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MODELLING OF GEORELIEF USING DTM - THE INFLUENCE OF POINT CONFIGURATION OF INPUT POINTS FIELD ON POSITIONAL AND NUMERIC ACCURACY

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The accuracy of modelling of georelief and its geometric structure using DTM depends on the properties of input discrete points field of altitudes (DPFA) and on the properties of approximing functions used in DTM. The subject of this paper are the properties of input discrete points field of altitudes and their influence on the positional and numeric accuracies of modelling of individual georelief parameters from viewpoint of their interdisciplinary applications. The input discrete points field must fullfil two basic conditions: the condition of repre- sentativeness and the condition of correct mutual configuration of points of discrete points field from which the primary triangle net is derived. The value of positional shift of calculated data if the condition of correct configuration of points of input discrete point fields of altitude is not fulfilled is also presented.

Key words: set of morphometric georelief parameters, georelief geometric structure, representativeness, configuration, positional accuracy, numeric accuracy, primary triangels net, altitudes, slope of georelief, normal curvature, vertical curvature, horizontal curvature of georelief, normal forms, horizontal forms of georelief, total forms of georelief

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1. THE PROBLEM OUTLINE

The introduction remark: Due to briefness in the text the topographic or terrain surface of georelief will be signed as **TSG**, the discrete points field of altitudes **DPFA**, the primary triangel net **PTN** and digital terrain model **DTM**.

Georelief is specific subsystem of landscape sphere that considerable influences the spatial differentiation of individual landscape components, the spatial differentiation of geoecological processes in landscape as well as landscape as entirety. However, it also considerable influences the individual spheres of human activities in space mainly in agricultural and forest spheres as well as the ones of transportation, industrial and settlement.

Therefore georelief from various wievpoints is not only the subject of study and modelling in many scientific disciplines but in the projection and technical praxes including military and management praxes as well. Due to this various digital georelief models (**D**igital **T**errain **M**odels - DTM) were developed that model georelief and its geometric structure mainly without time parameter in various dimensions and on various levels of spatial accuracy.

It means that georelief is modelled on the basis of geodetically, cartometrically or photogrametrically measured input data by the approximing functions of two variables without time parameter as static spatially differentiated system in the selected scale 1 : M_i and in the adequate distinctive level U_i from what the time interval of spatial actuality of input as well as calculated output data is derived.

Using DTM the resulting accuracy of georelief and its geometric structure modelling depends on:

- properties of input discrete points field of altitudes

- properties of approximing functions used in DTM.

The various different results during georelief modelling using DTM can be got when the selected region of georelief is modelled using DTM as follows:

1. from the same input representative discrete points field of altitudes (DPFA), however using various approximing functions (the differences in results are caused by the different properties of the used individual approximing functions),

2. using the same approximing function used in DTM however from various input representative discrete points field of altitudes (DPFA), while in this case the differences are caused by the different properties of individual input discrete points fields despite the fact that all of them fullfill the conditions of representativeness.

In the contribution the problem (2) of positional accuracy of georelief modelling and the set of its morphometric quantities from wievpoint of the properties of input representative discrete points field of altitudes (DPFA) is discussed. It is showed that the input representative discrete points field of altitudes (DPFA) must fullfill two basic conditions:

2a. the condition of representativeness

2b. the condition of correct configuration of points of discrete points field of altitudes (DPFA) from which the primary triangel net (PTN) is derived.

So, the subject of the contribution are the properties of input data and their influence on the calculation of numeric and positional accuraces of calculated data that in our case is the set of morphometric quantities of georelief that characterizes georelief geometric structure.

In the following parts of contribution in the sense of introduction remark we will use the abbreviations for the most frequent terms TSG, DPFA, PTN and DTM.

The total problem is outlined in the contributions of Kalak and Krcho (1983), Krcho (1964a, 1964b, 1973, 1975, 1977, 1990, 1992, 1993) where the problem was systematically solved, beginning by the mathematic formulation of the morphometric quantities set even with physical significance and ending by the modelling of total georelief geometric structure using the digital models. The part of solved problem was the problem of properties of input DPFA and its PTN in relation to the geometric structure of TSG and so to the positional and numeric accuraces of TSG modelling using DTM. This is important from wievpoint of interdisciplinary DTM applications in many scientific disciplines including civil and military practice. It has the significance during the modelling of dynamics and spatial differentiation of erosion-denudation and transport processes on georelief and on TSG resp. In relation to this we shall demonstrate that there exist considerable relations between spatial points distribution of input DPFA and its PTN and the geometric structure of TSG.

The resulting accuracy of modelling in the sense of the above mentioned depends on the properties of input set DPFN and its PTN and on the properties of approximing functions used in DTM. According to the density of points and their distribution the set of points DPFA must fullfill the criteria of representativeness and mutual configuration influencing PTN triangels.

The mentioned problem in relation to the contributions Krcho (1973 to 1991) is briefly documented from the selected modelled region of Ružiná at Lučenec (Inner Western Carpathians) in the form of computer outputs.

2. GEORELIEF AND ITS TSG - GEOMETRIC STRUCTURE OF TSG AND MODELLED SET OF MORPHOMETRIC QUANTITIES AS GEORELIEF PARAMETERS

We consider TPG in the Carthesian coordinates system < 0, x, y, z > as smooth surface described by the function in general form

$$z = f(x, y), \text{ resp. } z = z(x, y), \tag{1}$$

that is formed by the set of points

$$E^{g}_{RF} = \left\{ A^{g}_{i}(x_i, y_i, z_i) \right\}_{i \in I}, \tag{2}$$

where I is the index set and i is appropriatelly selected identification mark for ordered triple x_i, y_i, z_i . Simultaneously, the function (1) is the function of continuous scalar field of altitudes formed in the scalar basis (x, y) by the set

$$E_{RF} = \left\{ A_i\,(x_i, y_i), z_i \right\}_{i \in I}, \tag{3}$$

where I is the index set and i is appropriatelly selected identification mark for ordered couple (x_i, y_i) and coordinated scalar z_i of altitude. In the scalar basis the set (3) corresponds with the set (2) on TSG.

Note 1: In the set (2) z_i means the coordinate expressed in linear measures while in the set (3) it means the scalar; the scalar z_i in each point A_i (x_i, y_i) of scalar basis is converted to the coordinate so that the unit of length u_i is coordinated to the scalar unit z_i , so the coordinate $z_i = u_i \, . \, z_i$ and it is vertical to the plane (x, y). If the end points of coordinates z_i are signed as points A_i^g $(x_i, y_i, z_i) \in E_{RF}^g$ (2) then the points form the smooth TSG in Carthesian system.

About the function (1) that analytical form of which is not known we suppose that it is continually differentiable at least up to second order, so it has at least up to second order the continual and partial derivations expressed briefly in the form z_x, z_y, z_{xx}, z_{xy}, z_{yy} .

Even if the analytic form of the function (1) is not known its required properties are essentially important so that the approximing functions $z = P_i\,(x, y)$ $(i = 1, 2, ...)$ must be adequate and they substitute in DTM the function (1) where TSG and its geometric structure is modelled on the basis of input DPFA and using DTM.

The geometric structure of TSG is characterized by the set of morphometric quantities

$$G_{RF} = \{z, \Delta z, s_n, \gamma_N, A_N, (K_N)_n \equiv \omega\,, (K_N)_t\,, (K_V)_t, K_r, N_nF, N_tF\, K_rF, F, ...\} \quad (4)$$

that are considered as morphometric parameters of georelief while

z - **altitudes** expressed in < 0, x, y, z > as the basic quantity that carries the information about TSG and it is considered the basic parameter in many quantitative relations not only in geography but in other scientific disciplines as well,

Δz - **relative heights considered in the direction of slope curves (of orthogonal curves to contour lines)** - are the basic parameters for calculation of mathematic - physical understood heights configuration of georelief in the gravitation field of the Earth,

s_n - **the slope length of georelief in the direction of slope curves (of orthogonal curves to contour lines)** according the mark $\pm\,(K_N)_n$ divided into

$(s_n)_X$ - the length of slope on convex forms N_nF_X in the direction of orthogonal curves to contour lines where $(K_N)_n \equiv \omega > 0$,

$(s_n)_K$ - the length of slope on concave forms N_nFK in the direction of orthogonal curves to contour lines where $(K_N)_n \equiv \omega < 0$,

that are important parameters for calculation of spatial distribution of erosion - denudation and transport energy of modelling processes on georelief.

From the set G_{RF} (4) we present the quantities that directly influence the distribution of input DPFA points and the formation of triangels PTN. They are as follows:

γ_N - **the slope of TSG in the direction of slope curves (of orthogonal curves to contour lines)** from $|grad\ z|$ expressed by the relation

$$\gamma_N = \operatorname{arc}tg(\,|grad\ z|) = \sqrt{z_x^2 + z_y^2}\ , \quad (5)$$

graphically in the form of isolines field in modelled region of Ružiná expressed in the Fig. 1a,

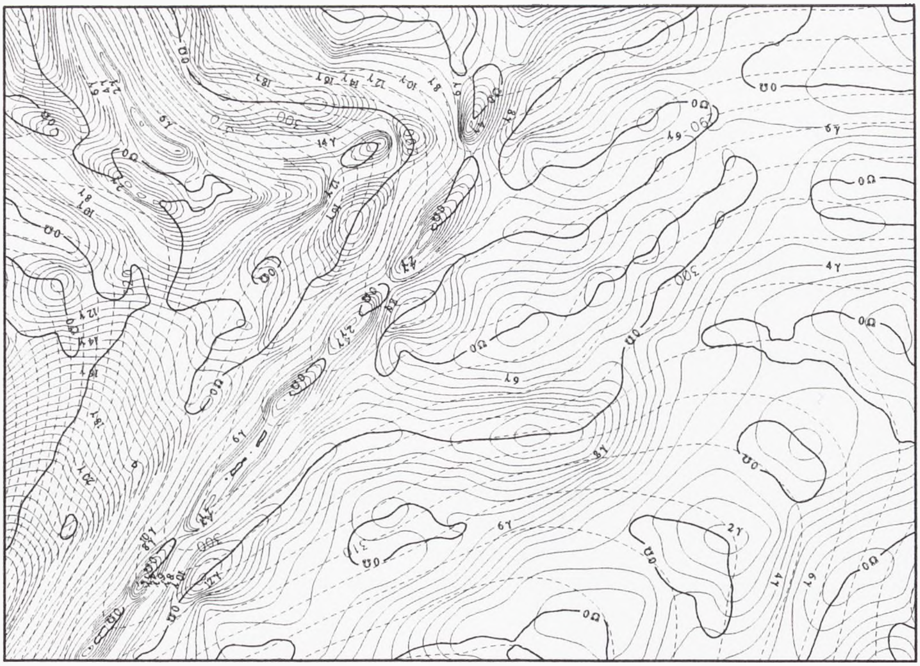

Fig. 1a. γ_N and $\Omega = 0$. Isoline field of slope γ_N of georelief in the direction of orthogonal curves to contour lines.

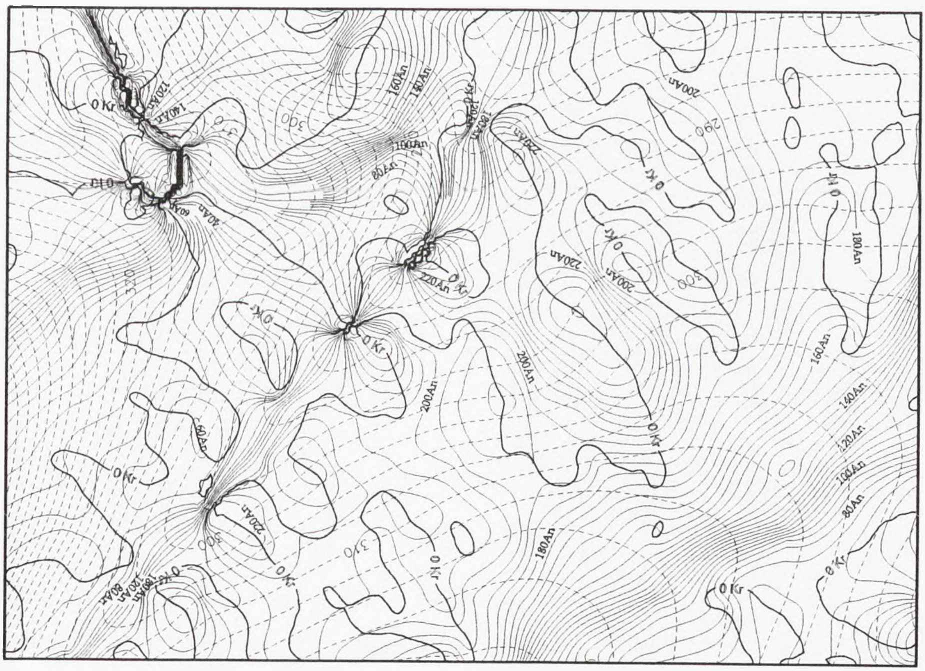

Fig. 1b. A_N and $K_r = 0$. Isoline field of the orientation of georelief against cardinal points.

A_N - **the orientation of TSG against cardinal points** determined by the vector -*grad z* and expressed from the coordinates of its unit vector $-\vec{n}^{0}$ so

$$A_N = \arccos\left(\frac{-z_x}{\sqrt{z_x^2 + z_y^2}}\right) = \arcsin\left(\frac{-z_y}{\sqrt{z_x^2 + z_y^2}}\right), \tag{6}$$

that is in the form of isolines field expressed at the Fig. 1b,

$(K_N)n$ **- the normal curvature of TSG in the direction of slope curves (of orthogonal curves to contour lines)** expressed by the relation

$$(K_N)_n \equiv \omega = -\frac{z_{xx}z_x^2 + 2z_{xy}z_x z_y + z_{yy}z_y^2}{(z_x^2 + z_y^2)\sqrt{(1 + z_x^2 + z_y^2)^3}}, \tag{7}$$

where $(K_N)_n \equiv \omega$ acquires the values $(K_N)_n > 0$, $(K_N)_n = 0$, $(K_N)_n < 0$ that quantitatively characterize the normal georelief forms N_nF in `the direction of orthogonal curves to contour lines; the radius of normal curvature $(R_N)_n = 1/(K_N)_n \equiv 1/\omega$ lies in the normal *N* to the TSG; the normal curvature $(K_N)_n \equiv \omega$ is in the form of isolines field from modelled region of Ružiná expressed at the Fig. 2,

Fig. 2. Ω Normal curvature of georelief in the direction of slope curves (of orthogonal curves to contour lines). Isoline field of the normal curvature of georelief in the direction of slope curves.

