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SELECTION AMONG ADVANCED MANUFACTURING TECHNOLOGIES USING FUZZY DATA ENVELOPMENT ANALYSIS

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ABSTRACT. Mathematical programming is one of the areas to which fuzzy set theory has been applied extensively. Data Envelopment Analysis (DEA) is a method of similar units based on an efficiency concept. In DEA, the same set of factors is measured for each unit, and there are multiple and non commensurate inputs and outputs. Efficiency is measured as the weighted sum of output over the weighted sum of input. Fuzzy mathematical programming formulates a mathematical programming problem with fuzzy parameters based on personal opinions or subjective judgements. Thus the decision maker is not forced into a precise formulation for mathematical reasons. In the paper, first, a fuzzy mathematical programming model with multiple attributes using data envelopment analysis is given. Then this model is employed for evaluating two alternative computer integrated systems (CIMs). The input attributes considered are first costs, management/engineering effort, riskiness. The output attributes considered are quality of results, net present worth, ease of use, adaptability and expandability. Solving the problem, the CIM alternative superior to the other is selected. The paper also includes a wide literature review for both fuzzy mathematical programming and data envelopment analysis.

1. Introduction

The term *advanced manufacturing system* (AMS) is broadly defined to include any automated (usually computer oriented) technology used in design, manufacturing/service, and decision support. A more common term is *computer-integrated manufacturing* (CIM) system. CIM regards manufacturing as an integrated process beginning with product design and ending with product delivery to a *satisfied* customer. The objective of CIM is to develop a digital computer data base that integrates manufacturing, design, and business function — such

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as purchasing, distribution, accounting, inventory control — into a cohesive system. The ideal CIM-controlled system will design and manufacture parts without disruption, from raw materials to the finished product. Clearly, CIM affects product fabrication and assembly, materials handling, inventory management, and maintenance as well as other important functions, such as product design, quality control, cost control, and customer service (Canada and Sullivan, [2]).

Today, most major corporations are struggling with their traditional investment justification procedures, because they are either *misapplied* or the information included in the calculations is *inadequate* for the multifaceted problems being tackled. Traditional investment justification procedures omit the difficult-to-measure benefits, such as *quality, flexibility, lead times, customer satisfaction*, and much more. If these benefits cannot be measured, such "intangible" benefits essentially are assumed not to exist. Major advantages of advanced manufacturing technology are therefore assumed away.

2. Fuzzy linear programming and literature review

Conventional mathematical programming problems maximize an objective function subject to constraints. In real world problems, however, constraints and objective functions are very fuzzy. Fuzzy mathematical programming formulates a mathematical programming problem with fuzzy parameters which are based on personal opinions or subjective judgments. The conventional model of linear programming (LP) can be stated as

$$\begin{aligned} &\text{maximize } f(x) = c^T x\,, \\ &\text{such that} \quad Ax \le b\,, \\ &\qquad\qquad\quad x \ge 0\,, \\ &\text{with} \quad c, x \in \mathbb{R}^n,\ b \in \mathbb{R}^m,\ A \in \mathbb{R}^{m\times n}. \end{aligned} \tag{1}$$

When the LP-decision has to be made in fuzzy environments, quite a number of possible modifications of Eq. (1) exist. In other words, by contrast to classical LP, "fuzzy LP" is not a uniquely defined type of model, but many variations are possible, depending on the assumptions or features of the real situation to be modeled.

In fuzzy LP, the role of the constraints can be different from that in classical LP, where the violation of any single constraint by any amount renders the solution infeasible. The decision maker might accept small violations of constraints, but might also attach different (crisp or fuzzy) degrees of importance to violations of different constraints. The decision maker can establish an aspiration

level, z, for the value of he objective function he/she wants to achieve and such that each of the constraints is modeled as a fuzzy set. A basic model for fuzzy LP can be given as (Zimmermann, [32]):
Find x such that:

$$\begin{aligned} c^T x &\tilde{\geq} z\,, \\ Ax &\tilde{\leq} b\,, \\ x &\geq 0\,, \end{aligned} \tag{2}$$

where $\tilde{\leq}$ denotes the fuzzified version of $\leq$ and has the linguistic interpretation *"essentially smaller than or equal"*. $\tilde{\geq}$ denotes the fuzzified version of $\geq$ and has the linguistic interpretation *"essentially greater than or equal"*.

