$. These two portfolios restrict the efficient frontier and its approximation is achieved by solving a series of the above problems for $E_* \leq E_P \leq E^*$. *Ballestero and Romero* (1996) define the index of profitability θ *and the* index of safety ψ, where

$$\theta = \frac{E_P - E_*}{E^* - E_*}, \qquad \psi = \frac{V^* - V_P}{V^* - V_*}$$

It is clear that $0 \leq \theta \leq 1$, $0 \leq \psi \leq 1$ and (V_*, E^*), respectively (1, 1), is so called ideal portfolio in the mean – variance space, respectively in the profitability – safety space. Analogically (V^*, E_*), respectively (0, 0), is the corresponding anti-ideal portfolio. Finally, these definitions result in the approximation of a normalized efficient frontier in the space profitability – safety.

The portfolio selection problem is very often examined as a multiple criteria optimization problem and techniques of compromise programming (Yu, 1985) are being used to find the efficient portfolio with a specified property. The set of efficient portfolios in the profitability – safety space can be formally written as the set

$$F = \{(\theta,\psi) \mid T(\theta,\psi) = 0,\ 0 \leq \theta \leq 1,\ 0 \leq \psi \leq 1\}$$

The efficient portfolio that minimise maximum deviation α from the ideal portfolio with anti-ideal portfolio as a reference portfolio can be find as the optimal solution of the problem in the form

$$\min \alpha$$

subject to

$$\alpha \geq 1 - \theta$$
$$\alpha \geq 1 - \psi$$
$$(\theta,\psi) \in F$$

or, equivalently, can be identify as the intercept of the line $\theta = \psi$ with the efficient frontier in the profitability – safety space. It is so called well balanced portfolio in this space and its approximation is presented in Table 6a-b (for the assumed investment style) where 0.7021 ≈ 0.7161, respectively 0.7736 ≈ 0.7595. The solution of the above problem provide corresponding "exact" well balanced portfolios with (θ, ψ) = (0.707264, 0.707264), annualised return 16.68% and annualised risk 9.11% for returns in SKK and (θ, ψ) = (0.766154, 0.766154), annualised return 13.84% and annualised risk 4.88% for returns in local currencies.

The described analysis required to solve a series of optimisation problems and for an effective realisation in Excel we have created user functions for computation of portfolio expected return (*PortfolioReturn*) and portfolio variance of returns (*PortfolioVariance*), presented in Figure 4, and VBA procedure (*EFF*) that automatizes the formulation and the solving of a sequence problems with solver and provides an approximation of the efficient frontier. In the VBA procedure, which is described in Figure 5, the following Excel ranges are used:

ovciel – the cell where the expected return is stored,
ov_cieldh – the cell with a lower bound for expected return,
ov_cielhh – the cell with an upper bound for expected return,
Vahy – range where optimal weighs are stored,

Vahy_dh – range with lower bounds on weights,
Vahy_hh – range with upper bounds on weights,
eqi – the cell with the formula for the sum of equity investment weights,
eqih – the cell with an upper bound for equities investment,
bnd – the cell with the formula for the sum of bond investment weights,
bndh – the cell with an upper bound for bonds investment,
mmt – the cell with the formula for the sum of money market investment weights,
mmth – the cell with an upper bound for money market investment,
suma – the cell that contains the formula for sum of range „Vahy“,
ov – the cell that calls function PortfolioReturn,
so – the cell that calls function PortfolioVariance,
ad – the cell that defines a step change for an increase of return,
ciel – the beginning of the range where required returns are stored,
vynos – the beginning of the range where computed returns are stored,
riziko – the beginning of the range where computed risks are stored,
optvahy – beginning of the range where optimal weights are stored