***$(K_N)_t$* - the normal curvature of TSG in the direction of tangents to contour lines** expressed by the relation

$$(K_N)_t = -\frac{z_{xx}z_y^2 - 2z_{xy}z_x z_y + z_{yy}z_x^2}{(z_x^2 + z_y^2)\sqrt{(1 + z_x^2 + z_y^2)^3}}, \tag{8}$$

where $(K_N)_t$ acquires the values $(K_N)_t > 0$, $(K_N)_t = 0$, $(K_N)_t < 0$,
the radius of normal curvature $(R_N)_t = 1/(K_N)_t$ lies in the normal N to the TSG similarly as the $(R_N)_n$, however $(R_N)t \neq (R_N)_n$,

***$(K_V)_t$* - the vertical curvature of TSG in the direction of tangents to contour lines** expressed by the relation

$$(K_V)_t = -\frac{z_{xx}z_y^2 - 2z_{xy}z_x z_y + z_{yy}z_x^2}{(z_x^2 + z_y^2)(z_x^2 + z_y^2 + 1)}, \tag{8'}$$

where $(K_V)_t > 0$, $(K_V)_t = 0$, $(K_V)_t < 0$, while the relation between $(K_V)_t$ and $(K_N)_t$ has the form $(K_V)_t = (K_N)_t / \cos \gamma_N$; $(K_N)_t = (K_V)_t . \cos \gamma_N$, where

$$\cos \gamma_N = \frac{1}{\sqrt{z_x^2 + z_y^2 + 1}}, \tag{8''}$$

the radius of vertical curvature $RV = 1/(K_V)_t$ lies in the vertical perpendicular to the plane (x, y) and so parallel to the axis of Carthesian coordinates system $< O, x, y, z >$,

***K_r* - the horizontal curvature of TSG** determined by the relation

$$K_r = -\frac{z_{xx}z_y^2 - 2z_{xy}z_x z_y + z_{yy}z_x^2}{\sqrt{(z_x^2 + z_y^2)^3}}, \tag{9}$$

where $K_r > 0$, $K_r = 0$, $K_r < 0$, while the relation between Kr and $(K_N)_t$ (8) has the form $K_r = (K_N)_t / \sin \gamma_N$; $(K_N)_t = K_r . \sin \gamma_N$, where

$$\sin \gamma_N = \frac{\sqrt{z_x^2 + z_y^2}}{\sqrt{1 + z_x^2 + z_y^2}}, \tag{9'}$$

the radius of horizontal curvature $(R)_{Kr} = 1/ K_r$ lies in the plane of the contour line; the horizontal forms of georelief K_rF are quantitatively characterized by the horizontal curvature K_r (9); the horizontal curvature K_r in the form of isolines fields is expressed in graphs in the Fig. 3;

***N_nF* - the normal forms of TSG in the direction of slope curves (of orthogonal curves to contour lines)** quantitatively characterized by the value $(K_N)_n \equiv \omega$ (7) and according the mark $\pm$ $(K_N)_n$ they are divided in:

N_nF_X - the convex normal forms in the direction of orthogonal curves to contour lines where $(K_N)_n \equiv \omega > 0$,

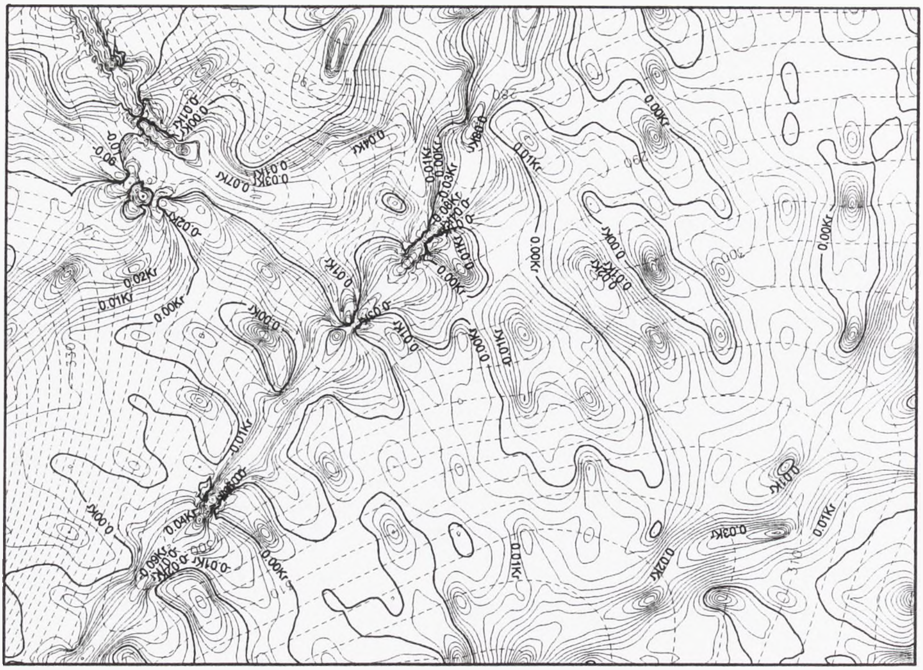

Fig. 3. K_r Isoline field of the horizontal curvature of georelief.

N_nF_K - the concave normal forms in the direction of orthogonal curves to contour lines where $(KN)n \equiv \omega < 0$,
while N_nF_X and N_nF_K are separated by the isoline $(K_N)_n \equiv \omega = 0$ in the equation

$$z_{xx}z_x^2 + 2z_{xy}z_xz_y + z_{yy}z_y^2 = 0, \qquad (10)$$

N_nF from mentioned modelled region of Ružiná are expressed in graphs at the Fig. 4.

***N_tF* - the normal forms of georelief in the direction of tangents to contour lines** quantitatively characterized by the value $(K_N)_t$ (8) and according the mark $\pm\ (K_N)_t$ internally divided into
N_tF_X - the convex normal forms in the direction of tangents to contour lines where $(K_N)_t > 0$,
N_tF_K - the concave normal forms in the direction of tangents to contour lines where $(K_N)_t < 0$,
while N_tF_X and N_tF_K are separated by the isoline $(K_N)_t = 0$ in the equation

$$z_{xx}z_y^2 + 2z_{xy}z_xz_y + z_{yy}z_x^2 = 0, \qquad (11)$$

K_rF* - *horizontal forms of TSG quantitatively characterized by the value K_r (9) and internally divided into
K_rF_X - the convex horizontall forms where $Kr > 0$ and so $(K_N)_t = K_r \sin \gamma_N > 0$
K_rF_K - the concave horizontal forms where $Kr < 0$, and so $(K_N)_t = K_r \sin \gamma_N > 0$

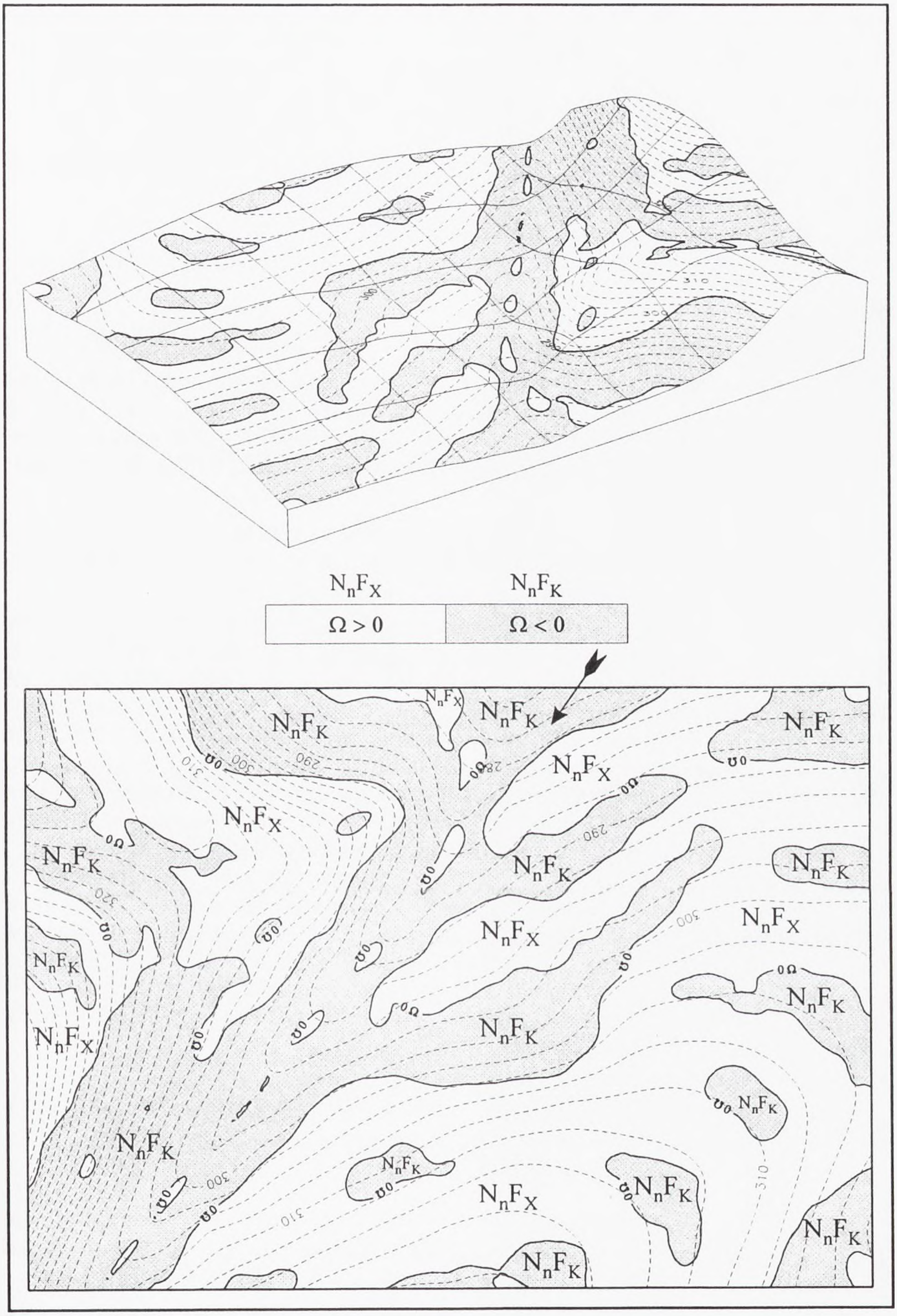

Fig. 4. Normal forms of georelief in the direction of slope curves (of orthogonal curves to contour lines).

while K_rF_X and K_rF_K are separated by the isoline $K_r = 0$ in the equation that is identical with the equation $(K_N)_t = 0$ (10) and therefore the forms N_tF [N_tF_X, N_tF_K] and the forms K_rF [K_rF_X, K_rF_K] are spatially identical, however according to the relations (9) internally quantitatively differentiated; the horizontal forms KrF are expressed in the Fig. 5.

F* - *the total forms of TSG quantitatively characterized $(K_N)_n$, K_r internally divided into

F_{XX} - convex - convex forms where is valid that $[(K_N)_n \equiv \omega > 0, K_r > 0]$

F_{KX} - concave - convex forms where is valid that $[(K_N)_n \equiv \omega < 0, K_r > 0]$

F_{KK} - concave - concave forms where is valid that $[(K_N)_n \equiv \omega < 0, K_r < 0]$

F_{XK} - convex - concave forms where is valid that $[(K_N)_n \equiv \omega > 0, K_r < 0]$.

The total geometric forms F [F_{XX}, F_{KX}, F_{KK}, F_{XK}] are separated by the isolines $(K_N)_n \equiv \omega = 0$ (10), as well as the isolines $(K_N)_t = 0 \equiv K_r = 0$ (11); see Fig. 6. They have essential interdisciplinary significance; from viewpoint of the subject of contribution they considerable determine the localization of points DPFA on TSG and the formation of triangel nets.

The note 2. The normal curvature $(K_N)_n$ is expressed also with the symbol ω due to the original contribution Krcho (1973) where it was derived and cartographically expressed on the coloured isoline map of normal georelief curvature.

The note 3. The set G_{RF} (4) in relation to the contribution Krcho (1964a, 1964b) was formulated in detail and derived in the contribution Krcho (1973) and later analyzed in detail in the contributions Krcho (1983a, b, 1986, 1990, 1991, 1992, 1993). Similarly, the problem of morphometric analysis of georelief was solved in the contributions Evans (1972). Both contributions i. e. Evans (1972), Krcho (1973) were issued independently, moreoves the contribution Krcho (1973) was issued in 1973. However, the contribution was sent to editorial in 1970, the issue was prepared in 1972. Unfortunatelly, due to the mathematic text, 10 coloured maps and 10 coloured charts and graphs supplemented, the preparation for issue took one year, so the contribution was published in 1973. However, we can state that the problem of morphometric analysis of georelief on the basis of geometric aspect of fields theories in connection to the contribution Šalamon (1961, 1963) was sketched and cartographically realized in the contributions Krcho (1964a, 1964b). In the contribution Krcho (1964b) the approach was documented on two coloured maps of the scale 1:5000 Košice - sever, Košice - juh and on the coloured Map of slope gradients in Košická kotlina (basin) in the scale 1:50 000.

3. THE REPRESENTATIVE INPUT SET OF POINTS DPFA FORMING THE SET ${}_{D}E_{RF}$ AND ITS ADEQUATE SET ${}_{D}E_{RF}$ AS THE BASIS FOR TSG MODELLING WITH USING DTM

In the sense of contributions Krcho (1973, 1986, 1990, 1991, 1992) the set of morphometric georelief quantities G_{RF} (4) characterizes the geometric georelief structure.