By substituting $\binom{-c}{A} = B$ and $\binom{-z}{b} = d$, Eq. (2) becomes:
Find x such that:

$$\begin{aligned} Bx &\tilde{\leq} d\,, \\ x &\geq 0\,. \end{aligned} \tag{3}$$

If each of the $(m+1)$ rows of Eq. (3) is represented by a fuzzy set, the membership functions of which are $\mu_i(x)$, then the membership function of the fuzzy set *"decision"* of model will be

$$\mu_{\tilde{D}}(x) = \min_i \{\mu_i(x)\}\,, \tag{4}$$

where $\mu_i(x)$ is the degree to which x satisfies the fuzzy inequality $B_i x \leq d_i$ (where B_i denotes the ith row of B).

Assuming that the decision maker is interested in a crisp *"optimal"* solution, the *"maximizing solution"* will be

$$\max_{x \geq o} \min_i \{\mu_i(x)\} = \max_{x \geq o} \mu_{\tilde{D}}(x)\,. \tag{5}$$

Here, $\mu_i(x)$ can be given as

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i\,, \\ \in [0,1] & \text{if } d_i \prec B_i x \leq d_i + p_i\,, \\ 0 & \text{if } B_i x \succ d_i + p_i\,, \end{cases} \quad i = 1, 2, \ldots, m+1\,, \tag{6}$$

and the simplest type of membership function assumed to be linearly increasing over the *"tolerance interval"* p_i is

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i\,, \\ 1 - \frac{B_i x - d}{p_i} & \text{if } d_i \prec B_i x \leq d_i + p_i\,, \\ 0 & \text{if } B_i x \succ d_i + p_i\,. \end{cases} \quad i = 1, 2, \ldots, m+1\,, \tag{7}$$

Introducing one new variable, Ψ, which corresponds essentially to $\mu_{\tilde{D}}(x)$, Eq. (2) becomes:

$$\begin{aligned} &\text{maximize } \Psi\,, \\ &\text{such that } \Psi p_i + B_i x \leq d_i + p_i\,, \quad i = 1, 2, \ldots, m+1\,, \\ &\qquad\qquad x \geq 0\,. \end{aligned} \tag{8}$$

Fuzzy mathematical programming has been used in many papers on various areas. Some recent papers are the following: Julien ([11]) investigates the application of fuzzy set and possibility theories for the representation of imprecise information in water quality management problems. Fuller and Fedrizzi ([7]) explore stability analysis in possibilistic programming by extending previous research results to possibilistic linear programs with continuous fuzzy number coefficients. Nakahara and Gen ([15]) propose a quantitative formulation of LP problems with fuzzy number coefficients, by using the ranking criteria proposed by themselves, and show an algorithm for solving the formulated problems in some cases. The optimization of an objective function with fuzzy number coefficients is formulated as the user oriented extension of the optimization of an objective function with real coefficients by the proposed ranking criteria. Carlsson and Fuller ([3]) introduce measures of interdependence between the objectives in order to provide a better understanding of the decision problems and to find effective and more correct solutions to multiple criteria decision making problems. Sakawa *et al.* ([19]) show that large-scale fuzzy LP problems can be reduced to a number of independent linear sub-problems, and the overall satisfying solution for the decision maker is directly obtained just only solving the sub-problems. Herrera and Verdegay ([10]) study some models for dealing with fuzzy integer LP problems which have a certain lack of precision of a vague nature in their formulation, and present methods to solve them with either fuzzy constraints or fuzzy numbers in the objective function or fuzzy numbers defining the set of constraints.

