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# CLAW-FREE AND GENERALIZED BULL-FREE GRAPHS OF LARGE DIAMETER ARE HAMILTONIAN

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ABSTRACT. A generalized $(i,j)$-bull $B_{i,j}$ is a graph obtained by identifying each of some two distinct vertices of a triangle with an endvertex of one of two vertex-disjoint paths of lengths $i, j$. We prove that every 2-connected claw-free $B_{2,j}$-free graph of diameter at least $\max\{7, 2j\}$ $(j \geq 2)$ is hamiltonian.

## 1. Introduction

In this paper we consider finite simple undirected graphs $G = \big(V(G), E(G)\big)$, and for definitions not defined here we refer to [2].

For a set $S \subset V(G)$, we denote by $N(S)$ the *neighbourhood* of $S$, i.e., the set of all vertices of $G$ which have a neighbour in $S$. If $S = \{x\}$, we simply write $N(x)$ for $N\big(\{x\}\big)$. For any subset $M \subset V(G)$, we denote $N_M(S) = N(S) \cap M$. If $H$ is a subgraph of $G$, we write $N_H(S)$ for $N_{V(H)}(S)$. The induced subgraph on a set $M \subset V(G)$ will be denoted by $\langle M \rangle$.

By $\mathrm{diam}(G)$ we denote the *diameter* of $G$, i.e., the largest distance of a pair of vertices $x, y \in V(G)$. A path with endvertices $x, y$ will be referred sometimes to as an $xy$-path. If $x, z$ are vertices at distance $\mathrm{diam}(G)$, then any shortest $xz$-path will be called a *diameter path* of $G$.

If $H_1, \dots, H_k (k \geq 1)$ are graphs, then a graph $G$ is said to be $H_1, \dots, H_k$*-free* if $G$ contains no copy of any of the graphs $H_1, \dots, H_k$ as an induced subgraph; the graphs $H_1, \dots, H_k$ will be also referred to in this context as *forbidden subgraphs*. Specifically, the four-vertex star $K_{1,3}$ will be also denoted by $C$ and called the *claw* and in this case we say that $G$ is *claw-free*. Whenever vertices of an induced claw are listed, its *centre*, (i.e., its only vertex of degree 3) is always

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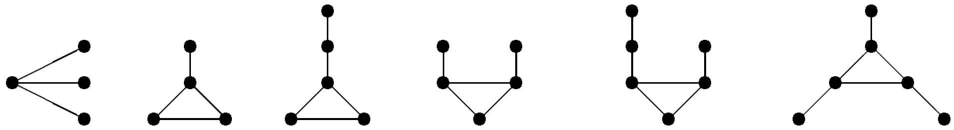


The claw $C$ $Z_1$ $Z_2$ The bull $B$ The wounded $W$ The net $N$

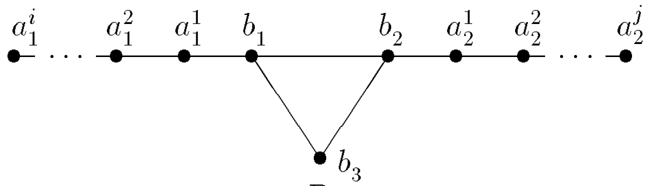
FIGURE 1

the first vertex of the list. Further graphs that will be considered as forbidden subgraphs are shown in Fig. 1.

There are many results dealing with hamiltonian properties in classes of graphs defined in terms of forbidden induced subgraphs (see, for example [9], [7], [10], [3], [4]). Bedrossian [1] (see also [8]) characterized all pairs $X, Y$ of connected forbidden subgraphs implying hamiltonicity.

