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# A BRIEF REVIEW OF OPTIMAL DESIGNS IN CONTINGENT VALUATION STUDIES

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ABSTRACT. When an organization such as a local or health authority or an environmental agency is considering making new or increased charges for a (new) amenity it might commission a survey in which a key question would be of the form: "Are you willing to pay an amount $X$ for the (new) amenity?". This illustrates one type of Contingent Valuation (CV) study, namely, a dichotomous choice CV study with the figure $X$ being a 'Bid' value. The paper will focus on both dichotomous choice (or single bounded) CV's and on double bounded CV's (in which a second bid is offered).

## 1. Introduction

A Contingent Valuation Study is essentially a Sample Survey of a relevant population, the primary aim of which is to estimate that population's willingness to pay (WTP) for some new (or possible previously free) amenity or it might be to estimate what increase in charges the population is willing to pay for an established amenity. These amenities fall into the category of non-market goods or services. Examples of such studies arise in the areas of health or welfare e.g., payment for (provision) of medical programs (see Donaldson [1993]); recreation (e.g., payment for fishing permits); and the environment (e.g., payment for pollution reduction programs).

The first such study focussed on pollution in the Delaware River Basin, USA in 1947. A more recent example is seen in Hanley [1989] which reported a study into the Willingness to Pay of visitors to a part of the Queen Elizabeth Forest Park in Central Scotland. There was interest in four aspects; wildlife, landscape, recreation and all combined. Four WTP questions were asked. For the last category this was: 'Suppose the government was considering selling the Queen Elizabeth Forest Park to a private forestry company. This would mean people would no longer be able to visit it. If the only way

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to prevent this happening was for the Forestry Commission to raise revenue by selling day tickets to visitors, how much would you be willing to pay, per person per visit?' This kind of question is known as an open ended question.

## 2. Criticisms of CV studies

An overriding criticism is that a CV question invites a hypothetical valuation, particularly if the respondent has not previously considered the issue in question. This could lead to biases of various kinds. Bishop and Heberlein [1979] construct a list of possibilities. Broadly speaking they fall into three categories: psychological, economic and statistical. Psychological biases include 'strategic bias' arising when a respondent 'overestimates' his/her WTP if (s)he suspects that payment will not become a reality, and 'free-riding' meaning WTP is 'underestimated' to keep real fees low. Economic biases include the 'embedding problem' arising if a respondent is unable to recognize other competing demands on a finite (recreational or environmental or health) budget. Finally CV studies are as subject to 'statistical' biases as any sample survey. For example length biased sampling is a potential problem with the study of Hanley [1989] since respondents were sampled on site. Two styles of enquiry were used: a self completion questionnaire and an interview. Those who stayed longer in the park were more likely to be interviewed.

## 3. Literature

These issues have given rise to a great deal of literature which will be reviewed on another occasion. Much of this appears in the following journals: Land Economics, American Journal of Agricultural Economics and The Journal of Environmental and Economic Management. This literature is rich in its use of statistical tools including regression, and binary regression methods since other potentially relevant questions are regularly included such as general questions on income and age. We shall also see the need for optimal regression designs. One apparent lack seems to be reference to methods of analyses developed in the survey methodology arena. The literature also includes a batch of 8 papers in Volume 34 of the Natural Resources Journal a legal publication, arising from contentious litigation concerning the use of CV studies in relation to the Exon Valdez Oil Spillage in Alaska.

## 4. Variations of the WTP question

With a view to resolving some of the criticisms of CV studies a blue ribbon panel was set up in the USA chaired by Arrow (the Nobel prize winning economist) and Solow. This produced a list of 15 recommendations for the conduct of CV studies. One of these stated that a dichotomous choice WTP question should be used, one of several alternatives to the open ended question which have evolved. These include:

(1) Closed ended format (or payment card):

The respondent is offered a list (possibly on a card) of possible payments and asked to identify the one closest to his/her maximum WTP. This variation was also used in the Hanley [1989] study. It produced higher mean WTP values than the open ended case for all four aspects of interest, 'significantly so' in the case of two of them.