3. Data development analysis (DEA): Literature review and fuzzy DEA (FDEA)

DEA is the optimization method of mathematical programming to generalize Farrell (1957) single-input/single-output technical efficiency measure to the multiple-input/multiple-output case by constructing a relative efficiency score as the ratio of a single virtual output to a single virtual input. It originally was developed by Charnes, Cooper, Rhodes ([4]) and extended by Banker, Charnes, Cooper (1984) to include variable returns to scale. So the basic DEA models are known as CCR and BCC. Since 1978, over 1000 articles, books and thesis have been published, and DEA has been rapidly extended by returns to scale, dummy, or categorical variables, discretionary and non-discretionary variables, incorporating value judgments, longitudinal analysis, weight restrictions, stochastic DEA, non-parametric Malmquist indices, technical change in DEA and many other topics. DEA models can be listed as follows: 1. Radial models (input orientation and output orientation), 2. Ratio models, 3. Additive models, 4. Multiplicative models, 5. Hyperbolic models (input orientation and output orientation), 6. Non-radial models (input orientation and output orientation), 7. Miscellaneous models (Emrouznejad, [6]). DEA is commonly used to evaluate of a number of producers. A typical statistical approach is characterized as a central tendency approach and it evaluates producers relative to an average producer. In contrast, DEA is an extreme point method and compares each producer with only the "best" producers. In the DEA literature, a producer is usually referred to as a decision making unit or DMU. Among the applications areas, there are health care, education, banks, manufacturing, benchmarking, management evaluation, fast food restaurants, retail stores. Some strengths of DEA are the following: DEA can handle multiple input and multiple output models; it does not require an assumption of a functional form relating inputs to outputs; DMUs are directly compared with a peer or combination of peers.; inputs and outputs can have very different units. Some limitations of DEA are the following: since DEA is an extreme point technique, noises (even a symmetrical noise with zero mean) such as measurement error can cause significant problems; DEA is good in estimating "relative" efficiency of a DMU but it converges very slowly to "absolute" efficiency; since DEA is a nonparametric technique, statistical hypothesis tests are difficult and are a focus of ongoing research; since a standard formulation of DEA creates a separate linear program for each DMU, large problems can be computationally intensive (Anderson, [1]). Anderson ([1]) gives DEA www bibliography sorted by years, application areas, theory/methodology, recent dissertations, respectively.

Some recently published papers are the following: Weber and Desai ([29]) demonstrate the use of DEA for measuring vendor performance and efficiency. Thore *et al.* ([26]) apply DEA to rank the efficiency of US computer companies during the period 1981–1990. Thompson *et al.* ([25]) analyze the efficiency and profitability of 14 integrated oil companies for the years 1980–1991, with a US focus. Ruggiero ([18]) extends the mathematical programming approach to frontier estimation known as DEA to allow for environmental variables that affect production. Stewart ([21]) contrasts the different approaches of DEA and multiple criteria decision making (MCDM) to superficially similar problems. The concepts of efficiency and Pareto optimality in DEA and MCDM are compared, and a link between the ratio efficiency definition in DEA and a distance measure in input-output space based on linear value functions is demonstrated.

Zhu ([31]) studies sensitivity analysis of the DEA ratio model (CCR) by means of modified versions of CCR model based on evaluation of a decision making unit (DMU) relative to a reference set grouped by all other DMUs. Thanassoulis ([24]) develops a DEA based method for setting targets for differentially effective schools. Cook *et al.* ([5]) apply DEA to a wide range of real world problems. A framework is presented for incorporating ordinal data factors into the standard ratio model. An application involving the prioritization of R&D projects is presented as a case in point where such ordinal factors appear in a natural way. Yeh ([30]) demonstrates an application of DEA in conjunction with financial ratios to help bank regulators in Taiwan not only to distinguish the efficient banks from the inefficient ones, but also to gain insight into various financial dimensions that somehow link to the bank's financial operational decisions.

The efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs to weighted inputs under the condition that similar ratios for every DMU are less than or equal to unity (Charnes *et al.*, [4]). DEA allows for the simultaneous analysis of multiple inputs to multiple outputs, a multifactor productivity approach. The general efficiency measure used by DEA can be in the best way summarized by Eq. (9):

$$\begin{gathered} E_k = \max\Big(\sum_r w_r O_{rk}\Big) \Big/ \Big(\sum_i w_i I_{ik}\Big), \\ \Big(\sum_r w_r O_{rj}\Big) \Big/ \Big(\sum_i w_i I_{ij}\Big) \le 1, \\ w_r, w_i \ge \varepsilon, \\ i = 1,2,\dots,m, \quad j = 1,2,\dots,n, \quad r = 1,2,\dots,t, \end{gathered} \tag{9}$$

where

E_k: the efficiency of DMU_k,

w_r: the weight assigned to the rth output by DMU_k,

w_i: the weight assigned to the ith input by DMU_k,

O_{rk}: the rth output produced by DMU_k,

I_{ik}: the ith input used by DMU_k,

O_{rj}: the rth output produced by DMU_j,

I_{ij}: the ith input used by DMU_j,

ε: a small positive scalar,

n: the number of DMUs,

t: the number of outputs,

m: the number of inputs.

Essentially each DMU is allowed to vary the weighting attached to each input and output in order to maximize its efficiency, subject to the constraint that no other DMU should consequentially obtain an efficiency greater than 1. For a full complement of efficiency ratings, this model has to be solved for each of the n-DMUs. It is often more appropriate to think of the efficiency problem as minimizing the ratio of inputs to outputs, and thus it is useful to utilise the reciprocal form of the above model. Converting this into LP form, the efficiency of DMU_k will be generated by solving the following:

$$\begin{aligned}
&E_k = \max\Bigl(\sum_r w_r O_{rk}\Bigr)\,,\\
&\Bigl(\sum_i w_i I_{ik}\Bigr) = 1\,,\\
&\sum_r (w_r O_{rj}) - \sum_i (w_i I_{ij}) \le 0\,,\\
&w_r\,, w_i \ge \varepsilon\,,\\
&i = 1,2,\ldots,m\,,\quad j = 1,2,\ldots,n\,,\quad r = 1,2,\ldots t\,.
\end{aligned} \tag{10}$$

The dual formulation of DEA is

$$\begin{aligned}
&E_k = \min \theta - \varepsilon \sum_i S_i^- - \varepsilon \sum_r S_r^+\,,\\
&\sum_j \lambda_j I_{ij} - S_i^- + \theta I_{ik} = 0\,,\\
&\sum_j \lambda_j O_{rj} - S_r^+ - O_{rk} = 0\,,\\
&\qquad \lambda_j\,,\ S_i^-,\ S_r^+ \ge 0\,,\\
&i = 1,2,\ldots,m\,,\quad j = 1,2,\ldots,n\,,\quad r = 1,2,\ldots,t\,,
\end{aligned} \tag{11}$$

where

S_i^-: the unused value of ith input of DMU_k,

S_r^+: the unused value of rth output of DMU_k,

θ: the coefficient determining how much DMU_k's inputs can be decreased,

λ_j: the intensity value taken by DMU_j.