**THEOREM A** [1]. *Let $X$ and $Y$ be connected graphs with $X, Y \neq P_3$, and let $G$ be a 2-connected graph that is not a cycle. Then, $G$ being $XY$-free implies that $G$ is hamiltonian if and only if (up to symmetry) $X = C$ and $Y = P_4, P_5, P_6, C_3, Z_1, Z_2, B, N$ or $W$.*

The results on hamiltonicity in $CP_i$-free, $CZ_i$-free and $CN$-free graphs were extended to larger classes (by characterizing the classes of nonhamiltonian exceptions) in [5] and [6] by using the closure concept introduced in [11]. A similar extension is possible in the class of $CB$-free graphs by introducing the class of $CB_{i,j}$-free graphs, where by $B_{i,j}$ $(i, j \geq 1)$ we denote the *generalized bull*, i.e., the graph obtained by identifying each of some two distinct vertices of a triangle with an endvertex of one of two vertex-disjoint paths of lengths $i, j$ (see Fig. 2).




$B_{i,j}$

FIGURE 2

However, as shown in [5], the closure method is not applicable in this class, since there are $CB_{i,j}$-free graphs such that their closure [11] is not $CB_{i,j}$-free.

It is easy to see that there are $CB_{i,j}$-free graphs of arbitrarily large diameter (a simple example can be obtained by taking $d+1$ vertex-disjoint cliques $K_0, K_1, \ldots, K_d$ and by adding all of the edges between consecutive cliques, namely $\{xy\colon x \in K_i,\ y \in K_{i+1},\ i = 0, 1, \ldots, d-1\}$).

In the main result of this paper we show that, for any $j \geq 2$, all 2-connected nonhamiltonian $CB_{2,j}$-free graphs have small diameter.

## 2. Main result

Before we prove the main result of the paper, Theorem 2, we first make some preliminary observations on shortest paths and their neighbourhoods.

Let $G$ be a claw-free graph, let $x, y \in V(G)$ and let $P\colon x = v_0 v_1 v_2 \dots v_k = y$ ($k \geq 3$) be a shortest $xy$-path in $G$. Let $z \in V(G) \setminus V(P)$.

(1) If $|N_P(z)| = 1$, then, since $G$ is claw-free, $z$ is adjacent either to $x$ or to $y$.

(2) If $|N_P(z)| \geq 2$ and $\{v_i, v_j\} \subset N_P(z)$, then, since $P$ is a shortest path, $|i - j| \leq 2$.

(3) By (1) and (2), $|N_P(z)| \leq 3$ for every vertex $z \in V(G) \setminus V(P)$. Moreover, if $2 \leq |N_P(z)| \leq 3$, then the vertices of $N_P(z)$ are consecutive on $P$.

This motivates the following notation:

$$N_i := \big\{z \in V(G) \setminus V(P)\colon N_P(z) = \{v_{i-1}, v_i, v_{i+1}\}\big\} \text{ for } 1 \leq i \leq k-1,$$
$$M_i := \big\{z \in V(G) \setminus V(P)\colon N_P(z) = \{v_{i-1}, v_i\}\big\} \text{ for } 1 \leq i \leq k,$$
$$M_0 := \big\{z \in V(G) \setminus V(P)\colon N_P(z) = \{v_0\}\big\},$$
$$M_{k+1} := \big\{z \in V(G) \setminus V(P)\colon N_P(z) = \{v_k\}\big\}.$$

Thus, by (1), (2) and (3), $N(P) \cup V(P) = \left(\bigcup\limits_{i=1}^{k} N_i\right) \cup \left(\bigcup\limits_{i=0}^{k+1} M_i\right) \cup V(P)$.

We further denote $S = V(P) \cup N(P)$ and $R = V(G) \setminus S$.

**Lemma 1.** *Let $j \geq 2$, let $G$ be a $CB_{2,j}$-free graph of diameter at least $\max\{7, 2j\}$ and let $P\colon v_0 v_1 v_2 \dots v_d$ be a diameter path in $G$. Then*

(i) *$\langle N_i \rangle$ is complete for $1 \leq i \leq d-1$ and $\langle M_j \rangle$ is complete for $0 \leq j \leq d+1$,*

(ii) *$M_i = \emptyset$ for $3 \leq i \leq d-2$,*

(iii) *$\langle N_i \cup N_{i+1} \rangle$ is complete for $1 \leq i \leq d-2$,*

(iv) *for every vertex $z \in R$ we have $N_P(z) = \emptyset$ and $N_S(z) \subseteq M_0 \cup M_1 \cup M_2 \cup M_{d-1} \cup M_d \cup M_{d+1}$.*

P r o o f. (i) If some $N_i$ or $M_i$ is not complete, then some $v_j$, $j \in \{i-1, i, i+1\}$, is a centre of an induced claw, a contradiction.