(2) Dichotomous Choice Format

The respondent is offered a single payment or bid and simply responds Yes or No according to his/her willingness to pay this bid. This format is also known as a Discrete Choice or Single Bounded question.

(3) Double Bounded Format

Here the respondent is offered two dichotomous choice questions. If the answer to the first bid is YES, a higher bid is offered in the second question, otherwise a lower bid is offered.

(4) Iterative Bidding

Here the respondent is offered a sequence of dichotomous choice questions, increasing or decreasing in bid-value offered according as the response to the first question is YES or NO respectively. The process stops when the response changes or the list of bids is exhausted.

One other variation on any WTP question is a WTA question which aims to identify a respondent's willingness to accept compensation for removal of a service or for foregoing a right to use an amenity. The Arrow panel recommended use of a WTP question and that this be of dichotomous choice format.

Clearly the binary responses to such a question require binary data techniques; in particular binary regression methods. Also the bids offered must be chosen. A distribution of bids across respondents is required; i.e., a binary regression design is needed.

## 5. Binary regression design

We give a brief review of results on optimal design for binary regression models. We consider the case of one design variable $x$ and a binary response $y$ such that $y \sim \mathrm{Bi}\big(1, \theta(x)\big)$, $\theta(x) = F(\alpha + \beta x)$, where $F(\cdot)$ is a chosen cumulative distribution function. This could represent the distribution of a standard latent variable $W$, i.e., $P(W \leq x) = F\big(\frac{x-\mu}{\sigma}\big) = F(\alpha + \beta x)$, $\alpha = \big(-\frac{\mu}{\sigma}\big)$, $\beta = \big(\frac{1}{\sigma}\big)$, where $\mu$ and $\sigma$ are the mean and standard deviation of $W$. Optimal designs for good estimation of $\alpha$, $\beta$, for standard cases of $F(\cdot)$ are long established in design literature. Ford, Torsney and Wu [1992] considered a wide class of distributions and used a general approach to determine local D-optimal and c-optimal designs. The latter criterion includes percentile estimation (of the distribution of $W$) which was developed by Wu [1988]. The general approach used by them is based on the idea that, under the parameter dependent linear transformation $z = \alpha + \beta x$, (the value of the random variable $Z = (\alpha + \beta W)$ if $W = x$) these nonlinear design problems transform to D-optimal or c-optimal design problems for weighted linear regression in $z$, the weight function being

$$w(z) = \frac{f^2(z)}{F(z)\big[1 - F(z)\big]}\,,$$

where $f(z) = F'(z)$ the density of $F(\cdot)$. The dependence of the transformation on the parameters illustrate that local optimal designs for nonlinear models depend on the true values of the parameters. Provisional estimates of them will be needed. This model in turn is equivalent to a two parameter linear regression model with two regressors forming the design vector

$$\underline{g}(z) = \sqrt{w(z)}\;(1, z)^T\,.$$

If the original design variable $x$ is restricted to an interval [c,d], this will transform to an interval [a,b] of values for $z$. Let

$$G = \big\{\underline{g}(z)\colon z \in Z = [a, b]\big\}\,.$$

We call $G$ the induced design space. An essential requirement is that the elements of G should be bounded. It turns out that, for a wide variety of choices of $F(\cdot)$, this is true for all real $z$, as Figure 1 illustrates. These depict closed convex loci for $Z = (-\infty, \infty)$. Ford *et al.* [1992] explored construction of D-optimal and c-optimal designs for all choices of $Z = [a, b]$. Earlier results mainly considered $Z = (-\infty, \infty)$ only.

Necessary and sufficient conditions for optimal designs are of course given by the general equivalence theorem. However there are equivalent geometrical characterizations in terms of $G$ for the above two criteria which Ford *et al.* [1992] exploited. In the case of D-optimality, which aims to minimize the determinant of the asymptotic covariance matrix of the parameter estimates, this characterization is that the support points of a D-optimal design are points of contact between $G$ and the smallest ellipsoid centred on the origin containing $G$. This idea was first conjectured by Silvey [1972] and proved by Sibson [1972], both of these being contributions to discussion of Wynn [1972].

