
RESEARCH ARTICLE

Frege on Identity and Co-Reference

Eros Corazza*

Received: 1 September 2020 / Revised: 26 October 2020 / Accepted: 28 December 2020

Abstract: In “Über Sinn und Bedeutung” (1892) Frege raises a problem concerning identity statements of the form $a=b$ and he criticizes the view he holds in the *Begriffsschrift* (1879, § 8). In building on a suggestion by Perry (2001/12, ch. 7) I will show how Frege’s *Begriffsschrift* account can be rescued and how Frege’s 1892 criticism of his *Begriffsschrift*’s position somewhat miss the point. Furthermore, the *Begriffsschrift*’s view can be developed to account in quite an elegant way to the so-called Frege’s Puzzle without committing to the sense/reference (Sinn/Bedeutung) distinction Frege introduces in “Über Sinn und Bedeutung”. To do so we have, though, to give up the idea that all the relevant information conveyed by the utterance of a simple sentence is encapsulated into a single content. I will show of this can be done in adopting a Perry-style pluri-propositionalist model of communication.

Keywords: Frege; Perry, identity co-reference.

1. Introduction

In “Über Sinn und Bedeutung” (1892) Frege raises a problem concerning identity statements of the form $a=b$ and he criticizes the view he holds in

* The University of the Basque Country. Ikerbasque, Basque Foundation for Science. Carleton University

 <https://orcid.org/0000-0001-8074-7777>

 The University of the Basque Country UPV-EHU, Donostia, Spain.

 eros.corazza@carleton.ca



the *Begriffsschrift* (1879, § 8). In building on a suggestion by Perry (2001/12, ch. 7) I will show how Frege's *Begriffsschrift* account can be rescued and how Frege's 1892 criticism of his *Begriffsschrift*'s position somewhat misses the point. Furthermore, the *Begriffsschrift*'s view can be developed to account in quite an elegant way to the so-called Frege's Puzzle without committing to the sense/reference (*Sinn/Bedeutung*) distinction Frege introduces in "Über Sinn und Bedeutung". To do so we have, though, to give up the idea that all the relevant information conveyed by the utterance of a simple sentence is encapsulated into a single content. I will show how this can be done in adopting a Perry-style pluri-propositionalist model of communication.

2. The *Begriffsschrift*'s account

The famous passage in the *Begriffsschrift* where Frege discusses identity, reads as follows:

Equality of content differs from conditionality and negation by relating to names, not to contents. Elsewhere, sign are mere proxies for their contents, and thus any phrase they occur in just expresses a relation between their various contents; but names at once appear *in propria persona* so soon as they are joined together by the symbol of equality of content; for this signifies the circumstance of two names' having the same content. Thus, along with the introduction of a symbol for equality of content, all symbols are necessarily given a double meaning—the same symbols stand now for their own content, now for themselves. (Frege 1879, § 8)¹

Frege then goes on to introduce the new notation: the three horizontal strokes symbol, ‘≡’, that stands for the identity of content. We need a sign for the identity of content because, in many cases, the same content can but be given by different names insofar as there must be different modes of

¹ I am adopting Geach's translation (in Geach & Black eds. 1952).

determination (*Bestimmungsweise*) for the same content.² To do so he proposes a geometrical example and he concludes that:

The name B thus has in this case the same content as the name A: and yet we could not antecedently use just one name, for only the answer to the question justify our doing so. The same point is determined in a double way: (i) it is directly given in experience, (ii) it is given as the point B corresponding to the straight line's being perpendicular to the diameter. To each of these two ways of determining it there answers a separate name. The need of a symbol for equality of content thus rests on the following fact: The same content can be fully determined in different ways; and *that*, in a particular case, *the same* content actually is given by *two ways of determining it*, is the content of a *judgment*. Before this judgment is made, we must supply, corresponding to the two ways of determination, two different names for the thing thus determined. The judgment needs to be expressed by means of a symbol for equality of content, joining the two names together. It is clear from this that different names for the same content are not always just a trivial matter of formulation; if they go along with different ways of determining the content, they are relevant to the essential nature of the case. In these circumstances the judgment as to equality of content is, in Kant's sense, synthetic. (Frege 1879, § 8)

In short, modes of determination are what triggers Frege to introduce the notion of identity of content, expressed by ‘ \equiv ’. For, if Frege were simply focusing on the content, as he does when using mathematical examples (see e.g. § 1 of the *Begriffsschrift*), he would merely use the ‘=’ sign.³ It is because the same content sometimes can be given only *via* different modes of determination, and thus by using two different names, that Frege appeals to the identity of content symbol ‘ \equiv ’.

² An interesting question would be to investigate how the notion of modes of determination of the *Begriffsschrift* relates to, and somewhat anticipates, the notion of modes of presentation of “Über Sinn und Bedeutung”. On this particular question see Simon (1995).

³ For a discussion on why Frege does not use mathematical examples when discussing the identity of content (expressed by ‘ \equiv ’) see May (2001).

What does a statement of the form $a \equiv b$ mean? If we stick with what Frege textually says in the passage quoted: “this signifies the circumstance of two names’ having the same content”. Thus, when the names ‘*a*’ and ‘*b*’ flank the ‘ \equiv ’ sign, it comes to mean that ‘*a*’ and ‘*b*’ have the same content.

