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BOOK REVIEW

Tian, J. P.: EVOLUTION ALGEBRAS AND THEIR APPLICATIONS. Lecture Notes in Math. Vol. 1921, Springer-Verlag, Berlin, 2008, xi, 125 p., ISBN 978-3-540-74283-8/pbk.

Algebras, associative and also nonassociative are very important objects of research in mathematics and applications, e.g., in mathematical genetics.

In this book there are introduced and investigated nonassociative algebras of a new class, namely evolution algebras, and discussed in detail many applications of evolution algebras in stochastic processes and genetics, and elsewhere.

An algebra A over a field K is called an evolution algebra if it admits a countable basis $\{e_i\}$ such that the multiplication table satisfies $e_i e_j = 0$, whenever $i \neq j$. The basis $\{e_i\}$, which is called the natural basis, plays a privileged role among all other bases. Evolution algebras considered here are commutative, but not in general power-associative, and possess some distinguishing properties that lead to many interesting mathematical results. An equivalent definition of evolution algebras is also given by generators and relations. This kind of algebras is motivated by evolution laws of genetics. In this spirit, the multiplication in an evolution algebra expresses self-reproduction of non-Mendelian genetics. If the elements e_i are viewed as alleles of organelle genes, the rule $e_i e_j = 0$, for $i \neq j$, reflects algebraically the uniparental inheritance phenomena, which arise in non-Mendelian genetics. Moreover, we have $e_i^2 = \sum_j a_{ij} e_j$, where a_{ij} is the probability that e_i produces e_j in the next generation.

It is worth to recall that nonassociative algebras have already been studied to understand and develop Mendelian genetics and this gave rise to the well-known theory of genetic algebras, which gives a mathematical formulation of Mendel's laws by representing sexual reproduction in genetics as multiplication in algebras. The most important references in this field are the following references: (cf. [W]), (cf. [LY]), (cf. [R]).

However, when we would like to consider asexual reproduction process, which is not Mendelian inheritance, we need a new mathematical idea to formulate and investigate algebras in non-Mendelian genetics. Actually, evolution algebras appear naturally and stem from this new idea.

Now we briefly describe the contents of the book. After an introductory Chapter 1, then Chapter 2 presents several concrete examples coming from biology, physics, and mathematics including topology and stochastic processes that serve as motivation for the foundation and the development of evolution algebras. All these phenomena suggest a common and intrinsic algebraic

structures, the evolution algebras.

Chapter 3 introduces the basic definitions and properties of evolution algebras, including some algebraic concepts, such as evolution operators, evolution subalgebras, the associative multiplication algebra and the derived Lie algebra of an evolution algebra, the important notions of periodicity, algebraic persistency and algebraic transiency are also investigated, which lead to a principal theorem about evolution algebras, the hierarchical structure theorem. This hierarchical structure shows that an evolution algebra is a mixed subject of algebras and dynamics. Using these tools, the author obtains a rough classification, the skeleton-shape classification of all evolution algebras.

Chapter 4 establishes the connections between evolution algebras and Markov chains by defining a type of evolution algebras determined by Markov chains. This correspondence takes a new perspective on Markov process theory and derives new algebraic properties for Markov chains. In other words, properties of Markov chains can be revealed by studying their corresponding evolution algebras. In particular, any general Markov chain has dynamical hierarchy and the probabilistic flow is moving with invariance through this hierarchy, and all Markov chains can be classified by the skeleton-shape classification of their induced evolution algebras. Some examples and applications are examined to show that algebraic versions of Markov chains also have advantages in computation of Markov processes.

In Chapter 5, the theory of evolution algebras is applied to biology. The author first reviews the basic aspects of non-Mendelian genetics and the inheritance of organelle genes, and gives a general algebraic formulation of non-Mendelian inheritance. Although the theory of evolution algebras is an abstract system, it gives insight into the substance of non-Mendelian genetics. For instance, evolution algebras can explain the heteroplasmy and the homoplasmy of organelle populations, and show that concepts of algebraic transiency and algebraic persistency are related to the biological transitory and stability, respectively. Coexistence of triplasmy in tissues of sporadic mitochondrial disorder patients is studied as well. Furthermore, once the algebraic structure of asexual progenies of Phytophthora infectans is obtained, one can make certain important predictions and suggestions to plant pathologists.

In the final chapter, to procced further study and better understanding of the significance of evolution algebras, the author lists some additional results he has obtained, and puts forward further specific research topics and open problems. For instance, a theorem on classification of directed graphs is proved. The author also poses a series of open problems about evolution algebras and graph theory, in order to analyze graphs algebraically. It is expected that graph theory can be used as a tool to study nonassociative algebras. Many other interesting and promising research topics in evolution algebras and group theory, knot theory, and Ihara-Selberg zeta functions are presented as further research topics. The most significant topic would have to develop a continuous evolution algebra theory for continuous time dynamical systems. In conclusion, evolution algebras provide a theoretical framework to unify many natural phenomena, and to allow us to see problems in several mathematical fields from a new perspective. This book will open a way, for future discussion and research, in the theory of evolution algebras and their relationships to other sciences.

Hence the book under review is suitable both for graduate students and researchers with interest in theoretical biology, genetics, Markov process, graph theory, and nonassociative algebras and their applications. The text contains a clear, detailed and self-contained exposition of evolution algebras. While reading this book we learn a large variety of highly interesting ideas and methods and enjoy beautiful results that introduce us into a fast developing new mathematical area.

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Miloslav Duchoň, Bratislava