Significant relations between geometric structure of TSG and the input points set DPFA and the formed PTN are manifested by the means of morphometric quantities $(K_N)_n$ (7), $(K_N)_t$ (8), K_r (9) and defined forms F_{XX}, F_{KX}, F_{KK}, F_{XK}. Therefore, the

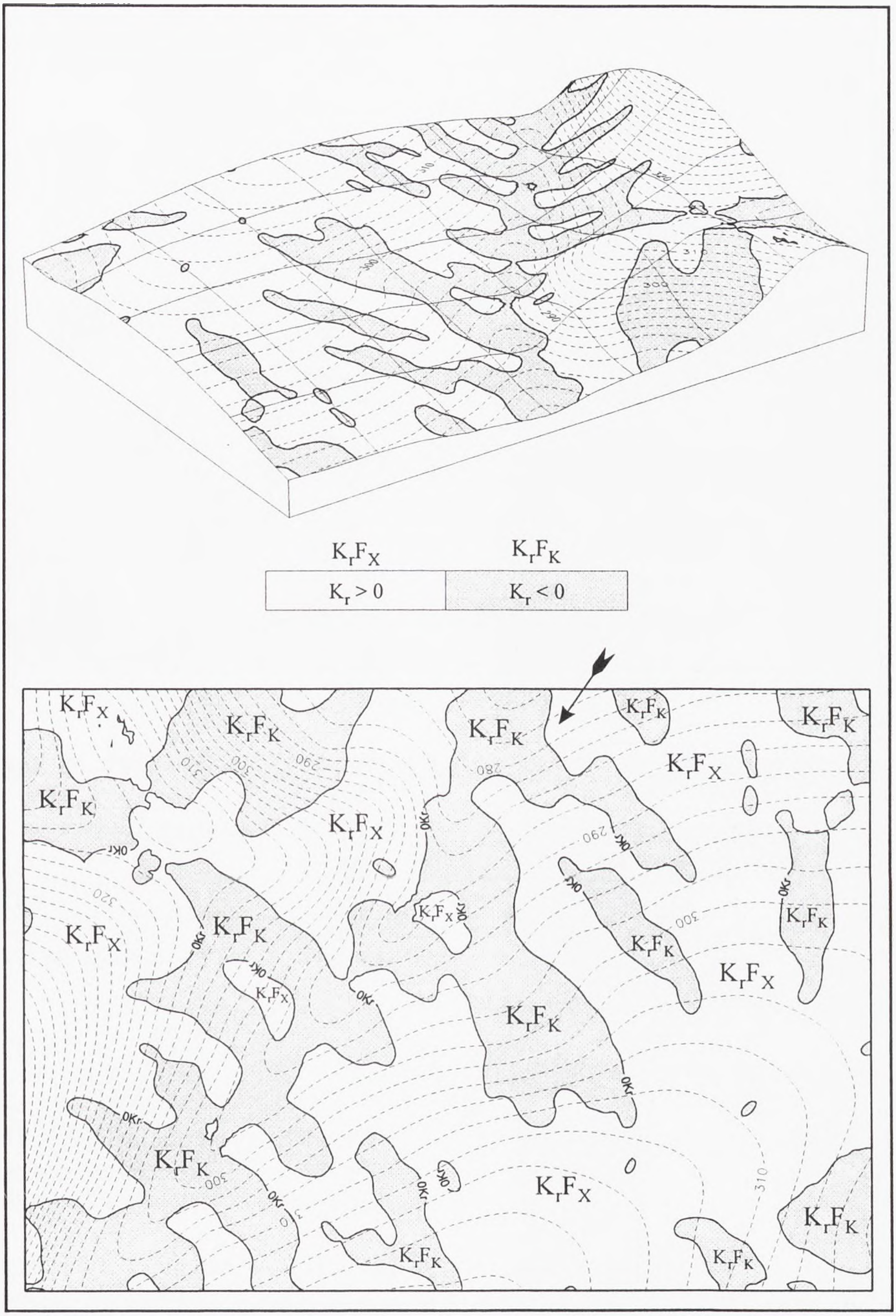

Fig. 5. Horizontal forms of georelief.

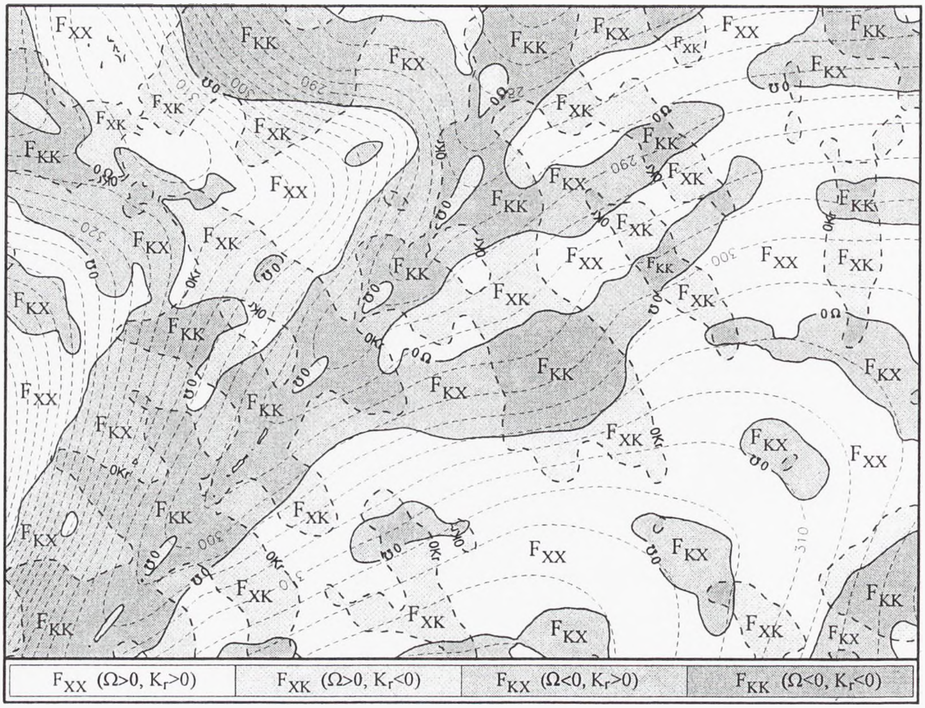

Fig. 6. Total geometric forms of georelief.

criteria for localization and configuration of points DPFA and the criteria for triangels PTN formation are based on.

The representative set of measured input points for DTM forms on TSG the set

$$ {}_D E^g_{RF} = \left[A^g_j \, (x_j, y_j, z_j) \right]^n_{j=1} , \qquad (12)$$

that in the plane (x, y) of Carthesian coordinate system $<O, x, y, z>$ as scalar basis of altitudes field corresponds the set

$$ {}_D E_{RF} = \left[A_j \, (x_j, y_j, z_j) \right]^n_{j=1} , \qquad (13)$$

where the scalar of altitude z_j is unambiguously coordinated to each point A_j (x_j, y_j). The points A_j (x_j, y_j), $z_j \in {}_D E_{RF}$ form in the scalar basis the representative DPFA. The points A_j (x_j, y_j, z_j) are nodal points of triangels PTN on TSG and their adequate points A_j (x_j, y_j), z_j in the scalar basis are the nodal points of triangels PTN in the scalar basis.

Let us suppose for theoretical and methodological reasons that the coordinates x_j, y_j, z_j points $A^g_j \in {}_D E^g_{RF}$ (12) are measured theoretically exactly, and they are appropriate to the function (1).

Then for ${}_{D}E_{RF}$ (12) is valid that

$$ {}_{D}E^{g}_{RF} = \left[A^{g}_{j}\,(x_j,y_j,z_j)\right]^{n}_{j=1} \subset E^{g}_{RF} = \left\{A^{g}_{i}\,(x_i,y_i,z_i)\right\}_{i\in I}. \tag{14}$$

For the adequate set ${}_{D}E_{RF}$ (13) is valid that

$$ {}_{D}E_{RF} = [A_j\,(x_j,y_j),z_j]^{n}_{j=1} \subset E_{RF} = \{A_i\,(x_i,y_i),z_i\}_{i\in I}, \tag{15}$$

where I is the index set and i is appropriatelly selected identification mark for ordered couple (x_j, y_j) and the coordinated scalar, z_j of the altitude, z_j.

From viewpoint of representativeness of spatial distribution of input points $A^{g}_{j} \in {}_{D}E^{g}_{RF}$ on TSG in the considered scale 1 : M_j and in its distinctive level U_j the relation between the density of the points and the geometric structure TSG expressed by the set (4) are important. The normal and horizontal curvatures $(K_N)_n \equiv \omega$, $(K_N)_t$, K_r as well as the average vertical curvature TSG between two arbitrary neighbouring points DPFA that is expressed by the symbol $(K_V)_{pr}$ play considerable role. We can briefly state that the point density of input DPFA depends on the value of TSG curvature so that it is statistically directly proportional. Because the points $A^{g}_{j} \in {}_{D}E^{g}_{RF}$ on TSG and their adequate points $A_j \in {}_{D}E_{RF}$ in the scalar basis are the terminal (nodal) points PTN, the length of sides and the size of triangels PTN will be statistically indirectly proportional to the density of points DPFA. It means that the length of triangel sides in PTN is in statistical relation with georelief curvature what is expressed in graphs at the Fig. 7a. The vertical curvature K_V (8) that is connected

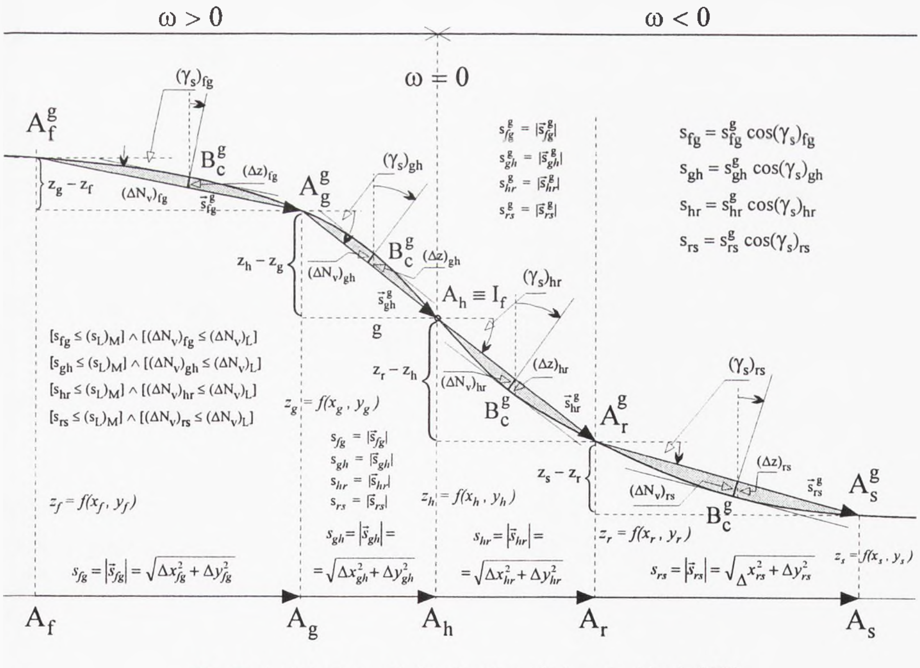

Fig. 7a. Profile of georelief in direction of triangels sides in PTN.

with the normal curvature $(K_N)_l$ in the direction of tangents to contour lines (8) is also considered.

Fig. 7a presents that if in the selected scale 1 : M$_i$ from viewpoint of its distinctive level U$_i$ and as a part of representativeness criterion **the admissible distance** $(\Delta N_V)_L$ is determined between the tangent point $B_c(x_c, y_c, z_c)_{ii}$ of arbitrary vertical section in the direction of adequate side s_{ii} of triangel and the side s_{ii} then will be valid:

- the bigger the vertical curvature $(K_V)_{ii}$ of georelief in its vertical section leading in the direction of each side s_{ii} of arbitrary triangel of triangels network, shorter side s_{ii} the under assumption that the condition $(\Delta N_V)_{ii} \leq (\Delta N_V)_L$ is preserved.

The couple of indexes $ii = fg, gh, hr, rs$ expresses the couples of consecutive numbers of individual points A_f^g, A_g^g A_h^g A_r^g A_s^g of triangels of triangel network determining the individual sides s_{ii} in the Fig. 7a as well as it identifies the individual vertical sections passing through the sides. Fig. 7a also indirectly seggests that on the vertical profile the distance $(\Delta N_V)_{ii}$ of the point $B_c(x_c, y_c, z_c)_{ii}$ from its side s_{ii} must not exceed in the considered scale 1 : M$_i$ "the limiting" value $(\Delta N_V)_L$ determined from viewpoint of its distinctive level U$_i$ as the part of representativeness criterion. It is expressed in detail for one triangel of triangel network in the Fig. 7b.

Due to the fact that the problem is not the subject of the contribution we shall be brief.