Hence a study of a plot of $G$ can be informative. We may be able to identify some support points, or at least draw inferences about their number. From Caratheodory's theorem we know that this number is 2 or 3. If there are two support points the optimal weights are $\frac{1}{2}$. If there are three support points there is an explicit solution for the optimal weights as first reported by Torsney and Musrati [1993]. If these points are $z_1, z_2, z_3$, the respective optimal weights $p_1, p_2, p_3$ are given by

$$p_i = D_i/(D_1 + D_2 + D_3)\,, \qquad i = 1, 2, 3\,,$$

where

$$\begin{aligned} D_i &= D_{jk}/(D_{ij} + D_{ik} - D_{jk})\,, \qquad && (i,j,k) = (1,2,3)\,, (2,1,3), (3,1,2)\,, \\ D_{ij} &= w(z_i)w(z_j)(z_i - z_j)^2, && (i,j) = (1,2), (1,3), (2,3)\,. \end{aligned}$$

These various results can often reduce the problem of constructing D-optimal designs to optimization of a function with respect to one or two variables. From the plots of $G$ it appears that there will be two support points for any $Z = [a, b]$ in the case of Figures 1-a, 1-b, 1-c but not 1-d. This is indeed the case.

The c-optimal criterion aims to minimize the asymptotic value of $\mathrm{Var}(\underline{c}^T\hat{\underline{\lambda}})$ where $\underline{c}^T\underline{\lambda}$ is a known linear combination of the unknown parameters $\underline{\lambda} = (\alpha, \beta)^T$. Under the transformation $z = \alpha + \beta x$, this transforms to another c-optimal criterion. The geometrical characterization of a c-optimal design, due to Elfving [1952] (see also Chernoff [1979]) is based on identifying the boundary of the convex hull of $G \cup \{-G\}$. The vector $\underline{c}$ extended if necessary will cut this at a point which is a convex combination of points in G and $\{-G\}$. These points are the support points and the convex weights are the optimal weights. Algebraic solutions for these weights are given in Kitsos, Titterington and Torsney [1988]. In the case of two parameter models there must exist a design with 1 or 2 support points. Ford *et al.* [1992] derived c-optimal designs for all vectors $\underline{c}$ and for all choices of $Z = [a, b]$, identifying, in particular, changes from one to two-point designs. It is clear from the convexity of the plots of $G$ that there will be one-point designs for many choices of $\underline{c}$. Wu [1988] extended this work to percentile estimation as noted above. A particular conclusion is that for the choices of $F(\cdot)$ considered, the optimal design for estimating the median is a one-point design taking all observations at the (current provisional estimate) of the median. This transforms to $z = 0$ in the case of the normal and the logistic choices of $F(\cdot)$.