In the *Begriffsschrift*, previous to the introduction of the sense/reference (*Sinn/Bedeutung*) distinction in his 1890 essays, Frege assumes that the content of a name is exhausted by what the name stands for. Thus, ‘Tully’, in the utterance of a simple sentence like “Tully is Roman” stands for Tully, the referent (or object) designated by the tokened name. Given that Tully is Cicero the names ‘Tully’ and ‘Cicero’ stand for the same object. There is a difference, though, between an utterance of “Tully is Roman” and one of “Cicero is Roman”. It is in order to capture this difference that Frege goes on to introduce the ‘ \equiv ’ symbol for identity of content. Let us consider:

- (1) Tully is Roman
- (2) Cicero is Roman

Since ‘Tully’ and ‘Cicero’ have the same content, (1) and (2), express the same content. If we adopt the notion of proposition, we could thus say that (1) and (2) express the same proposition, i.e. that Tully/Cicero is Roman.⁴ A constituent of such a proposition is the object (the referent or content of the names appearing in subject position). Consider now a simple, *modus ponens*, inference like:

- (3)
 - a. If Tully is Roman, then Tully is European
 - b. Tully is Roman
 - c. Therefore: Tully is European

From (3a) and (3b), though, we cannot infer:

- (4)
 - a. Therefore: Cicero is European

⁴ To be precise, though, we should talk about state-of-affairs or circumstances (as something that can obtain) when talking about the content of an utterance when interpreting Frege’s *Begriffsschrift* (see Mendelsohn 1982: 286). For simplicity sake I will talk about propositions, for the main point I am trying to articulate is independent of this particular interpretation of the *Begriffsschrift*.

Yet, ‘Tully’ and ‘Cicero’ have the same content. I surmise that Frege was driven by problems like this then he introduced in his logical notation the three stroke sign, ‘≡’. Thus, to infer (4a) from (3a) and (3b), we have to add the premise that ‘Tully’ and ‘Cicero’ have the same content, i.e.:

$$(5) \quad d. \text{ Tully} \equiv \text{Cicero}^5$$

Why did Frege introduce the ‘≡’ symbol and not employ the traditional equality sign, ‘=’? In other words, what is the difference between ‘≡’ and ‘=’? After all, in the first paragraph of the *Begriffsschrift* Frege uses the ‘=’ sign: “This indeterminateness makes it possible to express by means of letters the general validity of propositions; e.g.: $(a + b)c = ac + bc$ ”.⁶ We are thus entitled to assume that in the *Begriffsschrift* Frege operates with two distinct signs, ‘=’ and ‘≡’.⁷ In Frege’s *Begriffsschrift* we thus have a

⁵ In the last sentence of § 8 of the *Begriffsschrift* Frege claims that $\neg(A \equiv B)$ means: “the symbol A and the symbol B have the same conceptual content, so that A can always be replaced by B and conversely”. (Frege’s notation ‘ \neg ’ means, roughly, ‘it is a fact that’). I ponder that by “conceptual content” Frege means the inferential power names and other expressions exhibit in inferential reasoning and how they can or cannot be substituted salva veritate in such reasoning.

⁶ For a detailed discussion of Frege’s distinction between ‘=’ and ‘≡’ see Perry (2020). See also Mendelsohn (1982) and May (2001). Perry proposes an interpretation of the *Begriffsschrift* without taking into consideration § 8. He argues that the *Begriffsschrift*’s account can be developed to take into considerations the problems Frege pointed toward when he introduced the sense/reference distinction without appealing to Frege’s ‘≡’. My aim is more modest insofar as I think that ‘=’ and ‘≡’ can subsist together in a coherent picture that deals with some of Frege’s various insights. Actually, if I am right, both ‘=’ and ‘≡’ must enter the picture if our aim is to develop an account sensitive to the problems Frege’s pointed out in “Über Sinn und Bedeutung”. For an interesting discussion on how the *Begriffsschrift* relates to Frege’s mature work see also Simon (1995) who argues: “in ‘On Sense and Reference’ the different ways in which a referent is given allow identities to be informative. If we apply the same way of counting levels to *Begriffsschrift* as to ‘On Sense and Reference’, we indeed found three, not two. We have the sign, its content, and the way in which the content is determined by the sign ... If we do not actually have sense in *Begriffsschrift*, we seem to have the next best thing” (Simon 1995: 133).

⁷ Although in the *Begriffsschrift* Frege uses the ‘=’ sign only once (in the first quoted paragraph), in his “Applications of the ‘Conceptual Notation’” (written on

difference between (i) identity (expressed by ‘=’) and (ii) identity of content (expressed by ‘ \equiv ’). While the former is a relation between things, the latter is a relation between signs. Actually, identity of content cannot hold between objects other than certain linguistic objects. While it makes sense to say that Tully is identical to himself, it does not make sense to say that Tully (the object) has an identity of content to himself, or that he entertains a content identity to himself. While ‘=’ expresses a metaphysical (or ontological) relation, ‘ \equiv ’ expresses a linguistic relation. I reckon that Frege’s “Über Sinn und Bedeutung” criticism of his § 8 of the *Begriffsschrift* somewhat blurs this distinction. To this criticism I now turn.