The objective function in the dual formulation of DEA:

$$\max \varepsilon \sum_i S_i^- + \varepsilon \sum_r S_r^+ - \theta\,. \tag{12}$$

Then the fuzzy – LP for DEA:

$$\begin{aligned}
&\max \Psi\,,\\
&\Psi(c_u - c_l) - \varepsilon \sum_i S_i^- - \varepsilon \sum_r S_r^+ + \theta \le -c_l\,,\\
&\Psi p_I + \sum_j \lambda_j I_{ij} - S_i^- + \theta I_{ik} \le p_I\,,\\
&\Psi p_0 + \sum_j \lambda_j O_{rj} - S_r^+ - O_{rk} \le p_O\,,\\
&\lambda_j\,,\ S_r^+\,,\ S_i^- \ge 0\,,\\
&i = 1,2,\dots,m\,, \quad r = 1,2,\dots,t\,, \quad j = 1,2,\dots,n\,,
\end{aligned} \tag{13}$$

where

c_u: the upper bound for the attainment of the objective,

c_l: the lower bound for the attainment of the objective,

p_I: the amount that can exceed the right hand side of the input constraint,

p_O: the amount that can exceed the right hand side of the output constraint.

Weights can be easily incorporated into fuzzy programming by linearly weighting of the membership functions. The weighting of of the membership function for the constraints is done inversely proportional to the weight. Then the following linear programming formulation is obtained:

$$\begin{aligned}
&\max \Psi\,,\\
&\Psi(c_u - c_l) - \varepsilon \sum_i S_i^- - \varepsilon \sum_r S_r^+ + \theta \le -c_l\,,\\
&\Psi w_I p_I + \sum_j \lambda_j I_{ij} - S_i^- + \theta I_{ik} \le p_I\,,\\
&\Psi w_O p_O + \sum_j \lambda_j O_{rj} - S_r^+ - O_{rk} \le p_O\,,\\
&\lambda_j\,,\ S_r^+,\ S_i^- \ge 0\,,\\
&i = 1,2,\dots,m\,, \quad r = 1,2,\dots,t\,, \quad j = 1,2,\dots,n\,,
\end{aligned} \tag{14}$$

where

w_I: the weight assigned to the input constraint,

w_O: the weight assigned to the output constraint.

4. A numeric example

Evaluation of AMTs and other strategic manufacturing technologies has included a spectrum of mathematical and systems modeling approaches. The methodologies that have been used for justification of AMTs include decision theory (Miltenburg and Krinsky, [13]), dynamic programming (Monahan and Smunt, [14]), game theory (Gaimon, [8]), linear/goal programming (Suresh, [22]), multiattribute utility theory (Parsaei and Wilhelm, [16]), outranking approaches (Parsaei *et al.*, [17]), and risk and simulation analysis (Troxler and Blank, [27]). The analytical hierarchy process (AHP) is one of the most popular techniques to evaluate AMTs (Wabalackis, [28]). Multiple models have been most effectively integrated into larger decision support systems and expert systems (Suresh and Meredith, [23]).

Consider the following table including the values of the inputs and the outputs for two CIM alternatives. The given values for the qualitative attributes are the scores which are out of 100.

	INPUTS				OUTPUTS			
CIMs	First Costs	Management / Engineering Effort	Riskiness	Quality Results	NPV	Ease of Use	Adaptability	Expandability
CIM_I	\$600,000	80	70	55	1,250,000	40	50	40
CIM_{II}	\$950,000	50	40	85	2,300,000	80	90	85

Using crisp DEA, the following linear programming is obtained:

$$E_{\mathrm{CIM}_I} = \min\bigl(\theta - \varepsilon S_1^- - \varepsilon S_2^- - \varepsilon S_3^- - \varepsilon S_1^+ - \varepsilon S_2^+ - \varepsilon S_3^+ - \varepsilon S_4^+ - \varepsilon S_5^+\bigr)$$

or

$$E_{\mathrm{CIM}_1} = \max\bigl(\varepsilon S_1^- + \varepsilon S_2^- + \varepsilon S_3^- + \varepsilon S_1^+ + \varepsilon S_2^+ + \varepsilon S_3^+ + \varepsilon S_4^+ + \varepsilon S_5^+ + \theta\bigr)$$

s.t.