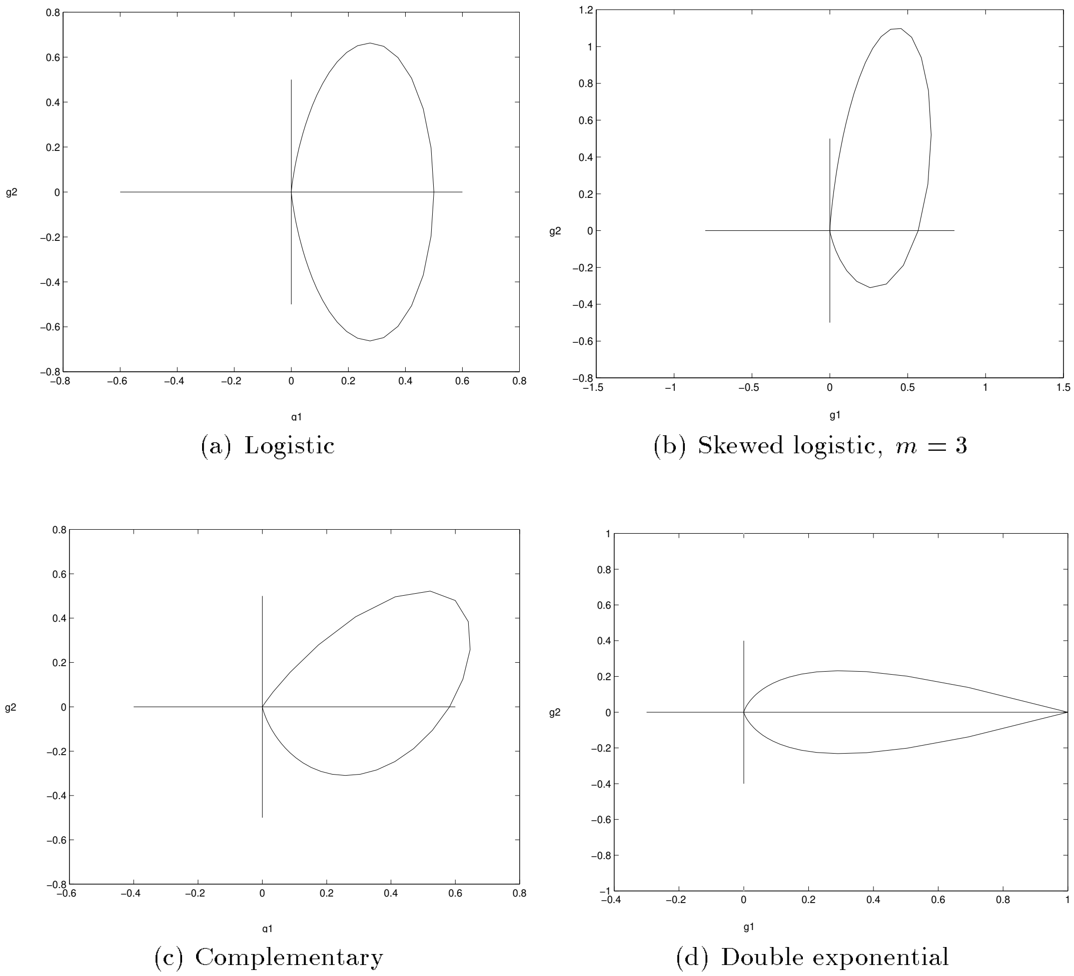


FIGURE 1. Plots of $G(Z)$ for two parameters binary weight functions, $g_1 = \sqrt{w(z)}$, $g_2 = z\sqrt{w(z)}$, and $-20\ leqz \leq 20$.

## 6. Design of single bid dichotomous choice CV studies

In a single bid dichotomous choice CV study a bid value, say $x$, must be chosen for a respondent. This is a design variable. An axiomatic assumption is that his/her response will be YES if WTP $\geq x$ and NO otherwise, where WTP is the respondent's true willingness to pay. In order to apply design theory we need to make assumptions about the distribution of WTP across the population of interest. Common assumptions in the CV literature have been that WTP or $\ln$ WTP has a logistic or normal distribution. The logistic distribution is given a utility function theory justification.

Let $G(\cdot)$ denote the cumulative distribution function of WTP, so that $G(x) = P(\text{WTP} \leq x)$. It is convenient to assume that

$$G(x) = G_0\left(\frac{h(x) - \mu}{\sigma}\right),$$

where $h(x)$ is an increasing function, $\mu$ and $\sigma$ can be interpreted as a location and a scale parameter of $h(\mathrm{WTP})$, and $G_0(\cdot)$ is a standardized distribution.

It is natural to focus on modelling the probability of a YES response.

$$\begin{aligned}
\Pr(\mathrm{YES}|\mathrm{BID} = x) &= \Pr(\mathrm{WTP} \geq x) \\
&= 1 - G(x) \\
&= 1 - G_0\Big(\frac{h(x) - \mu}{\sigma}\Big) \\
&= F\Big(\frac{\mu - h(x)}{\sigma}\Big) \\
&= F\big(\alpha + \beta h(x)\big) \\
&= F(z),
\end{aligned}$$

where

$$\begin{aligned}
z &= \alpha + \beta h(x), \\
\alpha &= \mu/\sigma, \quad \beta = (-1/\sigma), \\
F(z) &= 1 - G_0(-z).
\end{aligned}$$