About the input sets (12) and (13) resp. we suppose that they are representative from viewpoint of considered scale 1 : M and their distinctive levels. **The starting criterion of representativeness** of distribution of input sets ${}_DE_{RF}^g$ (12) and ${}_DE_{RF}$ (13) will be:

a. permissible length of sides of each triangel PTN where each side determined by the couple of terminal points A_r^g, $A_s^g \in {}_DE_{RF}^g$ $(r,s \in j = 1,2\ldots \text{while } r \neq s)$ on TSG is expressed by the positional vector

$$\vec{s}_{rs}^{\,g} = \Delta x_{rs}\,\vec{i} + \Delta y_{rs}\,\vec{j} + \Delta z_{rs}\,\vec{k}\ , \qquad (16)$$

what in the scalar basis (x, y) corresponds to the positional vector

$$\vec{s}_{rs} = \Delta x_{rs}\,\vec{i} + \Delta y_{rs}\,\vec{j} + 0\vec{k}\ , \qquad (16')$$

while $\Delta x_{rs} = x_s - x_r$, $\Delta y_{rs} = y_s - y_r$, $\Delta z_{rs} = z_s - z_r$; the length of side s_{rs}^g and s_{rs} are expressed by the absolute value of vectors (16), (16') i. e.

$$s_{rs}^g \equiv |\vec{s}_{rs}^{\,g}| = \sqrt{\Delta x_{rs}^2 + \Delta y_{rs}^2 + \Delta y_{rs}^2}\ , \qquad s_{rs} = |\vec{s}_{rs}| = \sqrt{\Delta x_{rs}^2 + \Delta y_{rs}^2}\ , \qquad (17)$$

what is expressed in graphs in the Fig. 7a and 7b.

b. the average vertical curvature $(K_V)_{pr}$ of the vertical cross-section passing through the points A_r^g, $A_s^g \in {}_DE_{RF}^g$, i. e. the average curvature of cross-section between the points A_r^g, $A_s^g \in {}_DE_{RF}^g$, on TSG and even the radius of average curvature $(R_V)_{pr} = 1/(K_V)_{pr}$ while

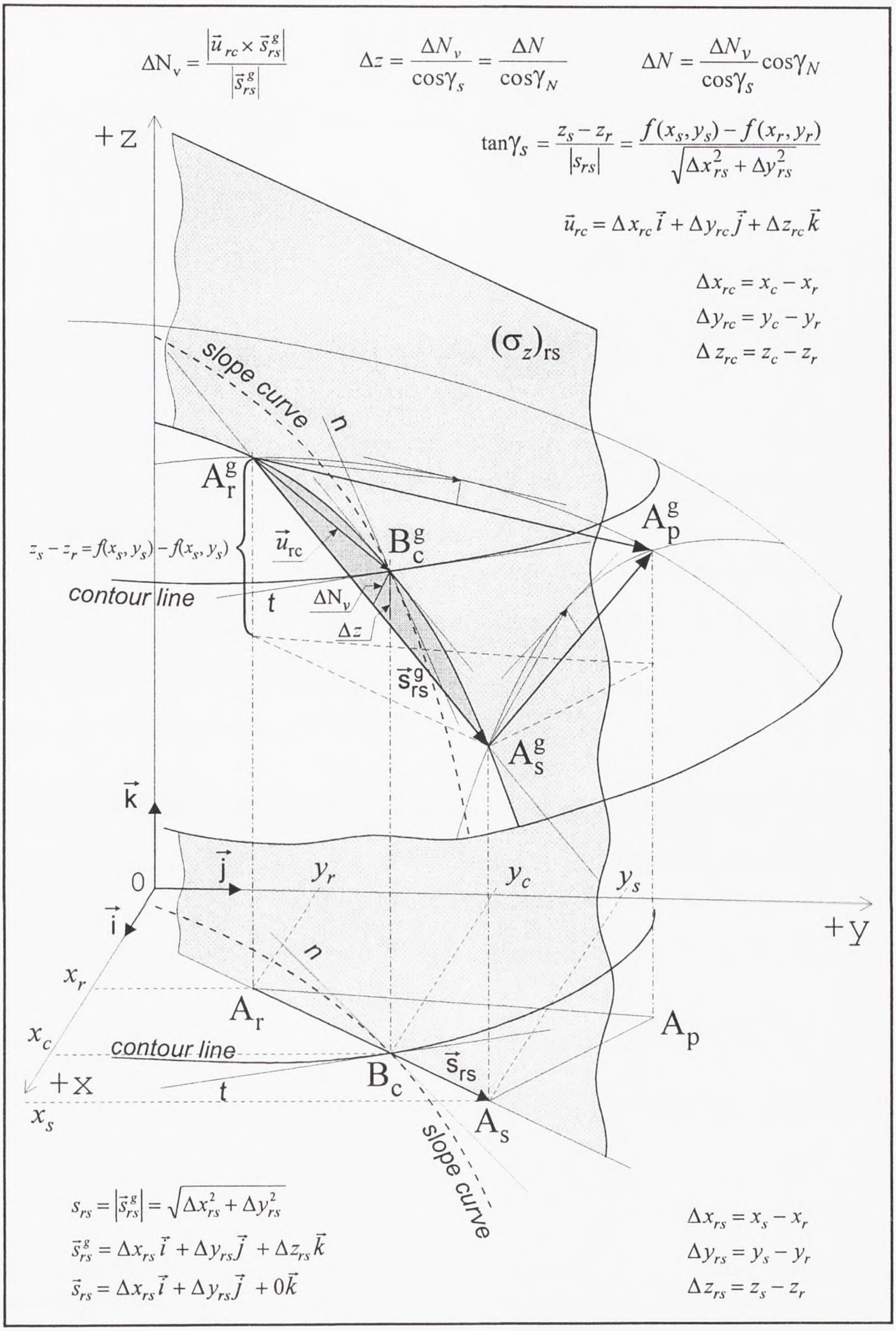

Fig. 7b. Determination of the representative length of the PTN triangle sides depending on vertical curvature K_v with $(\Delta N_V)_L$ limit value assessed.

$$(K_V)_{pr} = \frac{(\Delta\gamma_s)_{rs}}{(\Delta s)_{rs}} = \frac{(\Delta\gamma_s)_{rs}}{(R_V)_{pr}\Delta\gamma_{rs}} = \frac{arctg\left(\frac{dz}{ds}\right)_s - arctg\left(\frac{dz}{ds}\right)_r}{(R_V)_{pr}[\,arctg\left(\frac{dz}{ds}\right)_s - arctg\left(\frac{dz}{ds}\right)_r\,]}, \tag{18}$$

where $(\Delta\gamma_s)_{rs} = (\gamma_s) - (\gamma_s)_r$, while

$$(\gamma_s)_r = arctg\left(\frac{dz}{ds}\right)_r = arctg(z_x \cos\alpha_{rs} + z_y \sin\alpha_{rs})_r, \tag{19}$$

$$(\gamma_s)_s = arctg\left(\frac{dz}{ds}\right)_s = arctg(z_x \cos\alpha_{rs} + z_y \sin\alpha_{rs})_s, \tag{19'}$$

and $(\Delta s)_{rs}$ on the cross-section is the length of arc between two points $A_r^g, A_s^g \in {}_D E_{RF}^g$, on the circle with diameter of curvature $(R_V)_{pr}$.

c. the distance ΔN_V between the point $B_c^g(x_c, y_c, z_c)$ on TSG and the side s_{rs}^g of considered triangel on TSG determined by the relation

$$\Delta N_V = \frac{|\overrightarrow{u_{rc}} \times \overrightarrow{s_{rs}^g}\,|}{|\overrightarrow{s_{rs}^g}\,|}, \tag{20}$$

Fig. 7b where the terminal point $B_c^g(x_c, y_c, z_c)$ of the positional vector $\overrightarrow{u_{rc}}$ lies on the vertical cross - section between the points A_r^g, and A_s^g so $x_c \in (x_r, x_s)$, $y_c \in (y_r, y_s)$, $z_c \in (z_r, z_s)$, and due to this for the derivation $(dz, ds)_c$ in the point $B_c^g(x_c, y_c, z_c)$ in the direction of cross-section it is valid that

$$\left(\frac{dz}{ds}\right)_c = \frac{f(x_s, y_s) - f(x_r, y_r)}{\sqrt{\Delta x_{rs}^2 + \Delta y_{rs}^2}} = \frac{z_s - z_r}{\sqrt{\Delta x_{rs}^2 + \Delta y_{rs}^2}} = tg(\gamma_s)_c, \tag{21}$$

The positional vector

$$\overrightarrow{u_{rc}} = \Delta x_{rc}\,\vec{i} + \Delta y_{rc}\,\vec{j} + \Delta z_{rc}\,\vec{k}, \tag{22}$$

on the vertical cross-section is determined by the points $A_r^g(x_r, y_r, z_r)$, $B_c^g(x_c, y_c, z_c)$ the Fig. 7b, so

$$\Delta x_{rc} = x_c - x_r,\ \Delta y_{rc} = y_c - y_r,\ \Delta z_{rc} = z_c - z_r.$$

The variable quantity ΔN_V in the selected map scale 1 : M_i must not exceed the selected so called limiting distance $(\Delta N_V)_L$ valid for all input DPFA in modelled region as the constant. It means that $0 \le \Delta N_V \le (\Delta N_V)_L$. Between the distance ΔN_V of the point B_c^g on TSG and the side s_{rs}^g of considered triangle situated in vertical plane $(\sigma_z)_{rs}$ and the distance ΔN in the direction of normal N to TSG passing through the point B_c^g the relation $\Delta N = \dfrac{\Delta N_V}{\cos\gamma_s}\cos\gamma_N$ is valid.

The note 4. Under the word vertical cross - section we understand the intersection of vertical plane $(\sigma_z)_{rs} \perp (x, y)$ with TSG while the vertical plane $(\sigma_z)_{rs}$ is overloaded by the points A_r^g, A_s^g, so the points A_r^g, A_s^g and the point B_c^g situated on TSG also lie in vertical plane $(\sigma_z)_{rs}$; in vertical plane $(\sigma_z)_{rs}$ there lies the abscissa s_{rs}^g what can be seen in graphs of Fig. 7b.

Between the quantities s_{rs}^g, $(K_V)_{pr}$, $(R_V)_{pr}$, ΔN_V, $\Delta\gamma_{rs}$ are valid the relations as follows:

$$s_{rs}^g = \frac{2\sqrt{2(\Delta N_V)_L - (K_V)_{pr}(\Delta N_V)_L^2}}{\sqrt{(K_V)_{pr}}}; \qquad s_{rs}^g = 2\sqrt{2(R_V)_{pr}(\Delta N_V)_L - (\Delta N_V)_L^2}$$

$$(K_V)_{pr} = \frac{8(\Delta N_V)_L}{(s_{rs}^g)^2 + 4(\Delta N_V)_L^2}; \qquad (R_V)_{pr} = \frac{(s_{rs}^g)^2 + 4(\Delta N_V)_L^2}{8(\Delta N_V)_L}, \tag{23}$$

where the value of angle $(\Delta\gamma_s)_{rs} = (\gamma_s)_s - (\gamma_s)_r$ expressed from the relations (19), (19') in radians and due to the quantities $(\Delta N_V)_L$, $(K_V)_{pr}$, $(R_V)_{pr}$, is determined by the relation

$$(\Delta\gamma_s)_{rs} = 2\arccos[1 - (K_V)_{pr}(\Delta N_V)_L] ;$$

$$(\Delta\gamma_s)_{rs} = \arcsin\left(\sqrt{2(K_V)_{pr}(\Delta N_V)_L - (K_V)_{pr}^2(\Delta N_V)_L^2}\right). \tag{23'}$$

In the relations (23) and (23') in the variable quantity ΔN_V (20) admissible value is considered of $(\Delta N_V)_L$ that is selected for all DPFA as constant in considered scale 1 : M_i and due to its distinctive level U_i. Even the change of sides s_{rs}^g value depends on the change of average vertical curvature $(K_V)_{pr}$ and the selected value $(\Delta N_V)_L$. Therefore the quantity ΔN_V (20) and its determined limiting value $(\Delta N_V)_L$ is determinig quantity of essential significance for the lengths of sides s_{rs}^g of triangels PTN on TSG as well as for density of and their adequate points $A_j^g(x_j, y_j, z_j) \in {}_D E_{RF}^g$ because the longest admissible lengths of sides s_{rs}^g of triangels PTN hence the density of points DPFA will be in close relation with the vertical curvature $(K_V)_{pr}$.

If there, from the starting point A_r^g under gradual change of terminal point A_s^g in the selected direction σ_{rs} we gradually find the optimal length of triangel side with resulting terminal position of the point A_s^g, then in dependance of the changing length and average vertical curvature $(K_V)_{pr}$ even ΔN_V (20) is changed. Then the longest admissible distance of terminal point A_s^g from the starting point A_r^g is such length s_{rs}^g where $\Delta N_V = (N_V)_L$. Therefore for the length of triangle side s_{rs}^g PTN will be valid the condition

$$\Delta N_V \leq (\Delta N_V)_L \tag{24}$$

Because $(K_V)_{pr}$ acquires small values under planar regions of TSG which are often close to zero, it is also required to introduce the limiting distance of two points A_r^g, A_s^g expressed by the symbol $(S_L^g)_M$. Therefore, all S_{rs}^g measured in selected scale 1 : M$_i$ on TSG except for the previous condition (24) must fullfil the condition

$$s_{rs}^g \leq (S_L^g)_M \tag{25}$$

that in the scalar basis (*x*, *y*) corresponds with the condition

$$[s_{rs} = s_{rs}^g \cos(\gamma_s)_{rs}] \leq [(S_L)_M = (S_L^g)_M \cos(\gamma_s)_{rs}]. \tag{25'}$$

From (24) and (25) suggest that each two arbitrary neighbouring points A_r^g, $A_s^g \in {}_DE_{RF}^g$ on TSG and their adequate points A_r, $A_s \in {}_DE_{RF}$ in the scalar basis (*x*, *y*) in considered scale 1 : M$_i$ will be distanced if both conditions (24) and (25) are fullfilled i. e. if

$$[s_{rs}^g \leq (S_L^g)_M \wedge [\Delta N_V \leq (\Delta N_V)_L] \tag{26}$$

$$\{[s_{rs} = s_{rs}^g \cos(\gamma_s)_{rs}] \leq [(S_L)_M = (S_L^g)_M \cos(\gamma_s)_{rs}]\} \wedge [\Delta N_V \leq (\Delta N_V)_L]. \tag{26'}$$

It means that the measured s_{rs}^g must not exceed for $\Delta N_V = (\Delta N_V)_L$ the limiting length $(S_L^g)_M$ even in case if for $s_{rs}^g = (S_L^g)_M$ there is $\Delta N_V \leq (\Delta N_V)_L$ and on the contrary, it must be valid that $s_{rs}^g < (S_L^g)_M$ even in case if for s_{rs}^g there is $\Delta N_V \leq (\Delta N_V)_L$.

From the viewpoint of graphic distinctive level for ΔN_V if we determine the limiting value $(\Delta N_V)_L = 0.1$ mm then for the scale 1 : 2 000 there will be $(\Delta N_V)_L =$ 0.2 m, for the scale 1 : 5 000 there will be $(\Delta N_V)_L = 0.5$ m, for the scale 1 : 10 000 there will be $(\Delta N_V)_L = 1.0$ m, for the scale 1 : 25 000 there will be $(\Delta N_V)_L = 2.5$ m. Then in the sense of relations (16) to (23) and under mentioned conditions (26) in relation to $(K_V)_{pr}$, $(R_V)_{pr}$ the lengths of triangel sides s_{rs}^g PTN are formed that fullfil from viewpoint of TSG geometry the conditions of determined representativeness. The value of radius of average curvature $(R_V)_{pr}$ and the average vertical curvature $(K_V)_{pr}$, in relation to the length of sides s_{rs}^g (17) at the limiting value $(\Delta N_V)_L = 0.1$ mm are illustrated according to the relations (23) and (23') in the Tab. 1 in the scale 1 : 2 000 and in the Tab. 2 in the scale 1 : 5 000. In both Tables the length of sides s_{rs}^g (17) due to the selected $(\Delta N_V)_L = 0.1$ mm is expressed in milimetres (first column in Tab. 1 and 2) as well as in metres (second column in Tabs) due to the considered scale 1 : 2 000 and 1 : 5 000. The same way, $(\Delta N_V)_L = 0.1$ mm is selected due to both scales expressed in metres i. e. $(\Delta N_V)_L = 0.2$ m and $(\Delta N_V)_L = 0.5$ m.