Figure 4

User Functions *PortfolioReturn* and *PortfolioVariance*

```
Function PortfolioVariance(wtsvec, vcvmat)
' wtsvec – the range of the weights
' vcvmat – the range with the covariance matrix
Dim v1 As Variant
If wtsvec.Columns.Count > wtsvec.Rows.Count Then
   wtsvec = Application.Transpose(wtsvec)
End If
v1 = Application.MMult(vcvmat, wtsvec)
PortfolioVariance = Application.SumProduct(v1, wtsvec)
End Function
Function PortfolioReturn(retvec, wtsvec)
' retvec – the range with the expected returns
If Application.Count(retvec) = Application.Count(wtsvec) Then
If retvec.Columns.Count > retvec.Rows.Count Then
   retvec = Application.Transpose(retvec)
End If
If wtsvec.Columns.Count > wtsvec.Rows.Count Then
   wtsvec = Application.Transpose(wtsvec)
End If
PortfolioReturn = Application.SumProduct(retvec, wtsvec)
Else
 PortfolioReturn = –1
End If
End Function
```

Figure 5

Procedure for Efficient Frontier Approximation

```
Sub EFF()
' Keyboard Shortcut: Ctrl + Shift + X
SolverReset
Call SolverAdd(Range("Vahy"), 3, Range("Vahy_dh"))
Call SolverAdd(Range("Vahy"), 1, Range("Vahy_hh"))
Call SolverAdd(Range("eqi"), 1, Range("eqih"))
Call SolverAdd(Range("bnd"), 1, Range("bndh"))
Call SolverAdd(Range("mmt"), 1, Range("mmth"))
Call SolverAdd(Range("suma"), 2, 1)
Call SolverOk(Range("ov"), 1, 0, Range("Vahy"))
Call SolverSolve(True)
Range("ov_cielhh").Value = Range("ov").Value
SolverReset
Call SolverAdd(Range("Vahy"), 3, Range("Vahy_dh"))
Call SolverAdd(Range("Vahy"), 1, Range("Vahy_hh"))
Call SolverAdd(Range("eqi"), 1, Range("eqih"))
Call SolverAdd(Range("bnd"), 1, Range("bndh"))
Call SolverAdd(Range("mmt"), 1, Range("mmth"))
Call SolverAdd(Range("suma"), 2, 1)
Call SolverOk(Range("so"), 2, 0, Range("Vahy"))
Call SolverSolve(True)
Range("ov_cieldh").Value = Range("ov").Value
Range("ovciel").Value = Range("ov_cieldh").Value
SolverReset
Call SolverAdd(Range("Vahy"), 3, Range("Vahy_dh"))
Call SolverAdd(Range("Vahy"), 1, Range("Vahy_hh"))
Call SolverAdd(Range("eqi"), 1, Range("eqih"))
Call SolverAdd(Range("bnd"), 1, Range("bndh"))
Call SolverAdd(Range("mmt"), 1, Range("mmth"))
Call SolverAdd(Range("suma"), 2, 1)
Call SolverAdd(Range("ov"), 2, Range("ovciel"))
Call SolverOk(Range("so"), 2, 0, Range("Vahy"))
n = (Range("ov_cielhh").Value – Range("ov_cieldh").Value) / Range("ad").Value
ciel = Range("ovciel").Value
ad = Range("ad").Value
For i = 0 To n
Call SolverSolve(True)
Range("ovciel").Copy
Range("ciel").Offset(i, 0).PasteSpecial Paste: = xlValues
Application.CutCopyMode = False
Range("ov").Copy
Range("vynos").Offset(i, 0).PasteSpecial Paste: = xlValues
Application.CutCopyMode = False
```

```
Range("so").Copy
Range("riziko").Offset(i, 0).PasteSpecial Paste: = xlValues
Application.CutCopyMode = False
Range("Vahy").Copy
Range("optvahy").Offset(i, 0).PasteSpecial Paste: = xlValues
Application.CutCopyMode = False
ciel = ciel + ad
Range("ovciel").Value = ciel
Call SolverChange(Range("ov"), 2, Range("ovciel"))
Next i
SolverFinish
End Sub
```

The following part of the paper should be viewed as an illustration of the presented methodology in the Excel environment. The methodology assumes a Slovak investor with incomes and payments in Slovak crowns. It means that exchange rates are included into analysis as well. Efficient portfolio sets of capital market that consists of Slovak corporate bonds, Slovak government bonds, Slovak and selected foreign money market tools, selected foreign equities and bonds, and domestic equities are constructed. Through bounds on such asset classes as bills, bonds and equities a level of risk aversion of an investor is modelled.

