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SIMPLIFICATION OF PALEOSTRESS GRID SEARCH ALGORITHMS

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Abstract: The paper presents a way to apply the graphic solution for shear stress vector on a generally oriented plane, presented by Fry (1992), in a grid search stress method. The analytical form of the Fry's solution can be used in each of the loops of available grid search routines (e.g. Gephart 1990b; Hardcastle & Hills 1991), computing the reactivation of the fault data set by each of the tested stress tensors. It provides an alternative way of the stress computation, compared with both Bott's (1959) and Fleischmann & Nemčok's (1991) approach. This computation involves just a few simple equations omitting the time consuming goniometric functions.

Key words: shear stress calculation, palaeostress method.

Introduction

Since Carey & Brunier's (1974) numeric method for stress tensor computation a large number of numeric routines have been developed. These methods usually make three basic assumptions: 1 - that the stresses were sufficient to cause slip (obeying the Coulomb-Mohr concept), 2 - that the direction of slip immediately after failure was parallel to the direction of maximum shear stress acting immediately before failure (implying that local heterogeneities that might inhibit the free slip on each fault plane - including interactions with other fault planes - are relatively insignificant), and 3 - that the data reflect an uniform stress field (both spatially and temporally) in the region of study - this requires that there has been no post-slip deformation of the region which would alter the fault orientations.

As proposed by Fleischmann & Nemčok (1991), the methods can be divided into two characteristic groups: indirect inversion methods, which use slip on faults to find the average stress state (e.g. Carey & Brunier 1974; Angélier 1979, 1984; Etchecopar et al. 1981; Lisle 1987, 1988; Reches 1987), and direct inversion methods, which use some assumption about the stress configuration and test the slip vectors on the fault planes (Gephart & Forsyth 1984; Hardcastle 1989; Gephart 1990a, b; Hardcastle & Hills 1991; Fleischmann & Nemčok 1991).

In the direct inversion methods, some estimate of the stress tensor is given systematically to cover the possible range of expected positions (grid of stress tensors). Bott's (1959) formulas were used as a basic tool for the shear stress computation. The inversion approach is given by the description of misfit for a given stress tensor by the angular difference between predicted and observed plane and slip direction. It has been demonstrated by Gephart & Forsyth (1984) that direct inversion techniques may provide a more robust stress tensor evaluation than other methods. They can be improved and speeded up either by the new misfit description or by new shear stress approaches.

The goal of this paper is to implant the new numerical solution for the shear stress, proposed by Fry (1992) in the geometric form, to the grid search methods.

Method

The method of Fry (1992) takes the principal stress axes as co-ordinate axes. It finds a vector **v** lying in the $\sigma_1\sigma_3$ plane at an angle V = arctan $[((\sigma_3 - \sigma_2)n)/((\sigma_1 - \sigma_2)l)]$ from the plotted σ_1 direction (Fig. 1). The shear stress vector s is given by the

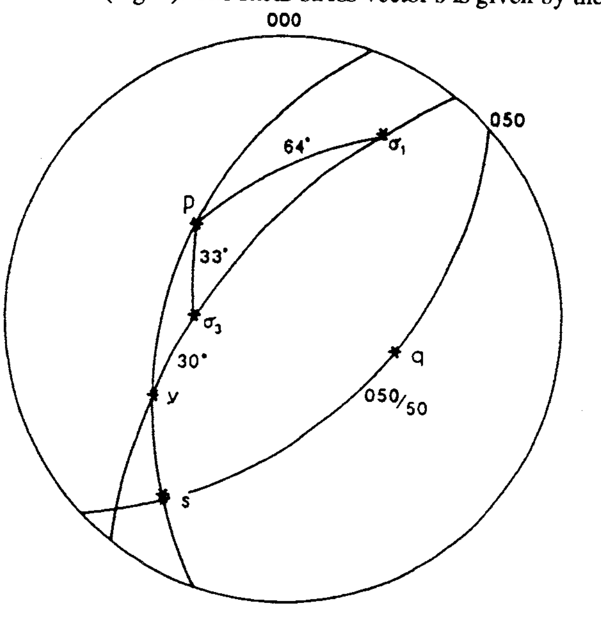

Fig. 1. The example used as Fig. 1 of Fry (1992), modified to accord with symbols used in this text. The projection is lower hemisphere equal-angle stereogram. The fault plane, strike 050, dip 50SE, is represented by its trace and by its normal **p**. The direction v, to be calculated, lies in the plane of greatest and least principal stress, where in this example σ_1 trends 030, plunge 18 and σ_3 trends 274, plunge 55. The angle between σ_1 and v is calculated from the angles from **p** to the principal stress axes and the stress ratio (in this example σ_1 = 400, σ_2 = 300, σ_3 = 210). In this case the calculated v lies in the upper hemisphere, so the direction labeled v is its opposite. **q** is the pole to the v, **p** plane, and the shear direction s is the pole to the plane containing **p** and **q**.