Tab. 1. Expression of the maximum allowed length of triangle sides in scale 1:2 000 along with the entry discrete point field and the still representative triangle plot made of it

$(\Delta N_V)_L = 0.1$ mm $\Rightarrow$ $(\Delta N_V)_L = 0.2$ m

s^g_{rs} [mm]	s^g_{rs} [m]	$(R_V)_{pr}$ [m]	$(K_V)_{pr}$ [m^{-1}]	$\Delta\gamma_{rs}{}^o$
4	8.00	40.1	0.024937	11.4496
5	10.00	62.6	0.015974	9.1624
6	12.00	90.1	0.011099	7.6366
8	16.00	160.1	0.006246	5.7284
10	20.00	250.1	0.004000	4.5831
12	24.00	360.1	0.002777	3.8194
14	28.00	490.1	0.002040	3.2738
16	32.00	640.1	0.001562	2.8646
18	36.00	810.1	0.001235	2.5464
20	40.00	1 000.1	0.001001	2.2918
25	50.00	1 562.6	0.000640	1.8334
30	60.00	2 250.1	0.000434	1.5279
40	80.00	4 000.1	0.000250	1.1459
50	100.00	6 250.1	0.000160	0.9167
60	120.00	9 000.1	0.000111	0.7639

Tab. 2. Expression of the maximum allowed length of triangle sides in scale 1:5 000 along with the entry discrete point field and the still representative triangle plot made of it

$(\Delta N_V)_L = 0.1$ mm $\Rightarrow$ $(\Delta N_V)_L = 0.5$ m

s^g_{rs} [mm]	s^g_{rs} [m]	$(R_V)_{pr}$ [m]	$(K_V)_{pr}$ [m^{-1}]	$\Delta\gamma_{rs}{}^o$
4	20.00	100.25	0.009975	11.4496
5	25.00	156.50	0.006389	9.1624
6	30.00	225.25	0.004439	7.6366
8	40.00	400.25	0.002498	5.7284
10	50.00	625.25	0.001599	4.5831
12	60.00	900.25	0.001111	3.8194
14	70.00	1 225.25	0.000816	3.2738
16	80.00	1 600.25	0.000625	2.8646
18	90.00	2 025.25	0.000494	2.5463
20	100.00	2 499.75	0.000400	2.2920
25	125.00	3 906.50	0.000256	1.8334
30	150.00	5 625.25	0.000178	1.5279
40	200.00	10 000.25	0.000100	1.1459
50	250.00	15 625.25	0.000064	0.9167
60	300.00	22 500.25	0.000044	0.7639

In the Tables in the scales 1 : 2 000 and 1 : 5 000 and due to the determined limiting value $(\Delta N_V)_L = 0.2$ m and $(\Delta N_V)_L = 0.5$ m the maximum admissible sides s^g_{rs} (17) lengths of triangels PTN are expressed where the input points field and the formed triangel net are representative. From the Tables it is also resulted that the last side s^g_{rs} in the Table 1 that fullfills the criteria (26) is the side

$$[s_{rs}^{g} = 100\text{m} = (S_L^g)_{M=2000}] \wedge [\Delta N_V = (\Delta N_V)_L = 0.2\text{m}]$$

and the the last side s_{rs}^{g} in the Table 2 that fullfills the criteria (26) is the side

$$[s_{rs}^{g} = 200\text{m} = (S_L^g)_{M=5000}] \wedge [\Delta N_V = (\Delta N_V)_L = 0.5\text{m}]$$

if in scale 1 : 2000 the limiting length of side $(S_L^g)_{M=2000}$ = 100 m is determined and in the scale 1 : 5000 the limiting length of side $(S_L^g)_{M=5000}$ = 200m is determined.

4. THE STRUCTURAL PROPERTIES OF TSG IN NEIGHBOURHOOD OF ITS ARBITRARY POINT AND THE CRITERIA FOR SPATIAL POINTS DISTRIBUTION DPFA AND TRIANGELS OF ITS PTN

The required properties of continual differentiality suggest that the function (1) can be extended into the Taylor's series. Due to this its structural properties can be studied in differentially small but even in final large neighbourhood of the point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ (2) on TSG and its adequate point $A_i(x_i, y_i)$, $z_i \in E_{RF}$ (3) in scalar basis.

In the sense of the contribution Šalamon (1963) and using the Taylor's expansion, two differentially small neighbourhoods determined by the relations

$$\mathrm{d}z = (z_x)_i\,\mathrm{d}x + (z_y)_i\,\mathrm{d}y \tag{27}$$

$$\mathrm{D}z = (z_x)_i\,\mathrm{d}x + (z_y)_i\,\mathrm{d}y + (1/2)[(z_{xx})_i\,\mathrm{d}x^2 + 2(z_{xy})_i\,\mathrm{d}x\mathrm{d}y + (z_{yy})_i\,\mathrm{d}y^2]. \tag{28}$$

can be coordinated to each point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ as the middle.

In these relations the partial derivations are related only to the selected point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ and its neighbourhood where they are considered as constants while the quantities dx, dy, dz are considered variable quantities. First linear relation (27) in the tangent plane to the TSG with the equation

$$Z - z_i = (z_x)_i\,(X - x_i) + (z_y)_i\,(Y - y_i) \tag{29}$$

expresses differentially small neighbourhood of selected tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ on TSG where the coordinates X, Y, Z of the tangent plane are changed in their entirety

$$X - x_i = \mathrm{d}x,\ Y - y_i = \mathrm{d}y,\ Z - z_i = \mathrm{d}z\,.$$

In the sense of the contribution Šalamon (1963) the linear equation (27) is related to TSG by the differential to the function of tangent plane to the middle point A_i^g. If the quantity dz in (27) is considered as the variable one then the equation (27) in the tangent plane (29) for the neighbourhood of tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ will express the linear elements as the parts of line passing through the neighbourhood. It seems to be that B. Šalamon called the neighbourhood the linear neighbourhood.

Second relation (28) due to the variable quantities dx, dy, dx^2, dy^2 is the quadratic function of two variables that in differentially small neighbourhood of tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ expresses in the coordinates system <0, x, y, z> the part of osculating paraboloid axis of which lies in the normal N to TSG passing through the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$.

If in the equation (28) we coordinate the significance of variable parameter to the quantity Dz then the equation (28) will express on the isolines passing the neighbourhood of point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ the curves elements of second order. The quadratic equation (28) in the sense of contribution Šalamon (1963) is the differential to the osculating paraboloid due to the middle point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ of neighbourhood that terminal is in this point and its axis is identical with the normal N to TSG passing through the point.

In the tangent plane (29) there lie the tangents to the normal cross-section in the point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ and pass in it through the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ on TSG. The normal curvature $(K_N)_m$ of the cross-sections in differential geometry is expressed by the parametric relation

$$(K_N)_m = \frac{1}{(R_N)_m} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} \cdot \frac{z_{xx} + 2z_{xy}m + z_{yy}m^2}{1 + z_x^2 + z_y^2 + 2z_x z_y m + (1 + z_y^2)m^2}\,, \qquad (30)$$

where m = dy/dx = tg α is variable parameter that is changed with the change of directional angle in definition region <0°; 90°), (90°; 270°), 270°; 360°> in the interval $(-\infty\,; +\infty)$; $(R_N)_m$ is the radius of normal curvature that lies in the normal N to TSG. For the value of parameter $m=n=k_n=z_x/z_y$ is defined $(K_N)_n \equiv \omega$ (7) and for the value of parameter $m=t= - z_x/z_y$ is defined $(K_N)_t$ (8) as well as due to the (9) even $K_r = (K_N)$t sin γ_N (9'), see Krcho (1973, 1983, 1990, 1991, 1992).

The properties of TSG that are observed from viewpoint of DPFA points distribution and the PTN forming from it, depend in each point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ and its neighbourhood on the sign of discriminant of second Gauss differential form

$$D_2 = \frac{z_{xx}z_{yy} - z_{zy}^2}{z_x^2 + z_y^2 + 1}\,.$$

If in the point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ there is $D_2 > 0$ then in (30) it will be valid that

1. for all values of parameter m there is $(K_N)_m > 0$, so $(K_N)_n > 0$, as well as $(K_N)_t = K_r \sin\gamma_N > 0$ or

2. for all values of parameter m there is $(K_N)_m < 0$, so $(K_N)_n < 0$, as well as $(K_N)_t = K_r \sin\gamma_N < 0$.

In both cases the Dupin indicatrix has the form of ellipse and osculating paraboloid has the form of eliptic paraboloid. All points on TSG where $D_2 > 0$ are considered elliptic points. Because in corresponce with $D_2 > 0$ the geometric forms F_{XX} are defined by the values $(K_N)_n > 0$; $(K_N)_t = K_r \sin \gamma_N > 0$ and the geometric forms F_{KK} by the values $(K_N)_n < 0$; $(K_N)_t = K_r \sin \gamma_N < 0$, the forms $F_{XX}, F_{KK} \in F$ are formed by the elliptic points. In both cases the Dupin indicatrix has the form of ellipse and osculating paraboloid has the form of eliptic paraboloid.

If there is in $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ $D_2 > 0$ then $(K_N)_m$ (30) with the change of parameter m acquires various signs. There can rise two possibilities:

1. for certain values m there are $(K_N)_m > 0$ as well as $(K_N)_n > 0$ and for certain values m $(K_N)_m < 0$ as well as $(K_N)_t = K_r \sin \gamma_N < 0$ or on the contrary.

2. for certain values m there are $(K_N)_m < 0$ as well as $(K_N)_n < 0$ and for certain values m $(K_N)_m < 0$ as well as $(K_N)_t = K_r \sin \gamma_N > 0$.

In the first case the forms F_{XK} are characterized by the forms $[(K_N)_n > 0 ; (K_N)_t < 0$ so $K_r < 0]$ and in the second case the forms F_{KX} are characterized by the forms $[(K_N)_n < 0 ; (K_N)_t > 0$ so $K_r > 0]$. The Dupin indicatrix in both cases has the form of dual set of hyperbols and the osculating paraboloid has the form of hyperbolic paraboloid. Due to this the geometric forms F_{XK}, F_{KX} are formed by the hyperbolic points.

If there is in $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ $D_2 = 0$ then $(K_N)_m$ (30) with the change of parameter m does not change the sign, however, for certain values m there is $(K_N)_m = 0$. The osculating paraboloid has the form of parabolic cylinder and the Dupin indicatrix has the form of two paralel lines. Due to this the points are called parabolic points. On TSG and in its scalar basis, using the points the isolines $(K_N)_n = 0$ as well as $(K_N)_t = K_r = 0$ are formed, that separate the individual total geometric forms F_{XX}, F_{XK}, F_{KK}, F_{KX}. In the forms the Dupin indicatrix has the form of two paralel lines. Presented total geometric forms are the key factor for localization of input points $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ as well as for the formation of PTN triangels.

The form of equation of osculating paraboloid (28) in the neighbourhood of the point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ is simplified when there is selected the coordinates system $<O' \equiv A_i, x', y', z'>$ due to the tangent plane (29) with the starting point $O'$and in the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ on TSG while the axis $z_N \equiv N$, so the axes x', y', lie in the tangent plane (29). In the coordinate system there will be $z_x = 0$, $z_y = 0$, so in the equation (27) there will be $dz = 0$ and the equation (28) will have the form

$$Dz = (1/2)[(z_{xx})_i\, dx^2 + 2(z_{xy})_i\, dxdy + (z_{yy})_i\, dy^2] \,. \tag{31}$$

Likewise, the form of parametric relation (30) is simplified so that it has the form

$$(K_N)_m = \frac{1}{1+m^2}(z_{xx} + 2z_{xy}m + z_{yy}m^2), \tag{32}$$

where the parameter $m = \mathrm{tg}\, v$, the angle is the angle in the tangent plane between the axis x' and the tangent of considered normal cross-section.

We consider on TSG the small however finally large neighbourhood of its arbitrary point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ e. g. in the extension

$$-15\text{m} \leq \Delta x \leq +15\text{m}; -15\text{m} \leq \Delta y \leq +15\text{m}$$

so the equations (27), (28) considered in the coordinate system $<O, x, y, z>$ will have the form

$$\Delta z = (z_x)_i \Delta x + (z_y)_i \Delta y \tag{33}$$

$$D_\Delta z = (z_x)_i \Delta x + (z_y)_i \Delta y + (1/2)[(z_{xx})_i \Delta x^2 + 2(z_{xy})_i \Delta x \Delta y + (z_{yy})_i \Delta y^2], \tag{34}$$

while the equation (33) in the tangent plane (29) expresses small but not infinitisimally small neighbourhood of selected tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ where the coordinates X, Y, Z are changed in the intervals $X - x_i = \Delta x$, $Y - y_i = \Delta y$, $Z - z_i = \Delta z$, and where Δz expresses the differences between the points coordinates of tangent plane neighbourhood and the coordinates of tangent point.

Small but not infinitisimally small large neighbourhoods of six selected points from individual geometric forms F_{XX}, F_{XK}, F_{KK}, F_{KX} in graphs of Fig. 8 from the selected part of modelled region of Ružiná. Due to the brief presentation two points with their neighbourhoods; the point A_1 and A_4 were selected in details.

If the significance of variable parameter Δz ($\Delta z < 0$, $\Delta z = 0$, $\Delta z > 0$) is coordinated to the quantity Δz then the equation (33) in the tangent plane (29) expresses the parts of lines passing through the selected finally large neighbourhood of the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$. The presented part of line from selected neighbourhood for $\Delta z = 0$ will pass through the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ while its outline $k \equiv k_t$ of tangent to contour line passing through the tangent point A_i^g. For the neighbourhood of two selected points A_1 and A_4 in Fig. 8 the course of lines (20) is expressed in Fig. 9a and 10a.