Note that the function $F(z)$ satisfies the properties of a cumulative distribution function. Also $\beta$ should be negative. Thus we have formulated a binary regression model, with the variable $z$ representing a standardized design variable, like that conceived by Ford *et al.* [1992]. Our CV design problem can be transformed to a weighted linear design problem with weight function

$$w(z) = \frac{f^2(z)}{F(z)\big[1 - F(z)\big]}.$$

Optimal designing here means choosing bids for respondents (possible values of WTP). These should be determined by a design or distribution which optimizes a chosen criterion. If the criterion is good estimation of the median by minimizing the asymptotic variance of its estimate the design is to take all observations at the currently believed value of the median. If however we wish good estimation of both parameters of the model by maximizing the D-optimal criterion then, for most choices of $F(\cdot)$, two bids should be used, 50 % of respondents being offered each bid. Note since the D-optimal design is invariant to linear transformation of design points, the D-optimal design for $\alpha$ $\beta$ is also D-optimal for $\mu$, $\sigma$. Also the bid values will correspond to values of $z$.

To completely define our problem we need to be clear about the design interval for $z$. Clearly WTP and hence $x$ is positive (although some of the CV literature entertain the notion of allowing for negative WTP). It follows that $h(x) = \ln(x)$

is unrestricted in value, in which case distributions such as the standardized normal or logistic are feasible choices of $F(\cdot)$. This of course implies that WTP is log-normal or log-logistic. However in the early CV literature raw WTP has been assumed to be normal or logistic.This corresponds to $h(x) = x$. Whether or not these are reasonable assumptions will in part depend on parameter values. Data quoted by Nyquist [1992] did not seem to be a good fit to the logistic yet the author based a simulation study on the parameter estimates.

Assuming that these distributions are a good fit there may still be implications for the design space, arising from the non-negativity constraint on $x$ if this imposes constraints on $h(x)$. The prime example is $h(x) = x$. Then $z = \alpha + \beta x \leq \alpha$, if $\beta < 0$. So a largest design interval for $z$ is $Z = (-\infty, b]$, $b = \alpha$. Possibly this should be further restricted to a finite interval $[a, b]$ a transformation of limits c and d on $x$ (whatever is $h(x)$, where $c$ is a minimum viable charge and $d$ is a maximum politically acceptable price).

However this issue has been ignored in the CV literature on design. These have effectively assumed $Z = (-\infty, \infty)$, in which case the standardized support points quoted may not transform back to positive WTP values. Kanninen [1993] for the logistic and Alberini [1995] for the normal report three (classes of) such designs namely: a D-optimal design, the design for minimizing the asymptotic variance of the estimate of the median (c-optimality) and designs (which depend on sample size) for minimizing the width of a fiducial interval estimate of the median. The c-optimal design is the one point design placing all weight at $z = 0$ and hence at the (currently known) median WTP-value. The others are symmetric designs in $z$ placing equal weight at values $\pm z^*$, where $z^*$ maximizes the relevant criterion over such symmetric designs. In the case of the D-optimal criterion these values are well known for the logistic and normal choices of $F(\cdot)$, namely $z^* = 1.543, 1.138$ respectively.

These same authors went on to consider Double Bounded CV studies to which we now turn.

## 7. On design of double bounded CV studies

Recall that a Double Bounded CV study presents each respondent with two bids, the second being higher or lower than the first according as the response to the first bid is Yes or No. The above two authors report constrained optimal designs for these bids under which the first bid is set equal to the currently known median i.e., $z = 0$ and the second bid is $+z^*$ or $-z^*$ according as the answer to the first bid is Yes or No; $z^*$ being chosen to optimise the relevant criterion. They report the values of $z^*$ for the criteria for which they reported single bid designs, and for the same distribution. A crucial further assumption was that this distribution was assumed to be the same at both bids. To distinguish this approach from the following we call it the Univariate Approach.