$$\begin{aligned}
600,000\,\lambda_{\mathrm{CIM}_I} + {}& 950,000\ \lambda_{\mathrm{CIM}_{II}} + S_1^- + 600,000\,\theta = 0\,,\\
80\,\lambda_{\mathrm{CIM}_I} + {}& 50\ \lambda_{\mathrm{CIM}_{II}} + S_2^- + 80\,\theta = 0\,,\\
70\,\lambda_{\mathrm{CIM}_I} + {}& 40\ \lambda_{\mathrm{CIM}_{II}} + S_3^- + 70\,\theta = 0\,,\\
55\,\lambda_{\mathrm{CIM}_I} + {}& 85\ \lambda_{\mathrm{CIM}_{II}} - S_1^+ - 55 = 0\,,\\
1,250,000\,\lambda_{\mathrm{CIM}_I} + {}& 2,300,000\ \lambda_{\mathrm{CIM}_{II}} - S_2^+ - 1,250,000 = 0\,,\\
40\,\lambda_{\mathrm{CIM}_I} + {}& 80\ \lambda_{\mathrm{CIM}_{II}} - S_3^+ - 40 = 0\,,\\
50\,\lambda_{\mathrm{CIM}_I} + {}& 90\ \lambda_{\mathrm{CIM}_{II}} - S_4^+ - 50 = 0\,,\\
40\,\lambda_{\mathrm{CIM}_I} + {}& 85\ \lambda_{\mathrm{CIM}_{II}} - S_5^+ - 40 = 0\,,
\end{aligned}$$

$$\lambda_{\mathrm{CIM}_{II}},\ \lambda_{\mathrm{CIM}_{II}},\ S_1^-,\ S_2^-,\ S_3^-,\ S_1^+,\ S_2^+,\ S_3^+,\ S_4^+,\ S_5^+ \geq 0\,.$$

The crisp solution to the above linear programming model is $\lambda_{\mathrm{CIM}_I} = 0.807$, $\lambda_{\mathrm{CIM}_{II}} = 0.102$, $S_1^- = 0.000$, $S_2^- = 7.820$, $S_3^- = 7.225$, $S_1^+ = 0.000$, $S_2^+ = 0.000$, $S_3^+ = 0.000$, $S_4^+ = 0.000$, $S_5^+ = 0.000$, $\theta = 0.9685$ and accepting $\varepsilon = 0.000000$, it is found that $E_{\mathrm{CIM}_I} = 0.9685$. In the fuzzy case, assume that the tolerance intervals of the constraints 4 and 6 are $p_4 = 10$ and $p_6 = 5$. In this case the constraints 4 and 6, respectively, become

$$\begin{aligned} &55\lambda_{\mathrm{CIM}_I} + 85\lambda_{\mathrm{CIM}_{II}} - S_1^+ - 65 = 0\,,\\ &40\lambda_{\mathrm{CIM}_I} + 80\lambda_{\mathrm{CIM}_{II}} - S_3^+ - 45 = 0\,. \end{aligned}$$

The solution of the LP above with the expanded constraints: $\lambda_{\mathrm{CIM}_I} = 0.979$, $\lambda_{\mathrm{CIM}_{II}} = 0.070$, $S_1^- = 1,900$, $S_2^- = 5.62$, $S_3^- = 5.18$, $S_1^+ = 0.000$, $S_2^+ = 9,750$, $S_3^+ = 0.000$, $S_4^+ = 0.000$, $S_6^+ = 9.900$, $\theta = 0.993$, and accepting $\varepsilon = 0.000000$, it is found $E_{\mathrm{CIM}_I} = 0.9918$. The fuzzy programming is obtained as

$$\max \Psi\,,$$

$$\theta - \varepsilon \sum_{i=1}^{3} S_i^- - \varepsilon \sum_{r=1}^{5} S_r^+ - 0.9685 \geq 0.0233\,\Psi\,,$$