The second relation (34) in the selected finally large neighbourhood of tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ expresses in coordinates system $<O, x, y, z>$ finally large part of osculating paraboloid. If we coordinate the significance of variable parameter $0 \leq D_\Delta z \leq 0$ to the quantity $D_\Delta z$ in (34) then through each selected value $D_\Delta z$ one of its isolines will be determined on the osculating paraboloid that will be gradually deviated with increasing distance from the point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ from its adequate contour line on TSG. Using the value $D_\Delta z = 0$, the isoline passing through the tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ is determined on the osculating paraboloid where it is connected with the contour line on TSG that passes through the point. The course of isolines (34) for the neighbourhood of points A_1, A_4 in Fig. 8 is expressed in Fig. 9b and 10b. In Fig. 9b there is expressed the course of isolines from the neighbourhood of points A_1, and in Fig. 10b there is expressed the course of isolines from the neighbourhood of points A_4.

The equation (31) expressed in the coordinates system $<O' \equiv A_i, x', y', z'>$ will

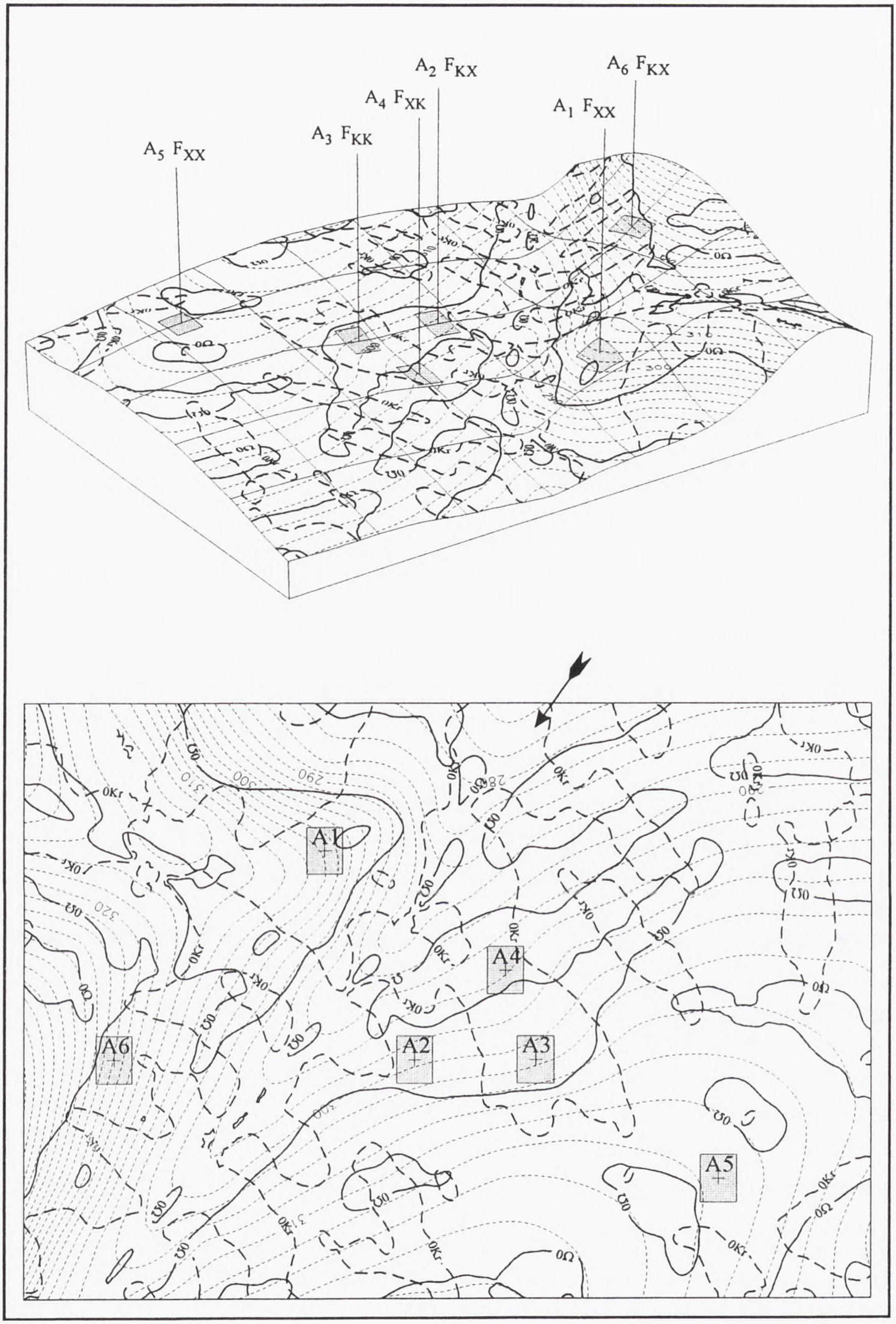

Fig. 8. Six points A_1, A_2, ..., A_6 selected in modelled region of Ružiná and their small though not infinitesimal neighbourhoods with analysed structural geometrical georelief characters.

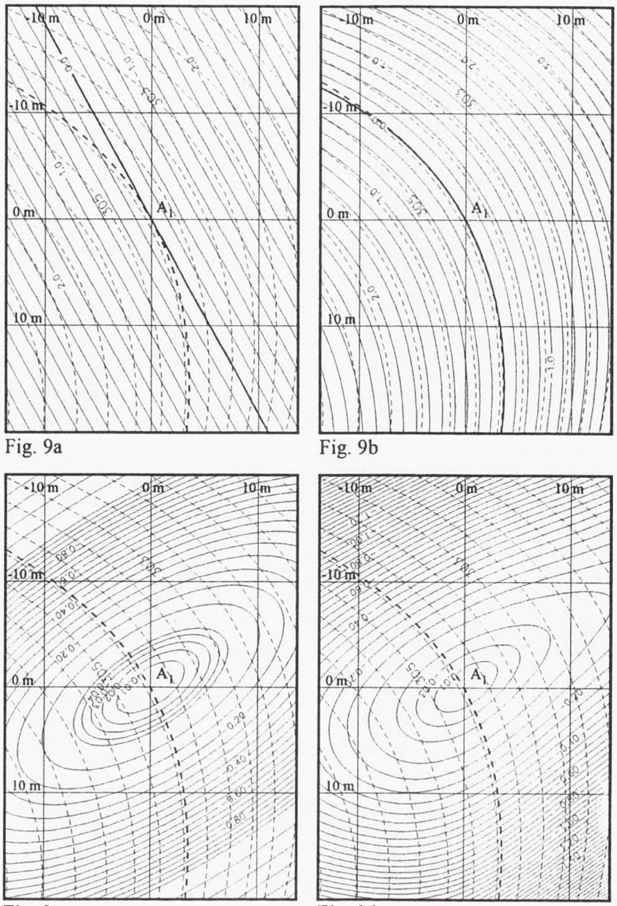

Fig. 9a Fig. 9b Fig. 9c Fig. 9d

Fig. 9. Structural geometrical properties of georelief in small neighbourhood of the selected point A_1 from Fig. 8.

have for finally large of neighbourhood of tangent point $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$ the form

$$D_\Delta z = (1/2)[(z_{xx})_i \Delta x^2 + 2(z_{xy})_i \Delta x \Delta y + (z_{yy})_i \Delta y^2] . \qquad (35)$$

Due to the fact that in the coordinate system there is $z_N \equiv N$ with the starting point $O' \equiv A_i^g$, the isolines of osculating paraboloid (35) determined by the variable parameter $D_\Delta z$ will have different course from the isolines determined by the equation (34). In the geometric forms $F_{XX}, F_{KK} \in F$ i. e. for $D_2 > 0$ they will have the form of ellipses with common middle in the tangent point A_i^g. This is expressed for the neighbourhood of point in Fig. 9c. In the geometric forms $F_{XK}, F_{KX} \in F$ i. e. for $D_2 < 0$ they will have the form of dual set of hyperbolas while the isolines $D_\Delta z = 0$ are the asymptoms passing through the point A_i^g. This is expressed for the neighbourhood of

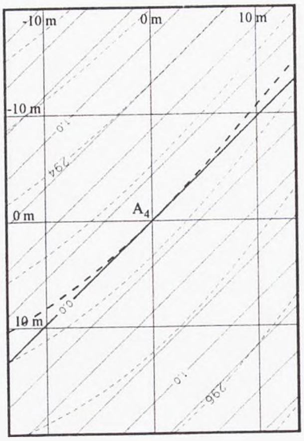

Fig. 10a

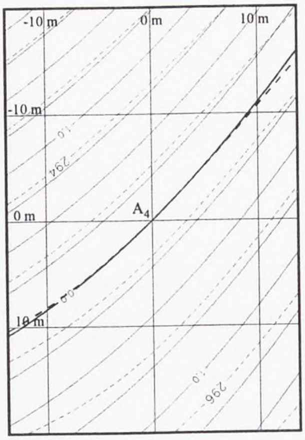

Fig. 10b

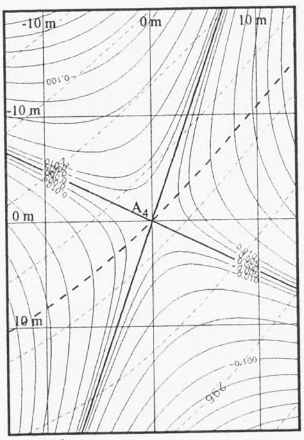

Fig. 10c

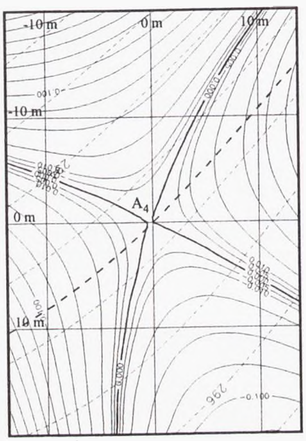

Fig. 10d

Fig. 10. Structural geometrical properties of georelief in small though not infinitesimal neighbourhood of the selected point A_4 from Fig. 8.

point A_4 in Fig. 10c. The form of isolines on the osculating paraboloid (35) is identical with the isolines (31).

Let us express in the coordinates system $< O, x, y, z >$ with starting point $O \equiv A_i^g$ in the selected finally large neighbourhood of arbitrary tangent point $A_i^g(x_i, y_i, z_i)$ on TSG the differences of coordinates $z - Z$ between the function TSG (1) and the tangent plane (29) and let us coordinate to the difference the significance of variable parameter $K_{\Delta z} = z - Z$. So, we get the equation of isolines of altitudes differences

$$[f(x,y) - z_i] - [(z_x)_i (X - x_i) + (z_y)_i (Y - y_i)] = K_{\Delta z}, \qquad (36)$$

where $0 \le K_{\Delta z} \le 0$.

One isoline is determined by the equation (36) for each value $K_{\Delta z}$. The isolines in the selected finally large neighbourhood of arbitrary tangent point $A_i^g(x_i, y_i, z_i)$ have similar course to the isolines of osculating paraboloid (35), however there are not the isolines of osculating paraboloid as the equations (35) and (36) suggest. Therefore, with increasing distance from the tangent point $A_i^g(x_i, y_i, z_i)$ from the isolines of osculating paraboloid they are more deviated. This is expressed for the neighbourhood of point A_1 at the Fig. 9d and for the neighbourhood of point A_4 at the Fig. 10d. The poperties of isolines (36) will be of essential significance for location of measured points $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$, as well as the formation of the PTN triangels.

5. THE CRITERIA FOR LOCALIZATION OF INPUT SET ${}_DE_{RF}$ POINTS FOR DTM AND THE CRITERIA FOR PTN FORMATION FROM INPUT POINTS $A_i^g(x_i, y_i, z_i) \in E_{RF}^g$

Each triangel of PTN with sequence number s=1, 2, ... on TSG is determined by the triple of points

$$A_e^g(x_e, y_e, z_e) \; ; A_f^g(x_f, y_f, z_f) \; ; A_g^g(x_g, y_g, z_g) \; , \tag{37}$$

where $(A_e^g, A_f^g, A_g^g) \in {}_DE_{RF}^g$ while from viewpoint of four basic geometric forms F_{XX}, F_{XK}, F_{KK}, F_{KX} each triple (37) must fullfil the mentioned conditions $D_2 > 0$, $D_2 = 0$, $D_2 < 0$ as entirety. But, due to the conditions $D_2 > 0$, $D_2 < 0$ the specific position has the condition $D_2 = 0$ that determines the dividing line among individual forms F_{XX}, F_{XK}, F_{KK}, F_{KX} that are formed by the isoline $(K_N)_n \equiv \omega = 0$, $(K_N)_t = K_r = 0$. Due to this, in the individual points of triple (37) there are admissible the combinations either $D_2 > 0$, $D_2 = 0$, or $D_2 < 0$, $D_2 = 0$, but not the combination $D_2 > 0$, $D_2 < 0$.

Because from all four forms, for the geometric ones F_{XX}, $F_{KK} \in F$ it is valid $D_2 > 0$ and for the geometric ones F_{KX}, $F_{XK} \in F$ there is valid $D_2 < 0$, then during the formation of each triangel PTN determined by the triple (37) it is valid that:

1. for $D_2 > 0$ i. e. for F_{XX}, $F_{KK} \in F$ in individual points of triple (37) are admisible combinations

1a. $D_2 > 0$ for whole triple (37) (internal triangel in F_{XX} or F_{KK})

1b. $D_2 > 0$ for two points of the triple (37) and $D_2 = 0$ for one point of the triple (the marginal triangel in F_{XX} or F_{KK} with two points internal $D_2 > 0$ and one point marginal $D_2 = 0$),

1c. $D_2 > 0$ for one point of the triple (37) and $D_2 = 0$ for two points of the triple (the marginal triangel in F_{XX} or F_{KK} with one point internal $D_2 > 0$ and two points marginal $D_2 = 0$),

2. for $D_2 < 0$ i. e. for F_{KX}, F_{XK} there are admisible combinations

2a. $D_2 < 0$ for whole triple (37) (internal triangel in F_{KX} or F_{XK})

2b. $D_2 < 0$ for two points of the triple (37) and $D_2 = 0$ for one point of the triple (the marginal triangel in F_{KX} or F_{XK} with two points internal $D_2 < 0$ and one point marginal $D_2 = 0$), 2c. $D_2 < 0$ for one point of the triple (37) and $D_2 = 0$ for two points of the triple (the marginal triangel in F_{KX} or F_{XK} with one point internal $D_2 < 0$ and two points marginal $D_2 = 0$).