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intersection of the v, p plane with the fault plane (Fig. 1). The detailed justification of this solution is given in Fry (1992).

Numerical implementation of Fry (1992)

The reduced stress tensor (sensu Angélier 1989) and fault plane are the known data. Principal stress orientations provided as trend T_σ and plunge P_σ are converted to direction cosines $(l_\sigma, m_\sigma, n_\sigma)$:

$$l_\sigma = \sin T_\sigma \cos P_\sigma$$
$$m_\sigma = \cos T_\sigma \cos P_\sigma \quad (1)$$
$$n_\sigma = -\sin P_\sigma$$

and give components of unit vectors:

$$\sigma_1 = (x_1, y_1, z_1)$$
$$\sigma_3 = (x_3, y_3, z_3) \quad (2)$$

Each fault plane given by the dip direction D and dip d provides the trend T_n and plunge P_n of its normal p as follows:

$$T_p = D + 180$$
$$P_p = 90 - d \quad (3)$$

They provide directional cosines of the normal p following equations (1):

$$l_p = \sin(D + 180)\cos(90 - d)$$
$$m_p = \cos(D + 180)\cos(90 - d) \quad (4)$$
$$n_p = -\sin(90 - d)$$

Satisfying the unit vector equation:

$$l_p^2 + m_p^2 + n_p^2 = 1 \quad (5)$$

Information on magnitudes of principal stresses is provided as a stress ratio. Etchecopar et al. (1981) use $R = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ with a range $0 \le R \le 1$ providing reduced principal stress magnitudes $\sigma_1 = 1$, $\sigma_3 = 0$ and $\sigma_2 = R$. If input in this form, stress ratio is converted to the form:

$$E = -R/(1 - R) \quad (6)$$

or it can be input directly as E, where $E = (\sigma_3 - \sigma_2)/(\sigma_1 - \sigma_2)$. Then, cosines l and n of angles between p and σ_1 and p and σ_3, respectively (Fig. 1), are computed:

$$l = l_p x_1 + m_p y_1 + n_p z_1$$
$$n = l_p x_3 + m_p y_3 + n_p z_3 \quad (7)$$

Then a non-unit vector $\mathbf{v} = (v_1, v_2, v_3)$ lying in the $\sigma_1\sigma_3$ plane can be computed as:

$$\mathbf{v} = l\sigma_1 + En\sigma_3 = l\sigma_1 - [(\sigma_3 * Rn/(1 - R)] \quad (8)$$

As proven by Fry (1992), the shear stress vector is given by the intersection of the v, p and fault planes. In order to compute this intersection, the non-unit normal $\mathbf{q} = (q_1, q_2, q_3)$ of the v, p plane is first computed as the cross product of v and p vectors:

$$q_1 = v_2 n_p - v_3 m_p$$
$$q_2 = v_3 l_p - v_1 n_p \quad (9)$$
$$q_3 = v_1 m_p - v_2 l_p$$

The intersection of the v, p and fault planes is parallel to non-unit vector $\mathbf{s} = (s_1, s_2, s_3)$, given by the cross product of the fault plane normal p and the non-unit normal q of the v, p plane:

$$s_1 = q_2 n_p - q_3 m_p$$
$$s_2 = q_3 l_p - q_1 n_p \quad (10)$$
$$s_3 = q_1 m_p - q_2 l_p$$

The trend T_s and plunge P_s of the shear stress vector $\mathbf{s} = (s_1, s_2, s_3)$ are recomputed as:

$$T_s = \arctan(s_1/s_2)$$
$$P_s = \arcsin(s_3/(s_1^2 + s_2^2 + s_3^2)^{1/2}) \quad (11)$$

where the denominator in the equation with T_s cannot be 0. If the denominator equals 0, the trend is either 90° or 270° and will need to be treated specially.