(ii) Suppose $M_i \neq \emptyset$ for some $i$, $3 \leq i \leq d-2$. Then, since $d \geq 2j$, for any vertex $x \in M_i$ we have $\big\langle \{v_{i-3}, v_{i-2}, v_{i-1}, x, v_i, v_{i+1}, \dots, v_{i+j}\} \big\rangle \simeq B_{2,j}$ or $\big\langle \{v_{i-j-1}, v_{i-j}, \dots, v_{i-1}, x, v_i v_{i+1}, v_{i+2}\} \big\rangle \simeq B_{2,j}$, a contradiction.

(iii) Suppose $xy \notin E(G)$ for some $i$ with $1 \leq i \leq d-2$ and two vertices $x \in N_i$, $y \in N_{i+1}$. Then either $\big\langle \{v_{i-1}, x, v_{i+1}, y, v_{i+2}, v_{i+3}, \dots, v_{i+2+j}\} \big\rangle \simeq B_{2,j}$ or $\big\langle \{v_{i+2}, y, v_i, x, v_{i-1}, v_{i-2}, \dots v_{i-1-j}\} \big\rangle \simeq B_{2,j}$, a contradiction.

(iv) By the definition of $P$ and $R$, we have $N_P(z) = \emptyset$ for every vertex $z \in R$. Since $G$ is claw-free, we have also $N_{N_i}(z) = \emptyset$ for $1 \le i \le d-1$. □

We can now state the main result of the paper.

**THEOREM 2.** *Let $j \ge 2$ be an integer and let $G$ be a 2-connected $CB_{2,j}$-free graph of diameter $d \ge \max\{7, 2j\}$. Then $G$ is hamiltonian.*

**Remark.** From [1] we know that every 2-connected $CB_{1,1}$-free or $CB_{2,1}$-free graph is hamiltonian. The graph in Fig. 3 indicates that there are 2-connected nonhamiltonian graphs of diameter $d = 6$ that are $CB_{2,j}$-free for any $j \ge 2$. The example in Fig. 4 shows that there are 2-connected nonhamiltonian graphs which are $CB_{2,j}$-free and have diameter $d = 2j-1$ for any $j \ge 3$. Hence the requirement $d \ge \max\{7, 2j\}$ in Theorem 2 is sharp.

Moreover, the example in Figure 5 indicates that there are 2-connected nonhamiltonian graphs of arbitrary diameter $d \ge 3$ which are $CB_{i,j}$-free for any pair $i, j$ such that $i \ge 3$, $j \ge i$. Hence the requirement $i = 2$ in Theorem 2 is also best possible.

It is easy to see that, in fact, each of the examples in Figures 3–5 yields an infinite family, since each of the vertical edges (marked in the figure by $K_i$) can be blown up to a clique of arbitrary order.

Proof of Theorem 2. Let $G$ be a 2-connected $CB_{2,j}$-free graph of diameter $d \ge \max\{7, 2j\}$, $j \ge 2$, and let $P\colon v_0v_1v_2\dots v_d$ be a diameter path in $G$. Let $M_i, N_i, S, R$ be as in Lemma 1. For $i \in \{0, 1, 2, d-1, d, d+1\}$ further denote $M_i^* = N_R(M_i)$.

We first make the following observation concerning the structure of $G$ "close" to the ends of $P$. Denote

$$S_0 = \bigcup_{i=0}^{2} \big(N_i \cup M_i \cup M_i^* \cup \{v_i\}\big), \quad R_0 = V(G) \setminus (S \cup S_0)$$

and

$$S_d = \bigcup_{i=d-2}^{d} \big(N_i \cup M_{i+1} \cup M_{i+1}^* \cup \{v_i\}\big), \quad R_d = V(G) \setminus (S \cup S_d)$$

(where we set $N_0 = N_d = \emptyset$). Then we have the following claims.