3. The “Über Sinn und Bedeutung”’s interpretation

The famous controversial passage under discussion starts as follows:

Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my *Begriffsschrift* I assumed the latter. (Frege 1892, 56-7)⁸

In this passage Frege considers only the ‘=’ sign and suggests that in his *Begriffsschrift* he understood it as a relation between signs. Yet, as we saw, in the paragraph under discussion of the *Begriffsschrift*, Frege did not discuss ‘=’; rather, he used the three stroke sign, ‘ \equiv ’. I am not accusing Frege of misunderstanding between linguistic (grammatical) phenomena and metaphysical ones. All I am claiming is that Frege somewhat “misunderstood” himself, i.e., that the interpretation offered in “Über Sinn und Bedeutung” of § 8 of the *Begriffsschrift* is misleading.⁹ To be precise, Frege seems to

the same year) in his notations Frege utilizes both ‘=’ and ‘ \equiv ’ (see Frege 1879b, 204-8). For instance, on page 205, Frege writes “we can regard ‘ $u+1 = v$ ’ as a function of u and v ...” and in a single notation on the same page Frege uses both ‘=’ and ‘ \equiv ’. This is further evidence that at the time of the *Begriffsschrift* Frege operated with both signs.

⁸ I am adopting Black’s translation (in Geach & Black eds. 1952).

⁹ To my knowledge the first who pointed out that the Frege of “Über Sinn und Bedeutung” mischaracterizes the view he holds in the *Begriffsschrift* is Angelleli who

argue that in the *Begriffsschrift* he interpreted what is ordinarily expressed as identity statements strictly in terms of his content-identity symbol and, thus, he adopted a particular analysis of statements involving ‘=’. The question we now face is why Frege, and most of his scholars following him, thought that Frege’s “Über Sinn und Bedeutung” view of § 8 of the *Begriffsschrift*’s is the correct one?¹⁰ My guess is that this misunderstanding is based on the fact that the Frege of “Über Sinn und Bedeutung”, unlike the Frege of the *Begriffsschrift*, (like most of his followers) thought that one ought to operate with either ‘=’ or ‘≡’ and that the two signs cannot coexist when we come to explain the problems Frege was after.

In “Über Sinn und Bedeutung” Frege presents us with the following problem when he rebuts his *Begriffsschrift*’s account. I quote the whole paragraph:

Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my *Begriffsschrift* I assumed the latter. The reason which seems to favour this are the following: $a=a$ and $a=b$ are obviously statements of different

suggests that in his later work Frege is not faithfully reproducing the semantics view he holds in the *Begriffsschrift*: “Frege himself in this respect has done injustice to his own text of 1879” (Angelleli 1967, 40). In his criticism of the *Begriffsschrift*’s in this passage, Frege undermines (or dismisses) the notion of modes of determination, the very notion that triggered him to introduce the identity of content symbol, ‘≡’.

¹⁰ The standard interpretation of Frege’s first paragraph of “Über Sinn und Bedeutung” has recently been questioned. Thau & Caplan (2001), for instance, argue that Frege never dismissed his *Begriffsschrift*’s interpretation of identity statements. For a criticism of Thau & Caplan’s interpretation see Dickie (2008) who argues that Frege’s *Begriffsschrift* solution differs from the one proposed in “Über Sinn und Bedeutung” insofar as Frege was concerned with two distinct puzzles. While in the *Begriffsschrift* Frege focuses on why a rational agent can understand two co-referential terms without realizing that they co-refer, in “Über Sinn und Bedeutung” Frege is concerned with inferences, i.e. why in a deductive proof we can provide justification in moving from the premises to the conclusion based on self-evident logical reasoning. The replacement of a term with a co-referential (or co-extensive) one in such a logical deduction may make the proof not logically self-evident and, thus, the two terms differ in cognitive value.

cognitive value; $a=a$ hold a priori ... while statements of the form $a=b$ often contain very valuable extension of knowledge and cannot always be established a priori. ... Now if we were to regard equality as a relation between that which the names ‘a’ and ‘b’ designate, it would seem that $a=b$ could not differ from $a=a$ (i.e. provided that $a=b$ is true). A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing. What we apparently want to state by $a=b$ is that the signs or names ‘a’ and ‘b’ designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted. But this relation would hold between the names or signs only in so far as they named or designated something. It would be mediated by the connection of each of the two signs with the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrary producible event of object as a sign for something. In that case the sentence $a=b$ would no longer be concerned with the subject matter, but only its mode of designation; we would express no proper knowledge by its means. But in many cases this is just what we want to do. If the sign ‘a’ is distinguished from the sign ‘b’ only as an object (here, by means of its shape), not as a sign (i.e. not by the manner in which it designates something), the cognitive value of $a=a$ becomes essentially equal to that of $a=b$, provided $a=b$ is true. (Frege 1892, 56-7)

If we break down this paragraph we have two main notions at work: cognitive significance and identity. Identity, we are told, is a relation “in which each thing stands to itself but to no other thing”. In that case, though, we cannot distinguish between statements of the form $a=a$ and statements of the form $a=b$. For, if the latter is a true statement, it would express the very same thing, i.e. that a (or b) is identical to itself. Frege argues that in his *Begriffsschrift* he assumed that in such cases what we assert is a relation between signs or names. But this cannot be the case, for we lose the subject matter and would express no proper knowledge. In uttering “Tully is Cicero” one is not talking about the names ‘Tully’ and ‘Cicero’, but about Tully/Cicero. Yet, as we saw, in the famous § 8 of the *Begriffsschrift* Frege does not discuss ‘=’, but ‘≡’.