Direct inversion

As discussed by Fleischmann & Nemčok (1991), the tensor search tests all possible stress orientations at all possible relative magnitudes. Magnitudes are simulated by the fixed reduced $\sigma_1 = 1$, $\sigma_3 = 0$ and $\sigma_2 = R$ values. The grid search can be prescribed by three numbers following Gephart (1990b). The σ_1 axis can be chosen as principal axis and described by its trend and plunge. The other two axes are fixed by a pitch of σ_2 in the plane normal to σ_1. The grid search is then implemented by selecting a number of specific primary σ_1 directions on a fixed grid. In the plane perpendicular to each of these σ_1 directions, a number of specific σ_3, σ_2 directions are chosen. At each position of the stress tensor, values of R are tested in chosen increments through the <0, 1> interval.

The decision, which tensor is the best solution for the given fault database can be obtained by various algorithms; e.g. as a sum of deviations between various components of the shear stress vector s computed for each of the fault planes and the measured shear stress direction on each of the fault planes (Carey & Brunier 1974; Angélier 1979). Usually, a check by Mohr's construction is used to test, whether each of the stress configurations produces the slip on the faults obeying the Coulomb-Mohr criteria.

The suggested shear stress computation can be used for each of the tested stress states in the grid search routines to infer the shear stress vector on each of the faults of the database. This computation with a set of a few simple equations provides an efficient alternative way of the shear stress calculation, compared to existing routines (e.g. Bott 1959; Fleischmann & Nemčok 1991).

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References

Angélier J., 1979: Determination of the mean principal directions of stresses for a given fault population. *Tectonophysics*, 56, 17-26.

Angélier J., 1984: Tectonic analysis of fault slip data sets. *J. Geophys. Res.*, 89, 5835-5848.

Angélier J., 1989: From orientation to magnitudes in paleostress determinations using fault slip data. *J. Struct. Geol.*, 11, 37-50.

Bott M.H.P., 1959: The mechanics of oblique - slip faulting. *Geol. Mag.*, 96, 109-117.

Carey E. & Brunier B., 1974: Analyse théorique et numérique d'un modèle méchanique élémentaire appliqué l'étude d'une population de failles. *C. R. Hebd. Séances. Acad. Sci. Sér. D.*, 279, 891-894.

Etchecopar A., Vasseur G. & Daignières M., 1981: An inverse problem in microtectonics for the determination of stress tensors from fault striation analysis. *J. Struct. Geol.*, 3, 51-65.

Fleischmann K.H. & Nemčok M., 1991: Paleostress inversion of fault - slip data using the shear stress solution of Means (1989). *Tectonophysics*, 196, 195-202.

Fry N., 1992: Direction of shear. *J. Struct. Geol.*, 14, 253-255.

Gephart J.W., 1990a: Stress and the direction of slip on fault planes. *Tectonics*, 9, 845-858.

Gephart J.W., 1990b: FMSI: a FORTRAN program for inverting fault/slickenside and earthquake focal mechanism data to obtain the regional stress tensor. *Computers & Geosciences*, 16, 953-989.

Gephart J.W. & Forsyth D.W., 1984: An improved method for determining the regional stress tensor using earthquake focal mechanism data: Application to the San Fernando earthquake sequence. *J. Geophys. Res.*, 89, 9305-9320.

Hardcastle K.C., 1989: Possible paleostress tensor configurations derived from fault - slip data in eastern Vermont and western New Hampshire. *Tectonics*, 8, 265-284.

Hardcastle K.C. & Hills L.S., 1991: BRUTE3 and SELECT: Quickbasic 4 programs for determination of stress tensor configurations and separation of heterogeneous populations of fault - slip data. *Computers & Geosciences*, 17, 23-43.

Lisle R.J., 1987: Principal stress orientations from faults: an additional constraint. *Annales Tectonicae*, 1, 155-158.

Lisle R., 1988: ROMSA: a BASIC program for paleostress analysis using fault - striation data. *Computers & Geosciences*, 14, 255-259.

Means W.D., 1989: A construction for shear stress on a generally oriented plane. *J. Struct. Geol.*, 11, 625-627.

Reches Z., 1987: Determination of the tectonic stress tensor from slip along faults that obey the Coulomb yield condition. *Tectonics*, 6, 849-861.