Namely the monthly data on returns computed from the following indices (Bloomberg) for the period from October 1996 to September 2002 were used together with corresponding exchange rates:

- Bills: SBWMUD3U, SBWMEU3L, BBOR6M,
- Bonds: SDX – S, SDX – P, SBUSL, SBEGEU, JPEGCOMP, JPEECOMP, JPEGEURO, JPEEEUR,
- Equities: TPX30, INDU, SX5P, SKSM.

The procedure for an efficient frontier modelling consists of the two steps. At first parameters of investor's investment style are specified. It is illustrated in Figure 4. Investor chooses the type of style (conservative, moderate, balanced, growth, aggressive), types of the assets according to the currency of their shares (all assets, assets in USD, assets in EUR), currency for returns evaluations (Slovak crowns, local currencies) and levels of expected returns (historical averages, expected returns). For individual investment styles upper bounds on asset classes (bills, bonds and equities) and upper bounds on individual assets are specified. It is also assumed that short sales are nor allowed. In the second step the VBA procedure that is presented in Figure 8 is realised.

Before a description of results let us note that questions about profitability of using of exchange rates changes at various kinds of financial investments are a subject of frequent analysis and speculations as well. In general, for an international investor, fluctuations in asset prices must be converted from the local currency into a currency which portfolio performance is evaluated in. Exchange rates are therefore critical for measuring and comparing the returns from different countries. The presented procedure provide also a possibility to optimise portfolios of selected domestic and international financial market tools from the viewpoint of an investor with incomes and expenditures in the Slovak crowns.

The basic result of the procedure application is an approximation of the efficient frontier of assumed investment opportunities. For the parameters specified in Figure 6 the frontier is illustrated in Figure 7 and its normalized version after introducing indices of safety and profitability [5] is presented in Figure 8. Tables 6a and 6b present three selected portfolios of the efficient frontier for the specified investment style together with their probabilistic characteristics. Namely portfolio with global minimum variance, well balanced portfolio in the space safety – profitability and portfolio with maximum return are presented.

Figure 6

Investor Investment Style Definition

	A	B	C	D	E	F	G	H	I
2	**Investment style definition**				**Expected definitions**				
3		Value			Value	Style:	Assets:	Currency:	Returns:
4	Style:	2	Moderate		1	Conservative	ALL	SKK	Historical
5	Assets:	1	ALL		2	Moderate	USD	CURRENCY	Expected
6	Currency:	1	SKK		3	Balanced	EUR		
7	Returns:	1	Historical		4	Growth			
8					5	Aggressive			
9	**Styles parameters**								
10							**Expected calculations**		
11	**Upper bounds on asset classes**						Sheet	**Optimize**	
12							Procedure	**CTRL+SHIFT+X**	
13	Style	Money market	Bonds	Equities					
14	Conservative	0.3	0.7	0					
15	Moderate	0.2	0.65	0.15					
16	Balanced	0.15	0.4	0.45					
17	Growth	0.1	0.3	0.6					
18	Aggressive	0.05	0.2	0.75					
19									
20	**Upper bounds on assets**								
21		SBWMUD3U	SBWMEU3L	BBOR6M	SDX-S	SDX-P	SBUSL	SBEGEU	JPEGCOMP
22	Style	SBWMUD3L	SBWMEU3L	BBOR6M	SDX-S	SDX-P	USD GOV	EU GOV	EM GLOB USD
23	Conservative	0.30	0.30	0.30	0.70	0	0.7	0.7	0
24	Moderate	0.20	0.20	0.20	0.65	0.4	0.65	0.65	0
25	Balanced	0.15	0.15	0.15	0.40	0.3	0.4	0.4	0.3
26	Growth	0.10	0.10	0.10	0.30	0.2	0.3	0.3	0.2
27	Aggressive	0.05	0.05	0.05	0.00	0.2	0	0	0.2
28									
29	**Expected Returns**								
30		SBWMUD3L	SBWMEU3L	BBOR6M	SDX-S	SDX-P	USD GOV	EU GOV	EM GLOB USD
31	Annual	9.43%	6.05%	12.52%	12.34%	13.81%	5.75%	3.93%	15.27%
32	Monthly	0.75%	0.49%	0.99%	0.97%	1.08%	0.47%	0.32%	1.19%
33									