The localization of PTN triangels determined by the triple (37) in details from modelled region of Ružiná in the sense of mentioned conditions is expressed in Fig. 11. Fig. 11 brings the square net with 10m x10m sides for size comparison. The points DPFA as well as the PTN triangels fullfil all required criteria of representativeness mentioned so far. We demonstrate that from the viewpoint of accuracy of positional coordination of calculated data the criteria are the necessary but not sufficient condition.

The availability of mentioned combinations 1a, b, c, as well as 2a, b, c we illustrate on the basis of equation of altitudes difference isolines (36) in the neighbourhood of arbitrary tangent point A_i^g and we introduce the condition that the tangent plane in the equation (36) will be parallel to the diagonal plane determined by the triple (37).

The equation of the diagonal plane with TSG determined by the triple (37) has the form

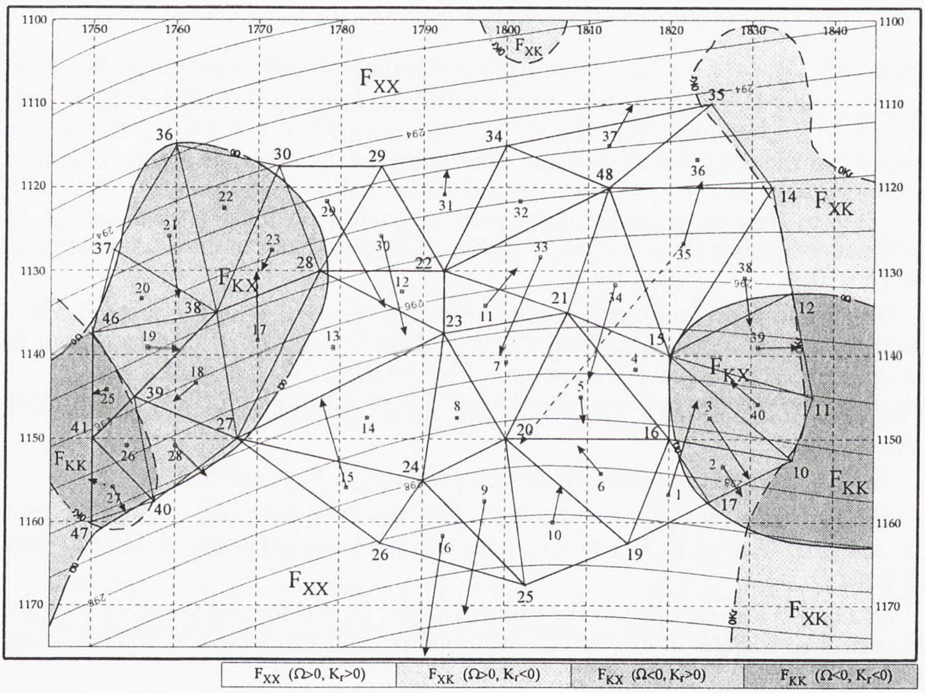

Fig. 11. Location of PTN triangles in the detail of the modelled region of Ružiná. The grid with size of the squares 10 m x 10 m facilitates the comparison.

$$\begin{vmatrix} x - x_{Ts} & y - y_{Ts} & z - z_{Ts} \\ \Delta x_{ef} & \Delta y_{ef} & \Delta z_{ef} \\ \Delta x_{eg} & \Delta y_{eg} & \Delta z_{eg} \end{vmatrix} = 0\,. \tag{38}$$

where $e, f, g \in j = 1, 2, ...$ are order number of points (37) where $e \neq f \neq g$,

$$x_{Ts} = (x_e + x_f + x_g)/3;\ y_{Ts} = (y_e + y_f + y_g)/3;\ z_{Ts} = (z_e + z_f + z_g)/3$$

are the coordinates of the centre of gravity of T_s of s - triangel of PTN and

$$\Delta x_{ef} = x_f - x_e\,;\quad \Delta y_{ef} = y_f - y_e\,;\quad \Delta z_{ef} = z_f - z_e$$

$$\Delta x_{eg} = x_g - x_e\,;\quad \Delta y_{eg} = y_g - y_e\,;\quad \Delta z_{eg} = z_g - z_e$$

The equation (38) has the normal form

$$N_x^o = (x - x_{Ts}) + N_y^o(y - y_{Ts}) + N_z^o(z - z_{Ts}) = 0\,, \tag{39}$$

where N_x^o, N_y^o, N_z^o are the coordinates of the unit vector of normal

$$\vec{N^o} = N_x^o\,\vec{i} + N_y^o\,\vec{j} + N_z^o\,\vec{k}$$

oriented to the external side of TSG while

$$N_x^o = \frac{D_x}{\rho};\ N_y^o = \frac{D_y}{\rho};\ N_z^o = \frac{D_z}{\rho};\ \text{where } \rho = \sqrt{D_x^2 + D_y^2 + D_z^2} \tag{39'}$$

and where

$$D_x = \begin{vmatrix} \Delta y_{ef} & \Delta z_{ef} \\ \Delta y_{eg} & \Delta z_{eg} \end{vmatrix} \qquad D_y = -\begin{vmatrix} \Delta x_{ef} & \Delta z_{ef} \\ \Delta x_{eg} & \Delta z_{eg} \end{vmatrix} \qquad D_z = \begin{vmatrix} \Delta x_{ef} & \Delta y_{ef} \\ \Delta x_{eg} & \Delta y_{eg} \end{vmatrix}\,.$$

Using the unit vector of normal $\vec{N^o}$ also the tangent plane to the TSG with the tangent point $A_i^g\ (x_i, y_i, z_i)$ is determined in the equation

$$N_x^o(x - x_i) + N_y^o(y - y_i) + N_z^o(z - z_i) = 0\,, \tag{40}$$

that is parallel to the diagonal plane (39). If the triple of points (37) on TSG correctly configurated then

$$x_i \equiv x_{Ts},\ y_i \equiv y_{Ts},\ z_i = z_{Ts} + \Delta z_{i,Ts};\ \Delta z_{i,Ts} = z_i - z_{Ts}, \tag{40'}$$

where $\Delta z_{i,Ts} = z_i - z_{Ts}$ is the distance of both planes in the direction of z axis. If the triple (37) is not correctly configurated then $x_i \neq x_{Ts}$, $y_i \neq y_{Ts}$ then the tangent point is the opposite side of the center of gravity $T_s\ (x_s, y_s, z_s)$ of s - triangel in the plane (x, y) positionaly shifted in $\Delta x_{iTs} = x_{Ts} - x_i$, $\Delta y_{iTs} = y_{Ts} - y_i$ while the vertical distance $\Delta z_{i,Ts}$ of both planes (39), (40) is maintained.

We shall demonstrate that even if the conditions of representativeness are fullfiled the input points $A_i^g \in {}_D E_{RF}^g$ are not usually correctly configurated what has direct negative consequences the positional and numeric accuracies of calculated data what is detailed expressed in the viewport at the Fig. 12a, b, c, d as well as 13a, b, c, d. Let us adjust both equations (39), (40) to the form

$$\frac{D_x}{D_z}(x - x_{Ts}) + \frac{D_y}{D_z}(y - y_{Ts}) + (z - z_{Ts}) = 0 \tag{41}$$

$$\frac{D_x}{D_z}(x - x_i) + \frac{D_y}{D_z}(y - y_i) + (z - z_i) = 0 \,, \tag{42}$$

so, in both equations (39), (40) for individual coefficients and due to the (39) it is valid that

$$\frac{N_x^o}{N_z^o} = \frac{D_x}{D_z} - (z_x)_{Ts} = -(z_x)_i \,; \quad \frac{N_y^o}{N_z^o} = \frac{D_y}{D_z} - (z_y)_{Ts} = -(z_y)_i \,. \tag{43}$$

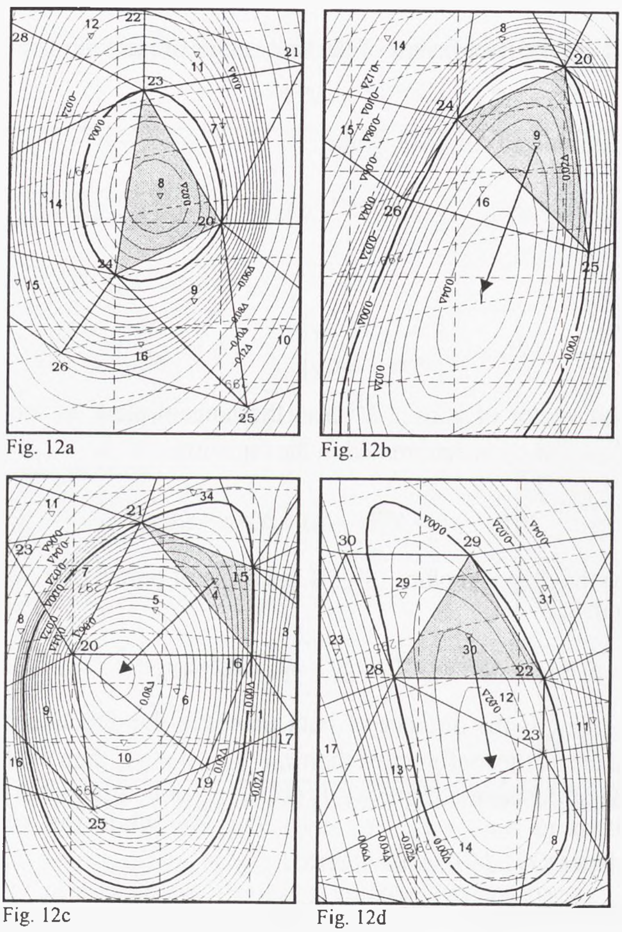

Fig. 12a Fig. 12b Fig. 12c Fig. 12d

Fig. 12. Four chosen PTN triangles from Fig. 11 in geometrical forms Fxx ($D_2 > 0$) with location shift of the tangent point A_i expressed.

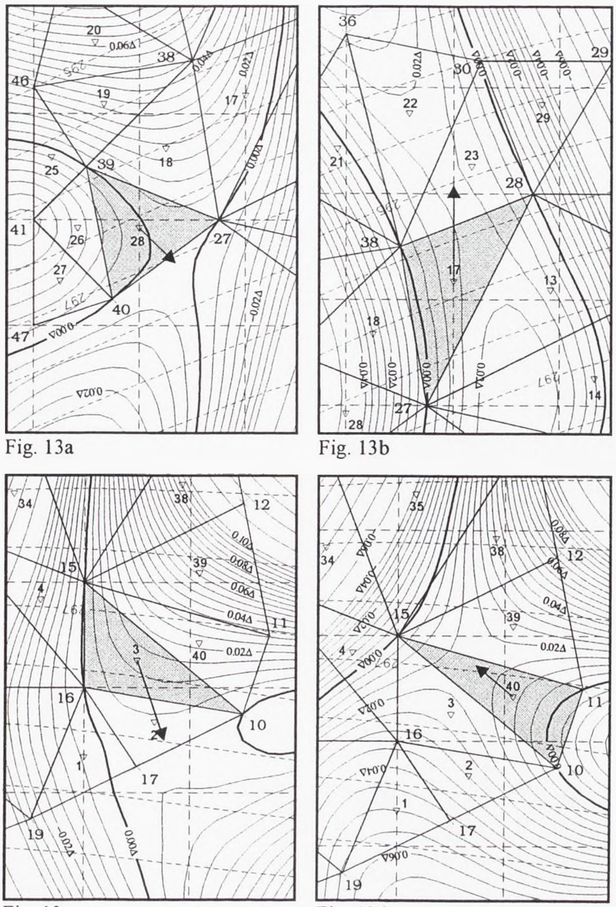

Fig. 13a Fig. 13b Fig. 13c Fig. 13d

Fig. 13. Four chosen PTN triangles from Fig. 11 in geometrical forms F_{XK}, F_{KX} ($D_2 < 0$) with location shift of the tangent point A_i expressed.

The tangent plane (42) due to the (43) is identical with the equation of tangent plane that is contained in the relation (36).

Therefore the relations for calculation of numeric values $(\gamma_N)_{Ts}$, $(A_N)_{Ts}$ in gravity centres of triangels of PTN derived and gradually detailed analyzed in the contributions Krcho (1977, 1983, 1986, 1990, 1992 - 93) have the form

$$|drad\, z|_{Ts} = tg(\gamma_N)_{Ts} = \frac{\sqrt{D_x^2 + D_y^2}}{D_z} = \sqrt{(z_x)^2_{Ts} + (z_y)^2_{Ts}} \tag{44}$$

$$\cos(A_N)_{Ts} = \frac{D_x}{\sqrt{D_x^2 + D_y^2}}\ ; \quad \sin(A_N)_{Ts} = \frac{D_y}{\sqrt{D_x^2 + D_y^2}}\ , \tag{45}$$

are valid due to the parallelism of both planes (41), (42), given by the common unit vector of normal $\vec{N^0}$ as well as for the tangent point $A_i^g \in E_{RF}^g$ in the tangent plane (42) with TSG regardless of validity (40') and also the tangent plane contained in the relation (36).

We can express for neighbourhood of the triangel the altitudes relations $\Delta z = z - Z$ between TSG (1) and the diagonal plane (41) in the equation form

$$f(x, y) - (-\frac{D_x}{D_z}X - \frac{D_y}{D_z}Y + \frac{D_x}{D_z}X_{Ts} + Z_{Ts}) = C_{\Delta z} \ , \qquad (46)$$

where $0 \leq C_{\Delta z} = z - Z \leq 0$ has the significance of variable parameter what is the equation of isolines field of altitudes differences so that we shall present the relation between spatial distribution of triple terminal points (37) of s - triangel of PTN and the position of the tangent point $A_i^g(x_i, y_i, z_i)$ of the plane (40). In the equation (46) X, Y, Z are the coordinates of points in the plane (41) and x, y are the coordinates of adequate points on TSG. Each isoline is determined by the value $C_{\Delta z}$. The isolines $C_{\Delta z}$ in the neighbourhood of tangent point $A_i^g(x_i, y_i, z_i)$ have identical course to that of isolines $K_{\Delta z}$ (36) but they are only shifted in values in $\Delta z_{i.Ts} = z_i - z_{Ts}$. The isoline $C_{\Delta z} = 0$ is formed by the intersection point of diagonal plane (41) with TSG and it passes through the points (37) triple.