Alberini [1995] relaxes this assumption. He considers the notion that a respondent has two WTP-values, $\mathrm{WTP}_1$ and $\mathrm{WTP}_2$ at the two bids respectively. Hence we call this the Bivariate Approach. There would be a justification for this if there was a time lag between offering the two bids thereby allowing for a change in opinions. Alternatively some argue that the respondent may react to the first bid, resulting in a revision of their opinions. The author assumes a bivariate normal distribution for $(\ln \mathrm{WTP}_1,\ \ln \mathrm{WTP}_2)$ with a common mean $\mu$, a common standard deviation $\sigma$ and a correlation $\rho$. Thus $\exp^{\mu}$ is median WTP. Let $x_1$ and $x_2$ be the two bids to be offered. Then standardized design variables (w.r.t. $\mu$, $\sigma$) are $z_i = \left[(\ln x_i) - \mu\right]/\sigma$. For fixed or known $\rho$ then, this is a two-parameter model. Alberini [1995] determines constrained c-optimal designs under which $z_1 = 0$ for all respondents and for YES responses to this bid

$$z_2 = \begin{cases} +z^*, & \text{with probability } \lambda\,, \\ -z^*, & \text{with probability } 1-\lambda\,, \end{cases}$$

while for NO responses

$$z_2 = \begin{cases} -z^*, & \text{with probability } 1-\lambda\,, \\ +z^*, & \text{with probability } \lambda\,. \end{cases}$$

with $z^*$ and $\lambda$ are chosen optimally. Their values depend on $\rho$. Note that an implication is that some of those who respond YES to the first bid may be offered a lower bid and vice versa.

## 8. Designs for second bids

We now advocate an alternative approach based on the idea that a design for the second bid of a double bounded CV question should, where possible, be based on the conditional distribution of WTP given the response, YES or NO, at the first bid; that is the c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$ above should take the relevant (standardized) conditional forms. We have explored this idea for both the univariate and bivariate approaches. We summarize some results since details are beyond the scope of this paper.

Suppose in the univariate approach that the first bid is $x$. Then our standardized design variable is $z = \alpha + \beta h(x)$ where $h(x)$ is an increasing function, $\alpha = \mu/\sigma$ and $\beta = (-1/\sigma)$ Let $c = z$. Suppose that the answer to the first bid is NO, so that $\mathrm{WTP} < x$. Hence the random variable $U = \alpha + \beta h(\mathrm{WTP}) > c$ if $\beta < 0$. In this case the design interval must be $Z = [a, b]$ where $a$ is strictly greater than $c$. We have considered the case $b = \infty$. Depending on the value of c sometimes there are two supports, sometimes three (of which one is $a$).

For the bivariate approach we assume that, for some increasing function $h(\cdot)$, the pair $h(\mathrm{WTP}_1)$, $h(\mathrm{WTP}_2)$ has a joint distribution indexed by a common location parameter $\mu$, a common scale parameter $\sigma$, and a third parameter, say $\rho$, measuring correlation or possibly some other form of dependence. Further we assume that the standardized variables $Z_i = (h(\mathrm{WTP}_i) - \mu)/\sigma$, $i = 1, 2$, have a joint distribution indexed only by $\rho$. The results obtained suggest that the D-optimal design for the second bid conditional on the response at the first bid has two support points as is the case in the single bounded approach. Note no restriction is imposed on the design interval by the response to the first bid.

## 9. Future work

We have reviewed with some minor criticisms the use of optimal designs in the case of CV studies with dichotomous choice questions and have briefly reported on new designs for the 2nd question of a double bounded dichotomous choice question conditional on the response to the first bid.

There remains much to do including deriving designs, in respect of 'D-optimal' and 'median' oriented criteria, of the following kinds :

(i) Designs for both stages of a double bounded CV study-univariate and bivariate cases. This will involve theory of designs for multivariate responses.

(ii) Designs for the bivariate approach when the dependence parameter is treated as unknown. This is a consideration for both the univariate and bivariate approach.

(iii) Designs for the bivariate approach to double bounded CV studies when different location and scale parameters are assumed at the two bids. This would seem a natural extension of the common location/scale case although the choice of criterion is possibly unclear. Good estimation of the parameters of the second bid may be of greater importance $c$.

(iv) Designs when finite limits are imposed on bids for both the double bounded and single bounded cases.

(v) Designs for optimal bids when other explanatory variables are included in a model for WTP.

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