**CLAIM 1A.** *The subgraph $\langle S_0 \rangle$ satisfies one of the following:*

(i) $N_{R_0}(S_0) = \emptyset$, $N_2 \ne \emptyset$, *and for any $x_2 \in N_2$ there is a hamiltonian $x_2v_2$-path $P_0$ in $\langle S_0 \rangle$,*

(ii) $N_{R_0}(S_0) \ne \emptyset$, $M_1^* = M_2 = M_2^* = \emptyset$, *and* $N_{S_0}(R_0) \subset M_0^*$.

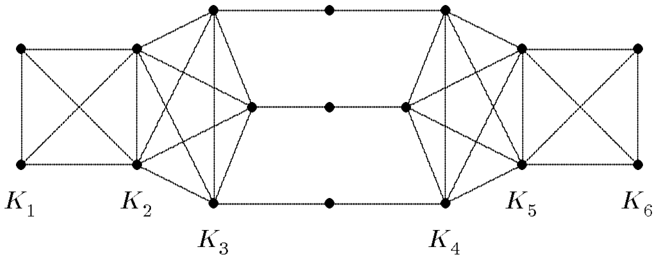



FIGURE 3

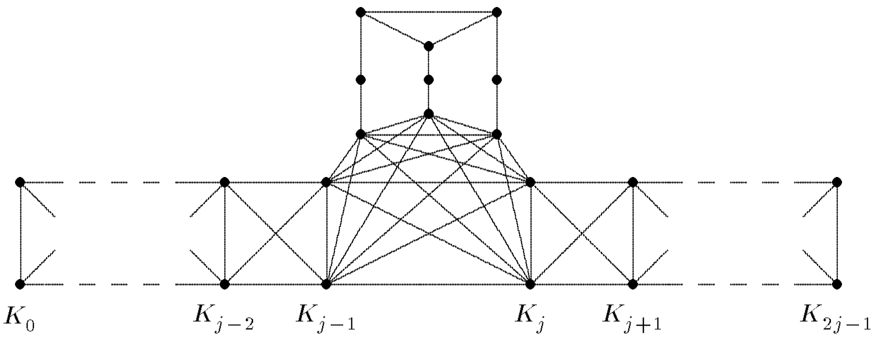



FIGURE 4

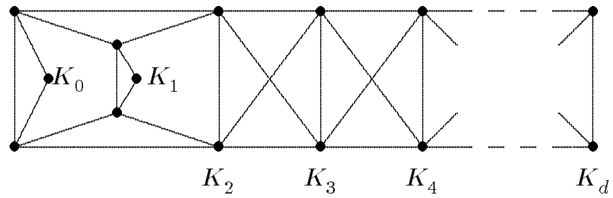



FIGURE 5

**CLAIM 1B.** *The subgraph $\langle S_d \rangle$ satisfies one of the following:*

(i) $N_{R_d}(S_d) = \emptyset$, $N_{d-2} \neq \emptyset$, *and for any $x_{d-2} \in N_{d-2}$ there is a hamiltonian $x_{d-2}v_{d-2}$-path $P_d$ in $\langle S_d \rangle$,*

(ii) $N_{R_d}(S_d) \neq \emptyset$, $M_d^* = M_{d-1} = M_{d-1}^* = \emptyset$, *and $N_{S_d}(R_d) \subset M_{d+1}^*$.*

By symmetry, it is sufficient to prove Claim 1a. We distinguish two cases.