Figs. 12a, b, c, d, as well as 13a, b, c, d present selected triangels from the Fig. 11 with their neighbourhoods determined by the intersection points for $C_{\Delta z} = 0$ given by the equation (46). At the Fig. 12a, b, c, d there are presented four selected triangels from geometric forms F_{XX} (D2 > 0) from what two triangels are determined by the triples

8 (20, 23, 24) ; 9 (20, 24, 25)

are internal triangels and two triangels determined by the triples

4 (15, 21, 16) ; 30 (22, 29, 25)

are marginal triangels of the forms F_{XX} . The sequence numbers s = 8, 9, 4, 30 before brackets express the sequence numbers of triangels PTN and the triples in brackets express the sequence numbers of triples of their terminal points.

The triangel at the Fig. 12a with the sequence number s = 8 determined by the triple of points A_{20} , A_{23} , $A_{24} \in {}_D E_{RF}^g$ is under the condition (40') so the tangent point (x_i, y_i, z_i) lies above the gravity point T_8. In triangels s = 9, 4, 30 from the Fig. 12b, c, d the condition (40') is not fullfilled, so due to this the tangent point $A_i^g(x_i, y_i, z_i)$ in each one is positionally shifted. The value of positional shift in Fig. is expressed by the positional vectors in the form of arrows.

Fig. 13a, b, c, d presents four selected triangels from geometric forms F_{KX} (D_2 < 0) determined by the triples

3 (15, 16, 10) ; 28 (27, 39, 40)

40 (15, 10, 11) ; 17 (27, 28, 38)

where s = 3, 17, 28, 40 are the sequence numbers of triangels PTN and triples in brackets are the sequence numbers of e, f, g terminal points A_e^g A_f^g A_g^g of triples (37). Figures suggest that in none of the triangels the condition (40') is fullfilled.

In the geometric forms F_{XX} or F_{KK} ($D_2 > 0$) under mentioned conditions 1a, b, c there is valid that $C_{\Delta z} = 0$ is closed curve where the triangel given by the triple (37) as well as the tangent point $A_i^g(x_i, y_i, z_i)$ of the tangent plane (42) with TSG lie inside. For F_{XX} there is valid that inside the area there is $C_{\Delta z} > 0$ with one maximum $C_{\Delta z\ max}$ in the tangent point A_i^g (Fig. 12a, b, c, d). For F_{KK} there is valid that inside the area there is $C_{\Delta z} < 0$ with the minimum $C_{\Delta z\ min}$ in the tangent point A_i^g . In both forms F_{XX} , F_{KK} the isolines $C_{\Delta z}$ in the neighbourhood of each triangel PTN delimitated by the isoline $C_{\Delta z} = 0$ have the form close to ellipses. If (40') is valid then in F_{XX} the tangent point will lie above the gravity centre T_s of s - triangel PTN (Fig. 12a) and in F_{KK} the tangent point will lie below the gravity centre T_s . If (40') is not valid then in F_{XX} F_{KK} the tangent point $A_i^g(x_i, y_i, z_i)$ will be positionally shifted against the gravity centre T_s (Fig. 12b, c, d).

In the geometric forms F_{KX} or F_{XK} ($D_2 < 0$) under mentioned conditions 2a, b, c there is valid that the intersection $C_{\Delta z} = 0$ is composed from two parts. One part passes through two points from triple (37) and one part passes through remaining third point from the triple (37). The tangent plane (42) cuts TSG in two curves that are crossed in the point $A_i^g(x_i, y_i, z_i)$. There is the value of parameter $C_{\Delta z}$ (46) in the point $C_{\Delta z} = z_i - z_{Ts}$ and the value of parameter $K_{\Delta z}$ (36) in the point $K_{\Delta z} = 0$. The point is the saddle point for other isolines $C_{\Delta z}$ and $K_{\Delta z}$. The isolines in its close neighbourhood have the form close to hyperbolas. If (40') is valid then in the forms F_{XK} the saddle point lies above the gravity centre T_s and in the forms F_{KX} the saddle point lies below the gravity centre T_s of the s - triangel of PTN. If (40') is not valid then in F_{XK} F_{KX} the saddle point will be positionally shifted against the gravity centre T_s (Fig. 13a, b, c, d).

The representative PTN appropriate for ($D_2 > 0$) i. e. for $F_{XX}, F_{KK} \in F$ for criteria 1a, 1b, 1c and for ($D_2 < 0$) i. e. for $F_{KX}, F_{XK} \in F$ for criteria 2a, 2b, 2c are presented at the Fig. 14.

CONCLUSION

The criteria of representativeness for input points DPFA distribution from viewpoint of accuracy of positional coordination is only the necessary condition. There is also the condition of appropriate points DPFA configuration so that all formed triangels PTN will fullfil the condition (40').

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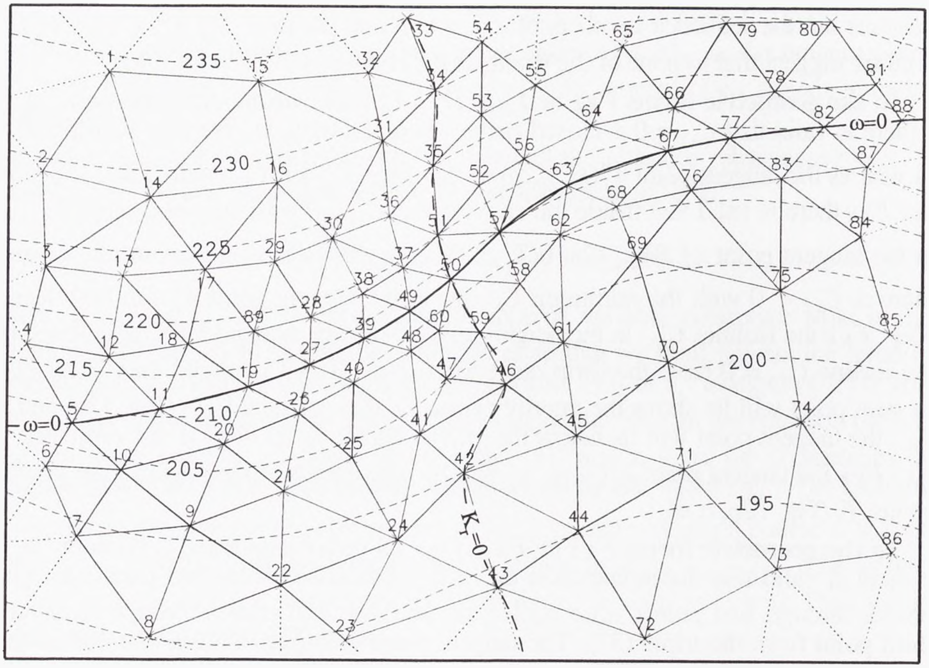

Fig. 14. Representative PTN and distribution of triangles according to the single overall geometrical forms F_{XX}, F_{KX}, F_{KK}, F_{XK}.

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MODELOVANIE GEORELIÉFU POMOCOU DTM - VPLYV KONFIGURÁCIE BODOV VSTUPNÉHO BODOVÉHO POĽA NA POLOHOVÚ A NUMERICKÚ PRESNOSŤ

V práci je načrtnutý problém polohovej a numerickej presnosti modelovania množiny morfometrických veličín georeliéfu pomocou DTM z hľadiska vlastností reprezentatívneho vstupného diskrétneho bodového poľa výšok (DBPV). Výsledná presnosť modelovania georeliéfu a jeho geometrickej štruktúry v podstate závisí:

- od vlastností reprezentatívneho vstupného diskrétneho bodového poľa výšok (DBPV),
- od vlastností aproximujúcich funkcií obsiahnutých v DTM, ktorým modelujeme georeliéf.

Rôzne, navzájom odlišné výsledky pri modelovaní georeliéfu pomocou DTM môžeme totiž dostať vtedy, ak vybranú oblasť georeliéfu pomocou DTM modelujeme:

1. z toho istého vstupného reprezentatívneho diskrétneho bodového poľa výšok (DBPV), avšak rôznymi aproximujúcimi funkciami (rozdiely vo výsledkoch sú v tomto prípade spôsobené odlišnými vlastnosťami jednotlivých použitých aproximujúcich funkcií),

2. tou istou aproximujúcou funkciou použitou v DTM, avšak z rôznych vstupných reprezentatívnych diskrétnych bodových polí výšok (DBPV), pričom v tomto prípade sú rozdiely spôsobené rôznymi vlastnosťami jednotlivých vstupných diskrétnych bodových polí, a to aj napriek tomu, že všetky spĺňajú podmienky reprezentatívnosti.

V práci je rozobraný problém súvisiaci s obsahom bodu 2, a to problém polohovej presnosti modelovania georeliéfu a množiny jeho morfometrických veličín z hľadiska vlastností reprezentatívneho vstupného diskrétneho bodového poľa výšok (DBPV). Potvrdzuje sa, že reprezentatívne vstupné diskrétne bodové pole výšok (DBPV) musí spĺňať dve základné podmienky:

2a. podmienku reprezentatívnosti rozloženia bodov vstupného diskrétneho bodového poľa výšok (DBPV),

2b. podmienku vhodnej (správnej) konfigurácie bodov reprezentatívneho vstupného diskrétneho bodového poľa výšok (DBPV), z ktorého je potom zostrojená trojuholníková sieť (PTS).

Kritériá reprezentatívnosti pre rozloženie bodov vstupného diskrétneho bodového poľa výšok (DBPV) sú z hľadiska presnosti polohového priradenia jednotlivých vypočítaných hodnôt výšok a ich vrstevnicového poľa, ako aj množiny morfometrických veličín a ich izočiarových polí, iba podmienkou nutnou, ale nie postačujúcou. K tejto prvej podmienke ako nutnej však pristupuje ešte ďalšia, a to podmienka vhodnej konfigurácie bodov reprezentatívneho vstupného diskrétneho bodového poľa výšok (DBPV) a to tak, aby všetky vytvorené trojuholníky trojuholníkovej siete spĺňali podmienku, že dotykový bod dotykovej roviny s georeliéfom paralelnej so sečnou rovinou určenou trojicou vrcholových bodov ľubovoľného trojuholníka leží nad, alebo pod ťažiskom tohto ľubovoľného trojuholníka. Táto podmienka je v texte práce vyjadrená vzťahom (40'). Tento problém je v práci graficky vyjadrený na obr. 11, 12a, b, c, d a na obr. 13a, b, c, d.

V práci sú zároveň uvedené pravidlá pre tvorbu trojuholníkov trojuholníkovej siete z bodov vstupného reprezentatívneho diskrétneho bodového poľa výšok z hľadiska celkových geometrických foriem georeliéfu F_{XX}, F_{KX}, F_{KK}, F_{XK}. Sú od seba navzájom oddelené izočiarami nulovej normálovej krivosti georeliéfu $(K_N)n \equiv \omega = 0$ v smere spádových kriviek a izočiarami nulovej horizontálnej krivosti georeliéfu $K_r = 0$.

Obr. 1a. γ_N a $\omega = 0$. Izočiarové pole sklonov γ_N georeliéfu v smere spádových kriviek.

Obr. 1b. A_N a $K_r = 0$. Izočiarové pole orientácie georeliéfu voči svetovým stranám.

Obr. 2. Ω. Normálová krivosť georeliéfu v smere spádových kriviek. Izočiarové pole normálovej krivosti georeliéfu v smere spádových kriviek.

Obr. 3. K_r. Izočiarové pole horizontálnej krivosti georeliéfu.

Obr. 4. Normálové formy georeliéfu v smere spádových kriviek.

Obr. 5. Horizontálne formy georeliéfu.

Obr. 6. Celkové geometrické formy georeliéfu.

Obr. 7a. Profil georeliéfu na spádovej krivke v smere trojuholníkových hrán primárnej trojuholníkovej siete (PTN).

Obr. 7b. Určenie reprezentatívnej dĺžky strán trojuholníkov PTN v závislosti od vertikálnej krivosti K_V pri stanovenej "limitnej" hodnote $(\Delta N_V)_L$.

Obr. 8. Šesť vybraných bodov A_1, A_2, ..., A_6 v modelovom území Ružiná a ich malé, ale nie infinitezimálne okolia, v ktorých sú analyzované štruktúrne geometrické vlastnosti georeliéfu.

Obr. 9. Štruktúrne geometrické vlastnosti georeliéfu v malom okolí vybraného bodu A_1 z obr. 8.

Obr. 10. Štruktúrne geometrické vlastnosti georeliéfu v malom, ale nie infinitezimálnom okolí vybraného bodu A_4 z obr. 8.

Obr. 11. Lokalizácia trojuholníkov PTN z detailu modelovej oblasti Ružiná. Na obr. je pre porovnanie veľkosti súčasne vykreslená štvorcová sieť o veľkosti strán 10m x 10 m.

Obr. 12. Štyri vybrané trojuholníky PTN z obr. 11 v geometrických formách F_{XX} (D_2>0) s vyjadreným polohovým posunom dotykového bodu A_i.

Obr. 13. Štyri vybrané trojuholníky s obr. 11 v geometrických formách F_{XK}, F_{KX} (D_2<0) s vyjadreným polohovým posunom dotykového bodu A_i.

Obr. 14. Reprezentatívna PTN a rozloženie jej trojuholníkov podľa jednotlivých celkových geometrických foriem F_{XX}, F_{KX}, F_{KK}, F_{XK}.

Tab. 1. Tab. 1. Vyjadrenie maximálnej dovolenej dĺžky strán trojuholníkov v mierke 1:2000, pri ktorých je vstupné diskrétne bodové pole a z neho zhotovená trojuholníková sieť ešte reprezentatívna.

Tab. 2. Tab. 1. Vyjadrenie maximálnej dovolenej dĺžky hrán trojuholníkov v mierke 1:5000, pri ktorých je vstupné diskrétne bodové pole a z neho zhotovená trojuholníková sieť ešte reprezentatívna.