$$\begin{array}{rrrrr} & 600,000\,\lambda_{\mathrm{CIM}_I} + & 950,000\lambda_{\mathrm{CIM}_{II}} + S_1^- + & 600,000\theta = 0\,, & \\ & 80\,\lambda_{\mathrm{CIM}_I} + & 50\lambda_{\mathrm{CIM}_{II}} + S_2^- + & 80\theta = 0\,, & \\ & 70\,\lambda_{\mathrm{CIM}_I} + & 40\lambda_{\mathrm{CIM}_{II}} + S_3^- + & 70\theta = 0\,, & \\ 75 - & 55\,\lambda_{\mathrm{CIM}_I} - & 85\lambda_{\mathrm{CIM}_{II}} + S_1^+ & & \geq 10\Psi\,, \\ & 1,250,000\,\lambda_{\mathrm{CIM}_I} + & 2,300,000\lambda_{\mathrm{CIM}_{II}} - S_2^+ - & 1,250,000 = 0\,, & \\ 50 - & 40\,\lambda_{\mathrm{CIM}_I} - & 80\lambda_{\mathrm{CIM}_{II}} + S_3^+ & & \geq 5\,\Psi\,, \\ & 50\,\lambda_{\mathrm{CIM}_I} + & 90\lambda_{\mathrm{CIM}_{II}} - S_3^+ - & 50 = 0\,, & \\ & 40\,\lambda_{\mathrm{CIM}_I} + & 85\lambda_{\mathrm{CIM}_{II}} - S_5^+ - & 40 = 0\,, & \end{array}$$

$$\psi, \lambda_{\mathrm{CIM}_I}, \lambda_{\mathrm{CIM}_{II}},\ S_1^-,\ S_2^-,\ S_3^-,\ S_1^+,\ S_2^+,\ S_3^+,\ S_4^+,\ S_5^+ \geq 0\,.$$

The solution of the fuzzy programming problem is $\psi = 0743$, $\lambda_{\mathrm{CIM}_I} = 0.653$, $\lambda_{\mathrm{CIM}_{II}} = 0.215$, $\theta = 0.9934$, $S_1^- = 0.000$, $S_2^- = 16.482$, $S_3^- = 15.228$, $S_1^+ = 0.000$, $S_2^+ = 60,750$; $S_3^+ = 0.000$, $S_4^+ = 0.000$, $S_5^+ = 0.000$, and accepting $\varepsilon = 0.000000$, it is found that $E_{\mathrm{CIM}_I} = 0.9873$. This is the solution that has the highest degree of membership in the decision set. The relative efficiency of CIM_{II} is 0.9873 and it is not an effective system.

5. Comparing DEA with other multicriteria decision making approaches

DEA may have a disadvantage in terms of the method's rationale if the DM is unfamiliar with LP concepts. But the concepts of productivity and relative productivity (on which the efficiency measures are based) are not difficult to comprehend by organizational decision makers, because most organizations rely significantly on ratio and productivity measures for performance. AHP is relatively intuitive, making the decision-making process less complex by decomposing the decisions to pairwise comparisons. Multiattribute utility theory (MAUT) is also a relatively simple concept to understand. Two of the difficulties in its rationale are the assumptions of dependence and independence, which may make it slightly confusing from a rationale standpoint. In the outranking approaches, the rationale could be understood by decision makers, but the actual mathematics may require more effort for comprehension of the rationale. In terms of decision-maker information requirements, DEA could carry out an analysis with a small decision-maker effort. A weakness of the DEA approach is its dependency on the data set of alternatives and its elements. Whereas, AHP and MAUT evaluations are independent of the set of alternatives (Sarkis, [20]).

6. Conclusions

Data envelopment analysis is becoming an increasingly popular management tool. There are about one hundred papers published in the international journals in 1997. Fuzzy DEA will not force the decision-maker into a precise formulation for mathematical reasons. With fuzzy DEA, the decision-maker can consider the small violations of constraints and can attach different degree of importance to violations of different constraints. This subject may be developed by other researchers, to include all the DEA models such as radial models, multiplicative models, additive models, ratio models, hyperbolic models.

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