I now try to suggest how the two accounts can be combined in dealing with the difference between statements of the form $a=a$ (e.g.: “Tully is Tully”) and statements of the form $a=b$ (e.g.: “Tully is Cicero”).¹¹ While in “Über Sinn und Bedeutung” they would be represented as:

- (5) Tully = Tully
 (6) Tully = Cicero

in the *Begriffsschrift* they would be represented as:

- (7) Tully ≡ Tully
 (8) Tully ≡ Cicero

(8) reads as: ‘Tully’ and ‘Cicero’ have the same content. The two names are, therefore, co-referential. In linguistics we usually express co-referentiality using co-indexation. Hence, (8) can be represented as:

- (9) Tully₁ = Cicero₁

The three stroke sign of the *Begriffsschrift* can thus be represented by the subscript signifying co-referentiality and the latter differs from identity: ‘Tully’ and ‘Cicero’ are different names after all. (9) can thus be understood as encompassing both ‘=’ and ‘≡’. If my understanding is right, then with an utterance of “Tully is Cicero” a speaker/writer conveys two pieces of information: (i) that Tully is identical with Cicero *and* (ii) that ‘Tully’ and ‘Cicero’ are co-referential (they have the same content). In so doing we do not loose the subject matter, for we are talking about the object, Tully/Cicero, the subject matter of the utterance; we are talking *of* an object carrying two names.¹² At the same time, though, we also suggest that the names ‘Tully’ and ‘Cicero’ have the same content, *viz.* that they co-refer (as it is stressed by them sharing the same subscript). In short, with an utterance of an identity statement of the form $a=b$ we convey two pieces of information.¹³

¹¹ Given that Frege also consider definite descriptions to be proper names the same story could be told using “The Morning Star is the Evening Star”.

¹² For a discussion about identity in “Über Sinn und Bedeutung” and the notion of subject matter, see Corazza & Korta (2015).

¹³ For an interesting discussion about the difference between identity and co-referentiality see May (2012). For a discussion of the difference between the identity of

The obvious question that now comes to mind is: why did Frege not analyze a statement of the form $a=b$ the way I did above? I suspect that Frege could not envisage an interpretation along these lines because he was presupposing that all the relevant information ought to be encompassed into a single content. In the *Begriffsschrift* the utterances “Tully is Tully” and “Tully is Cicero” express the same content, i.e. that the object Tully/Cicero is identical to itself. This is the problem that Frege recognizes in “Über Sinn und Bedeutung”. For, if they express the same content, we cannot explain how the first is trivial, while the second may help us to expand our knowledge. This is the well-known and discussed Frege’s puzzle. It is also well-known that to solve this problem, i.e. the difference in cognitive significance between the two utterances, Frege introduces the sense/reference distinction. Though ‘Tully’ and ‘Cicero’ have the same reference (stands for the same thing) they express different senses. Senses are the constituents of the thought expressed by an utterance. While in the *Begriffsschrift* “Tully is Tully” and “Tully is Cicero” express the same proposition (have the same content), in “Über Sinn und Bedeutung” they express different thoughts and the latter is the bearer of cognitive significance.

The problem of what is the sense expressed by a tokened name has been largely discussed. It is not my intent to engage in this rich and often controversial debate. My aim is more limited. I merely want to show how we may reconcile our insight from the *Begriffsschrift*’s and “Über Sinn und Bedeutung”.¹⁴ To do so, though, we must give up the view that a single utterance comes equipped with a single content, be it a proposition or a

content of the *Begriffsschrift* and the notion of identity Frege develops in his mature period, see May (2001).

¹⁴ As far as I know, the first who suggested that Frege’s *Begriffsschrift* account can be made consonant with the one he proposed in “Über Sinn und Bedeutung” is Perry (2001/12, see in particular ch.7, section 3; see also Corazza’s 2003 review of Perry’s 2001), when he spelled out the critical referentialism framework and hints at how the reflexive content of an utterance captures the Fregean account in the *Begriffsschrift*, while the referential content deals with the problem of the subject matter Frege insists upon in “Über Sinn und Bedeutung”. More on this in the next section.

thought. The prevailing view is that an utterance can be (semantically) associated only to one content, be it a thought (“Über Sinn und Bedeutung”) or a proposition (*Begriffsschrift*). In the next section I will show how the *Begriffsschrift*’s view can be developed to deal with the problems Frege raises in the first paragraph of “Über Sinn und Bedeutung”. To summarize, we can agree with both the *Begriffsschrift*’s position that statements of the form $a=a$ and $a=b$ express the same proposition (have the same content) and the view proposed in “Über Sinn und Bedeutung” that they differ in cognitive significance (express different thoughts). To do so, though, we have to assume that statements like these come equipped with more than a single content or proposition. This can be done in adopting a Perry-style pluri-propositionalist model of communication.

4. Back to the *Begriffsschrift*

To understand Frege’s “Über Sinn und Bedeutung” discussion of his account of identity in the *Begriffsschrift* and the way he distinguishes between ‘ \equiv ’ and ‘ $=$ ’ we must take a detour. I suggested that the ‘ \equiv ’ symbol represents a linguistic relation, while the ‘ $=$ ’ sign a metaphysical one. Actually, Frege often answers semantics/grammatical concerns in relying on ontological (or metaphysical) distinctions. At the same time, though, Frege drives ontological distinctions based on the grammatical ones. When it comes to discuss the role of a name (*Eigenname*), for instance, Frege characterizes it as *what designates* an object, while he characterizes an object as *what is designated* by a name. The same holds with predicates or concept-words (*Begriffswort*). A predicate is what denotes a concept and a concept is what is referred to by a predicate. As Dummett puts it:

Frege’s use of the ontological term ‘object’ is strictly correlative to his use of the linguistic term ‘proper name’: whatever a proper name stands for is an object, and to speak of something as an object is to say there is, or at least could be, a proper name which stands for it. The question therefore naturally arises in which realm, the linguistic or the ontological, the principle of classification is to be applied. (Dummett 1973/1981, 55-6)

I conjecture that this is also what happened when Frege discussed identity in the *Begriffsschrift* and in “Über Sinn und Bedeutung”. While in the former he focuses on the semantics/grammatical relation, in the latter he focuses on the ontological one. And it is from his ontological perspective that in “Über Sinn und Bedeutung” Frege understands and undermines the identity account he proposes in § 8 of the *Begriffsschrift*. If I am right, though, both accounts can subsist. We have, though, to give up the view that all the relevant information conveyed by the utterance of a sentence is encompassed into a single content, be it a proposition or a thought. Frege, like many of his followers, committed what Barwise & Perry characterized as the fallacy of misplaced information, i.e.: “The idea that all the information in an utterance must come from its interpretation [the proposition expressed] we call the *fallacy of misplaced information*” (Barwise & Perry 1983, 38).

For ‘ \equiv ’ and ‘=’ to coexist we must avoid the fallacy of misplaced information. One way to do so is to accept the (Perry-inspired) view that an utterance comes equipped with different contents or truth-conditions.¹⁵ Let me illustrate the framework I endorse that allows us to avoid this fallacy. The position I propose can be characterized as *pluri-propositionalism*. For, a single utterance comes equipped with variegated contents or propositions. This, though, does not amount to say that in producing an utterance a speaker ends up expressing (or saying) a multitude of propositions. It simply means that many propositions (or truth-conditions) are *available* when we come to analyze a communicative interaction. Propositions are abstract entities that, although they have no causal power, play important classificatory roles. This framework can be viewed as a reaction to *mono-propositionalism*, or to what Korta (2007) characterizes as the dogma of mono-propositionalism.¹⁶ That is, the view that, if we discount implicatures and

¹⁵ See Perry’s (1988, 2001/12) critical referentialism (see also Korta & Perry’s critical pragmatics, 2011).

¹⁶ If my interpretation is correct Mendelsohn’s critique of Frege’s *Begriffsschrift* theory, i.e. that “names in BG [Begriffsschrift] were systematically ambiguous; they stood for their objects they customarily denoted everywhere save when they occurred at either end of the ec [equality of content, ‘ \equiv ’] symbol, at which place they stood for themselves” (Mendelsohn 1982, 285), does not affect the reconstruction I am

presuppositions, there is one and only one proposition associated with the utterance of a sentence. This proposition is required to play variegated roles such as: representing the semantic content of the utterance, what the speaker said, the proposition expressed, the information transmitted, the content of attitudes (what is referred to by that-clauses), the output of semantics, the input for Gricean reasoning, and so on and so forth. No unique proposition can play all these different roles. In what follows I will offer a brief justification for this conjecture.

Pluri-propositionalism, as I take it, is a hybrid between the *Begriffsschrift* and “Über Sinn und Bedeutung”—i.e. the view that the content of a tokened name is the object it refers to *and* the view that names contribute in *conveying* some descriptive information. Thus, the utterance of a simple sentence containing a proper name, on top of expressing a proposition having the referent of the name as a constituent, also carries information about the way the speaker and/or hearer apprehends this proposition. This descriptive information captures, I will show, what in the *Begriffsschrift* Frege characterizes as the modes of determination of the content and, thus that Frege’s *Begriffsschrift* account already has all the relevant tools to deal with the main problems Frege highlights in the first paragraph of “Über Sinn und Bedeutung”.

To quickly illustrate the pluri-propositionalist framework I defend, let us consider a simple scenario. When seeing John, Sue tells him: “Your shoe is untied”. John thinks “My shoe is untied”, and stoops to tie it. A case of observation, leading to communication, leading to action. But what is communicated? The traditional answer is: a proposition, i.e. that John’s shoe is untied. But the duties that fall upon this proposition are weighty. It must get at what Sue observed and said, what John understood and thought, and the reason for John’s action. Why did not Sue tell John: “John’s shoe is untied”? She would have said the same thing after all, *viz.*, that John’s shoe is untied. If, instead of addressing John using the possessive ‘your’, Sue

proposing. For, in whichever utterance names appear they stand for their customary content (object); yet, at the same time, as we will now see, they get mentioned in the reflexive content. The Perry-inspired view I am defending does not assume that names, or utterances for that matter, are ambiguous insofar as they are associated with different contents. Perry’s critical referentialism is not an ambiguity thesis.

addressed him using his name, John could well not stop to tie his shoe. For, on top of the fact that in a face-to-face situation (of this sort) it is unconventional to address someone using his or her proper name, John may think that Sue was not telling him that his shoe is untied, but that the shoe of someone else sharing his name is untied. John could also be amnesiac and not know that his name is ‘John’, and so on and so forth. In processing Sue’s utterance “Your shoe is untied” John, as a competent speaker of English, understands that his own shoe is untied. If, instead of talking to John, Sue were talking to Jane, to pass the same message, she could not say “Your shoe is untied”, for she would be telling Jane that her shoe is untied. Rather, she would say: “John’s shoe is untied”. Jane could thus direct her attention toward John without bothering about her own shoe. How can we explain these simple communicative situations that trigger different actions?