Style_setup / Optimize / Style / Graph / EF-normlized / Graph-norm / Statistics / Returns

Figure 7

Efficient Frontier of the Investment Style

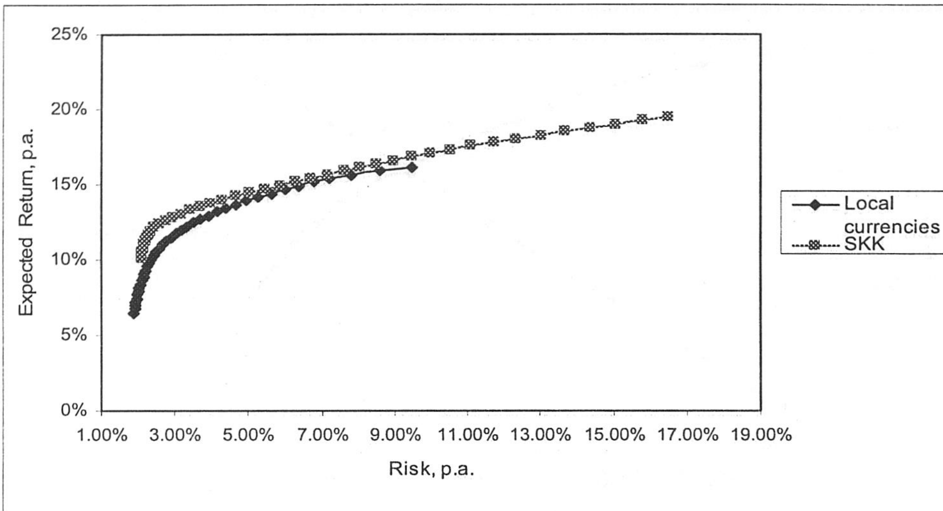


Table 6a

Selected Efficient Portfolios and their Characteristics (with Returns in SKK)

Style	Moderate	Currency	SKK	Portfolio		
Assets	ALL	Returns	Historical	GMV (in %)	P–S Balanced (in %)	Maximum Return (in %)
Expected return, p.a.				10.09	16.60	19.49
Standard deviation, p.a.				2.12	8.98	16.51
Minimum return (5% significancy level)				6.61	1.83	–7.67
Probability of nonpositive return				0.00	3.22	11.89
Probability for return less than 10%				48.22	23.11	28.27
Probability for return more than 20%				0.00	35.25	48.78
Index of profitability (P)				0.00	70.21	100.00
Index of safety (S)				100.00	71.61	0.00
Money Market				20.00	20.00	20.00
			SBWMUD3L	0.00	0.00	0.00
			SBWMEU3L	0.00	0.00	0.00
			BBOR6M	20.00	20.00	20.00
Bonds				65.00	65.00	65.00
			SDX–S	28.95	0.00	0.00
			SDX–P	20.88	2.02	0.00
			USD GOV	0.00	0.00	0.00
			EU GOV	15.17	0.00	0.00
			EM GLOB USD	0.00	0.00	0.00
			EM GLOB EUR	0.00	0.00	0.00
			EM EE USD	0.00	18.86	50.00
			EM EE EUR	0.00	44.12	15.00
Equities				15.00	15.00	15.00
			TPXC30	0.99	0.00	0.00
			INDU	12.49	15.00	0.00
			SX5P	0.15	0.00	15.00
			SKSM	1.37	0.00	0.00