*Case 1: $M_2 \neq \emptyset$.*

We first show that $M_0^* \subset M_2^*$. If $M_0 = \emptyset$, then obviously $M_0^* = \emptyset \subset M_2^*$. Hence we may assume that $M_0 \neq \emptyset$. Then $\langle M_0 \cup M_2 \rangle$ is complete, since otherwise, for some two vertices $x \in M_0$, $y \in M_2$ such that $xy \notin E(G)$ we have $\langle\{x, v_0, v_1, y, v_2, v_3, \dots, v_{2+j}\}\rangle \simeq B_{2,j}$, a contradiction (note that both $\langle M_0\rangle$ and $\langle M_2\rangle$ are complete by Lemma 1 $(i)$). Suppose now that $yz \notin E(G)$ for two vertices $y \in M_2$, $z \in M_0^*$. Then $\langle\{x, z, y, v_0\}\rangle \simeq C$ for a vertex $x \in M_0$, a contradiction. This implies that $yz \in E(G)$ for every $y \in M_2$, $z \in M_0^*$. But then every vertex in $M_0^*$ has a neighbour in $M_2$, i.e., $M_0^* \subset M_2^*$, as required.

Next we show that $M_1^* \subset M_2^*$. We may assume that $M_1^* \neq \emptyset$. Let $z \in M_1^*$, i.e., $xz \in E(G)$ for some $x \in M_1$, and suppose that $zy \notin E(G)$ for some $y \in M_2$. If $xy \notin E(G)$, then $\langle\{z, x, v_1, y, v_2, v_3, \dots, v_{2+j}\}\rangle \simeq B_{2,j}$, and if $xy \in E(G)$, then $\langle\{x, z, y, v_0\}\rangle \simeq C$. Hence $zy \in E(G)$, implying $M_1^* \subset M_2^*$.

Thus, we conclude that $(M_0^* \cup M_1^*) \subset M_2^*$. Now, if $N_{R_0}(S_0) \neq \emptyset$, then $yz \in E(G)$ for some two vertices $y \in M_2^*$ and $z \in R_0$, but then, for a vertex $x \in M_2$, $\langle\{z, y, x, v_1, v_2, v_3, \dots v_{2+j}\}\rangle \simeq B_{2,j}$, a contradiction. Hence $N_{R_0}(S_0) = \emptyset$.

There is also no edge from $M_2^*$ to any of $M_i$, $i \geq 3$, since $M_i = \emptyset$ for $3 \leq i \leq d-2$ by Lemma 1 (ii), and an edge from $M_2^*$ to any of $M_{d-1}$, $M_d$, $M_{d+1}$ yields a $v_0v_d$-path of length at most $6$, contradicting the fact that $P$ is a diameter path and $d \geq 7$. Consequently, $N_2 \cup \{v_2\}$ is a cutset of $G$. Since $G$ is 2-connected, $N_2 \neq \emptyset$.

Summarizing, we already know that $\langle M_0 \cup M_2\rangle$ is complete, $(M_0^* \cup M_1^*) \subset M_2^*$ and, by Lemma 1 (iii), $\langle N_1 \cup N_2\rangle$ is complete. Moreover, it is easy to see that $N(x) \cap M_2^*$ is complete or empty for all $x \in M_2$, and if $M_0 \neq \emptyset$, then $M_0^* = M_2^*$ (otherwise we have a claw with centre in $M_2$). But then it is straightforward to check that in each of the possible cases (according to whether $M_0$, $M_1$, $N_1$ and $M_2^*$ are empty or nonempty) there is a hamiltonian $x_2v_2$-path in $\langle S_0\rangle$ for any $x_2 \in N_2$. Thus, we are in situation (i) of Claim 1a.

*Case 2:* $M_2 = \emptyset$.

We first consider the subcase when $M_0^* = M_1^* = \emptyset$. This immediately implies that $N_{R_0}(S_0) = \emptyset$ and, since $G$ is 2-connected, $N_2 \neq \emptyset$. Moreover, if $M_0 \neq \emptyset$, then, since $v_0$ cannot be a cutvertex, there is an edge $xy$ with $x \in M_0$ and $y \in M_1 \cup N_1$. In all these cases, it is easy to find a hamiltonian $x_2v_2$-path in $\langle S_0\rangle$ for any $x_2 \in N_2$, i.e., we are again in situation (i) of Claim 1a.