The traditional answer is that what we express and grasp in a communicative interchange is a proposition. The search for a single proposition is misguided. There is a structure of related propositions, that are not intrinsically equivalent, but equivalent in the circumstance, that does the job. What Sue sees can be captured by an existential or “Fregean” proposition: There is a man I see and a shoe he wears, and it is untied. But to get at the common element between what she says, “Your shoe is untied”, and what John understands, “My shoe is untied”, we seem to need a proposition about John, a so-called “Russellian” or “singular” proposition that is not a description of John, but John himself that is the common element. Similarly with Sue telling Jane: “John’s shoe is untied”. If Jane does not know whom Sue intends to talk about, by being a competent speaker of English and recognizing that ‘John’ is a proper name, she would nonetheless grasp an existential or “Fregean” proposition: There is someone named ‘John’ whose shoe is untied. To know whom Sue is talking about, Jane has to identify John and, thus, grasp a singular or “Russellian” proposition that is not a description of John but a proposition with John himself as a constituent.¹⁷

¹⁷ The idea that a single utterance may express more than one proposition is not new. When distinguishing between tone and sense, Frege already hinted at that: “But whilst the word ‘dog’ is neutral as between having pleasant or unpleasant associations, the word ‘cur’ certainly has unpleasant rather than pleasant associations and put us rather in mind of a dog with somewhat unkempt appearances. Even

To summarize, the pluri-propositionalist model can be spelled out, roughly, as follows. Utterances of simple sentences like:

- (10) Your shoe is untied
- (11) John's shoe is untied

come equipped with various contents. Their analysis starts by distinguishing between the reflexive and the referential (or official) contents. Thus, while (10a) and (11a) constitute the reflexive contents, (10b) and (11b) are the referential contents:

- (10) a. There is an individual x the speaker of (10) addresses by uttering the possessive 'your' & the speaker of (10) says that x 's shoe is untied
- b. That John's shoe is untied

if it is grossly unfair to the dog to think of it in this way, we cannot say that this makes the second sentence false. True, anyone who utters this sentence speaks pejoratively, but this is not part of the thought expressed. What distinguishes the second sentence from the first is of the nature of an interjection. It might be thought that the second sentence does nevertheless tell us more than the first, namely that the speaker has a poor opinion of the dog. In that case, the world 'cur' would contain an entire thought" (Frege 1897, 240-1, *italics added*). Bach (1999) and Neale (1999), for instance, argued that in uttering a sentence a speaker may say two things at once. Bach argues that in uttering "Tom is rich but he is honest" one expresses two propositions, i.e. (i) that Tom is rich and (ii) that there is a contrast between being rich and being honest. Corazza (2002) argues that utterances containing complex names also express more than one proposition. E.g. "The Virgin Mary is Jesus' mother" expresses the propositions that Mary was Jesus' mother and that Mary was a virgin. This helps us to deal with anaphoric pronouns linked with expressions composing the complex name such as 'she', 'one' and 'that color' in "Little [Red1 Riding Hood2]3 was so-called because she3 wore one2 of that color1". Without denying that in uttering a single sentence a speaker can express more than one proposition, i.e. she can say more than one thing at a time. The pluri-propositionalism I defend following Perry, though, is of a different nature. For it is committed to the view that each utterance comes equipped with various contents and that some of the latter (the reflexive contents) do not pertain to the Gricean what is said or Kaplanian content.

-
- (11) a. There is an individual x and a convention C such that: C is exploited by the speaker of (11); C permits one to designate x with ‘John’ & the speaker of (11) said that x ’s shoe is untied
 b. That John’s shoe is untied

By simply hearing an utterance of (10) or (11), a competent speaker would understand something like (10a) and (11a) even if she is unable to grasp who the speaker is and whom she or he designates with his or her use of ‘your’ and ‘John’. These are the *reflexive* contents of utterances of sentences like (10) and (11). They represent the conditions the referent must fulfill to be the individual the speaker refers to and intends to talk about. What the speaker (in our example, Sue) says, though, is not something about these contents. What she says is something about John’s shoe and what she says is true just in case John’s shoe is untied. What Sue expresses is the proposition that John’s shoe is untied. That is, in uttering (10) or (11) Sue expresses the proposition (10b)/(11b). Since the latter is the same, in uttering either (10) or (11) Sue said the same thing. But she said it in different ways, i.e. in exploiting different conditions that John’s shoe, the referent and propositional constituent, must fulfill, in the context of the utterance and communicative exchange, to enter the proposition expressed by Sue.

The traditional philosophical understanding of the truth-conditions of a given declarative utterance are the incremental conditions needed to judge whether it is true or false, once all the linguistic and contextual factors are fixed. In short, in our analysis we start from the product, *viz.* the utterance of a given sentence abstracted away from the context of the utterance. That is, we start from the meaning the utterance inherits from the sentence, the type. In so doing we quantify over meanings. We then proceed to fill in the missing ingredients from the actual circumstances in which the utterance occurs. In our analysis we can see that an utterance conveys many other relevant information. In other words, it is by starting to fill in more and more contextual information that the incremental truth-conditions (the official content) gets computed.¹⁸ This does not mean, though, that a

¹⁸ “It is fair to call these truth-conditions of [the note], because they are conditions such that, were they satisfied, [the note] would be true ... they are reflexive conditions, conditions on [the note] itself. The truth-conditions on which philosophers

speaker/hearer ought to be consciously aware of all the processing going from the pure reflexive content to the incremental one. Yet, they play an important *classificatory* role. In particular, they help us to classify what goes on in the speaker/hearer mind when she processes an utterance. In so doing it helps us to deal with problems pertaining to *cognitive* significance.¹⁹