Figure 8
Normalized Version of the Efficient Frontier

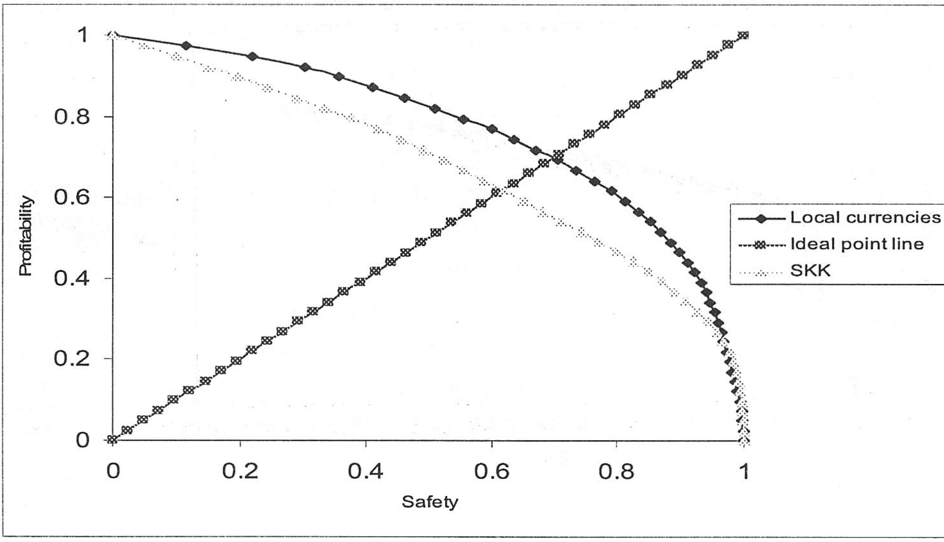


Table 6b
Selected Efficient Portfolios and their Characteristics (with Returns in Local Currencies)

Style	Moderate	Currency	SKK	Portfolio		
Assets	**ALL**	**Returns**	**Historical**	GMV (in %)	P–S Balanced (in %)	Maximum Return (in %)
Expected return, p.a				6.49	13.93	16.18
Standard deviation, p.a				1.90	4.94	9.48
Minimum return (5% significancy level)				3.37	5.81	0.59
Probability of nonpositive return				0.03	0.24	4.39
Probability for return less than 10%				96.73	21.29	25.73
Probability for return more than 20%				0.00	10.94	34.34
Index of profitability (P)				0.00	77.36	100.00
Index of safety (S)				100.00	75.95	0.00
Money Market				20.00	20.00	20.00
			SBWMUD3L	0.00	0.00	0.00
			SBWMEU3L	0.00	0.00	0.00
			BBOR6M	20.00	20.00	20.00
Bonds				65.00	65.00	65.00
			SDX–S	7.66	0.00	0.00
			SDX–P	10.79	39.31	1.25
			USD GOV	0.00	0.00	0.00
			EU GOV	46.55	0.00	0.00
			EM GLOB USD	0.00	0.00	0.00
			EM GLOB EUR	0.00	0.00	0.00
			EM EE USD	0.00	0.00	13.75
			EM EE EUR	0.00	25.69	50.00
Equities				15.00	15.00	15.00
			TPXC30	5.31	0.00	0.00
			INDU	4.70	7.13	0.00
			SX5P	0.02	7.87	15.00
			SKSM	4.96	0.00	0.00

4. Conclusions and Suggestions for Further Research

The first part of the paper extends an information about history of capital markets of selected countries published in Dimson, Marsh, Staunton (2001) as a result of modern portfolio theory applications. From the covariances between returns or excess returns, structures of portfolios with global minimum variance, structures of market portfolios, CML – slopes and corresponding betas of risky assets and on the base of analysis of mutual locations of efficient frontiers and capital market lines one can derive conclusions about an existence of dominance relations among analyzed capital markets, in general, or for specified ranges of risk and expected returns. The main goal of the paper is to provide and present to potential investor a methodology for modelling efficient frontiers and selection benchmarks – efficient portfolios with a special property (in our case it is in so called profitability – safety space) in well known Excel environment. It means that presented numerical applications should be viewed, more or less, as an illustration although real data were used.

The developed tools rely on Markowitz's portfolio selection model which is still very popular in the practice of fund management. However in the recent years one can observe radical changes in the investment environment. There exist various financial instruments with non-symmetric return distribution such as options and bonds. In addition recent statistical studies revealed that not all common stocks follow normal distribution, which was usually considered to be a valid assumption for these kinds of stocks. It follows that one can not fully rely on MV model and should look for other risk measures than variance and the methodology should be extended to capture such lower partial risk measures as lower semi-variance (lower semi-standard deviation), lower semi-absolute deviation, below target risk and conditional value at risk are presented.

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