Hence we suppose that $M_0^* \cup M_1^* \neq \emptyset$. Now, if $M_1^* \neq \emptyset$, then for any vertex $v \in M_1^*$, any shortest $vv_d$-path through $M_1$ has length $d+1$ (note that there is no path $vu_1u_2v_3 \dots v_d$ of length $d$ with $u_1 \in M_1$ and $u_2 \in N_2$, since otherwise we have $\langle\{u_1, v, v_0, u_2\}\rangle \simeq C$, a contradiction). Since $G$ has diameter $d$, there must be another $vv_d$-path $P'$ of length at most $d$. Let $w$ be the successor of $v$ on $P'$. If $w \in M_{d-1} \cup M_d \cup M_{d+1}$, then we get a $v_0v_d$-path of length at most $5$; hence $w \in R_0$. But then, for a vertex $x \in M_1$, $\langle\{w, v, x, v_0, v_1, \dots, v_{1+j}\}\rangle \simeq B_{2,j}$, a contradiction. Hence $M_1^* = \emptyset$, implying $M_0^* \neq \emptyset$.

Summarizing, we now have $M_2 = \emptyset$, $M_1^* = \emptyset$ and $M_0^* \neq \emptyset$. By the definition of $R_0$ and $M_i^*$ $(i = 0, 1, 2)$, there is no edge between $R_0$ and $M_0 \cup M_1 \cup M_2$, which implies that $N_{S_0}(R_0) \subset M_0^*$ (if nonempty). Thus, if $N_{R_0}(S_0) \neq \emptyset$, we are in situation (ii) of Claim 1a. Hence finally suppose that $N_{R_0}(S_0) = \emptyset$. Then $N(M_0^*) \subset M_0^* \cup M_0 \cup M_{d-1} \cup M_d \cup M_{d+1}$. If $N_{S_d}(M_0^*) \neq \emptyset$, we obtain a $v_0 v_d$-path of length $\ell < 7$, a contradiction. Hence $N(M_0^*) \subset M_0^* \cup M_0$, but then any vertex $x \in M_0^*$ is at distance at least $d+2$ from $v_d$, contradicting the fact that $P$ is a diameter path. Hence the claim follows. □

Suppose now that $S_0$ satisfies (i) of Claim 1a and $S_d$ satisfies (i) of Claim 1b. Then every $\{v_i\} \cup N_i$ is a cutset of $G$, and since $G$ is 2-connected, $N_i \neq \emptyset$ for $3 \leq i \leq d-3$. Let $P_i$ be a hamiltonian path in $\langle N_i \rangle$, $3 \leq i \leq d-3$. Then $x_2 P_0 v_2 P_3 P_4 \dots P_{d-3} x_{d-2} P_d v_{d-2} v_{d-3} \dots v_3 x_2$ is a hamiltonian cycle in $G$.

By symmetry, it remains to consider the case when $\langle S_0 \rangle$ satisfies (ii) of Claim 1a. Let $x \in M_0$. If $xy \in E(G)$ for some $y \in N_1$, then $\langle \{z, x, y, v_1, v_2, v_3, \dots, v_{2+j}\} \rangle \simeq B_{2,j}$ for a vertex $z \in M_0^*$, a contradiction. Hence $N_{N_1}(M_0) = \emptyset$. If $N(x) \cap (M_{d-1} \cup M_d \cup M_{d+1}) \neq \emptyset$, we get a $v_0 v_d$-path of length at most $d-1$. Since $M_i = \emptyset$ for $2 \leq i \leq d-2$, there is no $xv_d$-path in $\langle S \rangle$ of length at most $d$. Hence there is a shortest $v_d x$-path $P'$ of length $\ell$ such that $d-1 \leq \ell \leq d$ and $V(P') \cap R \neq \emptyset$. This immediately implies that $N_{R_d}(S_d) \neq \emptyset$, i.e., $\langle S_d \rangle$ satisfies (ii) of Claim 1b (specifically, the successor of $v_d$ on $P'$ is in $M_{d+1}$).