The utterance of a sentence like (1), “Tully is Roman”, can be analyzed as follows:

- (12) In uttering ‘Tully’ one refers to Tully
- (13) Tully satisfies ‘is Roman’

We can thus cash out the reflexive content of (1) as follows:

- (14) There is an individual x and a convention C such that:
 - (i) C is exploited by (1)
 - (ii) C permits one to designate x with ‘Tully’
 - (iii) x is Roman

The referential (official) content would correspond to the proposition expressed (roughly, the intuitive what is said or Kaplanian content):

traditionally focus are incremental; they are conditions on the subject matter; that is, what the world beyond the utterance must be like, for the utterance to be true; or, as I like to put it, what else, has to be true, given the linguistic and contextual facts about the utterance ... the conditions will not say much about the world independently of [the note]. However the familiar philosophical concept of truth-conditions corresponds to the case in which one knows a lot about [the note], so the incremental, what else must be the case for [the note] to be true, are conditions that pertain to the world outside [the note], not [the note] itself ... as you figure out more about [the note], fixing more of its linguistic properties, the conditions that had to be fulfilled for its truth become more focused on the world.” (Perry 2001/12, 93-4)

¹⁹ In the hands of the theoretician propositions, qua abstract entities, play an important classificatory role. It is in this sense that reflexive contents help to deal with problems pertaining to cognitive significance, i.e. what is going on in the speaker/hearer mind during a communicative interaction and, thus, what Frege in “Über Sinn und Bedeutung” comes to characterize as the modes of presentations. For more on this see Corazza (2018).

(15) That Tully is Roman

In short, the reflexive content captures Frege's *Begriffsschrift* view that, associated to an expression there must be a mode of determination of its content. In grasping the reflexive content, the hearer can start processing relevant information that may ultimately, if all goes well, enable her to grasp the official or referential content. In a nutshell, as communication goes, we can focus on the variegated contents an utterance can convey. If we now consider Frege's identity statements of the form $a = b$ like:

(16) Tully is Cicero

it can be analyzed as follows:

- (17) (i) There is an individual x and an individual y and conventions C and C^* such that:
 - (ia) C and C^* are exploited by (17)
 - (ib) C permits one to designate x with 'Tully' while C^* permits one to designate y with 'Cicero'
- (ii) $x = y$

(17) represents the reflexive content of (16). In this content the names get mentioned and it is stated that they are co-referential (i.e. have the same content), as the " $x = y$ " stresses. Once again, (17) encapsulates the *Begriffsschrift*'s identity of content sign, ' \equiv '. The official or referential content of (16) would simply be that Tully/Cicero is identical to itself. Since these contents, *qua* abstract entities, help us to classify what goes on in one mind they can give a way to deal with Frege's puzzle about cognitive significance and to explain people actions. For this reason, in "Über Sinn und Bedeutung" Frege introduces the notion of sense. Roughly, he assimilates senses to the modes of presentation of the objects referred to. As I take it, reflexive contents are what allows us to classify (from a theoretical viewpoint) how speakers cognize the referents. They help to classify the mental contents cognizers entertain when uttering or hearing a sentence. In that sense, *qua* classifiers of what goes on in speaker/hearer mental realm they help us to deal with problems pertaining to *cognitive* significance. As Kaplan puts it: "We use the manner of presentation, the character, to individuate psychological states, in explaining and predicting action" (Kaplan

1989, 532). It is in that sense that I argued that the reflexive contents are what help us to deal with problems pertaining to the *cognitive* significance of an utterance.

By now it should be clear how the *Begriffsschrift*'s account can be understood to counter the criticism Frege proposes in “Über Sinn und Bedeutung”. The *Begriffsschrift*'s identity of content sign, ‘≡’, is explained at the lever of the reflexive content, where the names flanking it get mentioned. On the other hand, the identity sign, ‘=’, of “Über Sinn und Bedeutung” gets analyzed at the level of the official or referential content. We can thus combine both accounts without rejecting the *Begriffsschrift*'s view. In particular, we can accommodate the view that: “along with the introduction of a symbol for equality of content, all symbols are necessarily given a double meaning—the same symbols stand now for their own content, now for themselves” (*Begriffsschrift*: § 8).

5. Conclusion

I hope to have shown that: (i) in “Über Sinn und Bedeutung” Frege somewhat mischaracterized the view he proposed in the famous § 8 of the *Begriffsschrift*, (ii) The *Begriffsschrift*'s account does not crumble under the criticism Frege proposes in “Über Sinn und Bedeutung”, (iii) ‘=’ and “≡” should both enter a plausible picture about communication, (iv) an identity statement like “Tully is Cicero” must be analyzed in appealing to both ‘=’ and ‘≡’. That is, by adopting what I characterized as the pluri-propositionalist model. It is by dismissing mono-propositionalism that Frege's *Begriffsschrift* account of identity of content and the one he presents in “Über Sinn und Bedeutung”, can both be incorporated to deal with a plausible theory of communication and handle some of the problems Frege was after without having to subscribe to the sense/reference distinction.

Acknowledgment

For discussions and/or comments I am grateful to Christopher Genovesi, Kepa Korta, María De Ponte, John Perry and a referee of this journal. Research for this paper has been partly sponsored by the Spanish ministry of economy

and competitiveness (FFI2015-63719-P (MINECO/FEDER, UE)); the Spanish ministry of science and innovation (PID2019-106078GB-I00 (MCI/AEI/FEDER, UE)) and the Basque Government (IT1032-16).

References

- Angelleli, Ignacio. 1967. *Studies on Gottlob Frege and Traditional Philosophy*. Dordrecht: D. Reidel. ISBN 13: 978-94-017-3175-1
- Bach, Kent. 1999. “The Myth of Conventional Implicature”. *Linguistics and Philosophy* 22: 327–66. DOI: [10.1023/A:1005466020243](https://doi.org/10.1023/A:1005466020243)
- Barwise, Jon & Perry, John. 1983. *Situations and Attitudes*. Cambridge Mass: MIT Press. ISBN: 0262021897, 9780262021890
- Corazza, Eros. 2002. “Description-Names”. *Journal of Philosophical Logic* 31 (4): 313–25. DOI: [10.1023/A:1019950905478](https://doi.org/10.1023/A:1019950905478)
- Corazza, Eros. 2003. “Review of John Perry 2001. Reference and Reflexivity”. Palo Alto: CSLI Publications. *Mind* 112 (445): 171–75.
DOI: [10.1093/mind/112.445.171](https://doi.org/10.1093/mind/112.445.171)
- Corazza, Eros. 2018. “Identity, Doxastic Co-Indexation, and Frege’s Puzzle”. *Intercultural Pragmatics* 15 (2): 271–90. <https://doi.org/10.1093/mind/112.445.171>
- Corazza, Eros & Korta, Kepa. 2015. “Frege on Subject Matter and Identity Statements”. *Analysis* 75 (4): 562–65. DOI: [10.1093/analys/anv073](https://doi.org/10.1093/analys/anv073)
- Dickie, Imogen. 2008. “Informative Identities in the Begriffsschrift and ‘On Sense and Reference’”. *Canadian Journal of Philosophy* 38 (2): 269–88.
<https://doi.org/10.1353/cjp.0.0015>
- Frege, Gottlob. 1879. *Begriffsschrift, Eine der Aritmetischen Nachgebildete Formalsprache des Reinen Denkens*. Halle: Nerbert
- Frege, Gottlob. 1879a. *Conceptual Notation and Related Articles. Translated and edited by Terrel Ward Bynum*. Oxford: Clarendon Press
- Frege, Gottlob. 1892. “Über Sinn und Bedeutung”/“Sense and Meaning”. In Gottlob Frege. 1952. Translations from the Philosophical Writings of G. Frege. In Peter Geach & Max Black (eds.) Oxford: B. Blackwell: 56–78
- Geach, Peter & Black, Max. 1952. *Translations from the Philosophical Writings of Gottlob Frege*. Oxford: Basil Blackwell. <https://b-ok.cc/book/639351/b28968>
- Korta, Kepa. 2007. “Acerca del Monopropositionalismo Imperante en Semantica y Pragmatica”. *Revista de Filosofía* 32 (2): 37–55. ISSN: 0034-8244
- Korta, Kepa & Perry, John. 2011. *Critical Pragmatics*. Cambridge: Cambridge University Press. DOI: <https://doi.org/10.1017/CBO9780511994869>
- May, Robert. 2001. “Frege on Identity Statements”. In Carlo Cecchetto, Gennaro Chierchia, Maria-Teresa Guasti (eds.). *Semantic Interfaces: Reference,*

- Anaphora, and Aspects*. Palo Alto: CSLI Publications: 1–51.
DOI: [10.1017/S0022226702221970](https://doi.org/10.1017/S0022226702221970)
- May, Robert. 2012. "What Frege's Theory of Identity is Not". *Thought* 1 (1): 41–8.
DOI: [1002/tht.6](https://doi.org/1002/tht.6)
- Mendelsohn, Richard. 1982. "Frege's Begriffsschrift Theory of Identity". *Journal of the History of Philosophy* 20 (3): 279–99. DOI: [10.1353/hph.1982.0029](https://doi.org/10.1353/hph.1982.0029)
- Neale, Stephen. 1999. "Coloring and Composition". In Kumiko Murasugi & Robert Stainton (eds.). *Philosophy and Linguistics*. Boulder CO: Westview Press: 35–82. ISBN-10: 0813390850; ISBN-13: 978-0813390857
- Perry, John. 1988. "Cognitive Significance and New Theories of Reference". *Nous* 22: 1–18. Reprinted in John Perry 2000. *The Problem of the Essential Indexical and Other Essays*. Palo Alto CA: CSLI Publications: 189–206.
<https://doi.org/10.2307/2215544>
- Perry, John. 2001/12. *Reference and Reflexivity* (2nd Edition). Palo Alto: CSLI Publications. ISBN 1-57586-433-9
- Perry, John. 2020. *Frege's Detour*. Oxford: Oxford University Press. ISBN 978-0-19-881282-1
- Simon, Peter 1995. "The Next Best Thing to Sense in Begriffsschrift". In John Biro & Petr Kotatko (eds.). *Frege: Sense and Reference One Hundred Years Later*. Dordrecht: Kluwer Academic Publishers: 129–40. ISBN: 978-94-011-0411-1
- Thau, Michael & Caplan, Ben. 2001. "What's Puzzling Gottlob Frege?". *Canadian Journal of Philosophy* 31 (2): 159–200.
<https://doi.org/10.1080/00455091.2001.10717564>