Let $v_d, v_{d+1}, \dots, v_{d+\ell} = x$ be the vertices of $P'$. If $\ell = d$, denote by $P''$ the path $v_{d+1} v_{d+2} \dots v_{2d} v_0$; otherwise set $P'' = P'$. It is apparent that $P''$ is also a diameter path. Denote by $\vec{C}$ the cycle (with an orientation) $v_0 v_1 \dots v_d v_{d+1} \dots v_{2d} (v_0)$ (of length $2d+1$ or $2d$, respectively). We show that $\vec{C}$ has no chord.

Since $P$, $P'$ and $P''$ are shortest paths, there is no chord $xy$ with $x, y \in V(P)$ or $x, y \in V(P'')$. By Lemma 1 (ii) (applied on $P$, $P'$ and $P''$), and since all the paths $P$, $P'$, $P''$ satisfy (ii) of Claim 1a, 1b, the only possible chords in $\vec{C}$ are the edges $v_{d-1} v_{d+1}$ and $xv_1$. But, if for example $v_{d-1} v_{d+1} \in E(G)$, then $v_{d+1} \in M_d$, implying $v_{d+2} \in M_d^*$, which contradicts Claim 1b (ii). Hence $v_{d-1} v_{d+1} \notin E(G)$ and, by symmetry, $xv_1 \notin E(G)$.

Now we observe that, for any $x \in V(G) \setminus V(\vec{C})$, $|N_{\vec{C}}(x)| \leq 3$. Immediately $|N_{\vec{C}}(x)| \leq 4$, since $G$ is claw-free and $\vec{C}$ is chordless. If $|N_{\vec{C}}(x)| = 4$, then $x$ has neighbours on both $P$ and $P''$. Since $M_i = \emptyset$ for $2 \leq i \leq d-1$ and no vertex in any $N_i$ can have a neighbour outside $S$, the only possibility (up to a symmetry) is $N_{\vec{C}}(x) = \{v_{d-2}, v_{d-1}, v_d, v_{d+1}\}$, but then $\langle \{v_{d+2}, v_{d+1}, x, v_{d-1}, v_{d-2}, \dots, v_{d-2-j}\} \rangle \simeq B_{2,j}$, a contradiction. Hence for every $x \in V(G) \setminus V(\vec{C})$ with $N_{\vec{C}}(x) \neq \emptyset$ we have $|N_{\vec{C}}(x)| \leq 3$ and, since $\vec{C}$ is chordless and $G$ is claw-free, $|N_{\vec{C}}(x)| \geq 2$. We can thus define analogously as before:

$$N_i^C := \{z \in V(G) \setminus V(\vec{C})\colon zv_{i-1}, zv_i, zv_{i+1} \in E(G)\},$$

$$M_i^C := \{z \in V(G) \setminus V(\vec{C})\colon zv_{i-1}, zv_i \in E(G)\}$$

for $1 \le i \le |V(\vec{C})|$ (indices are considered modulo $|V(\vec{C})|$).

By Lemma 1 (ii) (applied on $P$, $P'$ and $P''$), and by Claim 1a, 1b (ii), we have $M_i = \emptyset$ for $2 \le i \le d-1$ and $d+2 \le i \le 2d-1$ (and also $i = 2d$ if $x = v_{2d}$). But this and the fact that $\vec{C}$ has no chords implies, together with Lemma 1 (iv), that, for $t = \lceil \frac{d}{2} \rceil$, the path $P'''\colon v_t v_{t+1} \dots v_{t+d}$ is a shortest $v_t v_{t+d}$-path in $G$. Since $P'''$ has length $d$, $P'''$ is a diameter path, implying that, by Lemma 1 (iv) (applied to $P'''$), $M_d = M_{d+1} = \emptyset$. By symmetry, we also have $M_{2d-1} = M_{2d} = M_1 = \emptyset$. But then, by Lemma 1 (iv), $V(G) = \bigcup_{i=0}^{|V(\vec{C})|} (\{v_i\} \cup N_i)$. Let $P_i$ be a hamiltonian path in $\langle N_i \rangle$, $i = 0, \dots, |V(\vec{C})|$. Then $v_0 P_0 v_1 P_1 \dots v_{2d-1} P_{2d-1} v_{2d} (P_{2d} v_0)$ is a hamiltonian cycle in $G$.

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