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ESTIMATING THE SHORT RATE FROM THE TERM STRUCTURES IN THE VASICEK MODEL

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ABSTRACT. In short rate models, bond prices and term structures of interest rates are determined by the parameters of the model and the current level of the instantaneous interest rate (so called short rate). The instantaneous interest rate can be approximated by the market overnight, which, however, can be influenced by speculations on the market. The aim of this paper is to propose a calibration method, where we consider the short rate to be a variable unobservable on the market and estimate it together with the model parameters for the case of the Vasicek model.

1. Introduction

A discount bond is a security which pays its holder a unit amount of money at specified time T (called maturity). Let $P(t,T)$ be the price of a discount bond with maturity T at time t. It defines the corresponding interest rate $R(t,T)$ by the formula

$$P(t,T) = e^{-R(t,T)(T-t)}, \qquad \text{i.e., } R(t,T) = -\frac{\ln P(t,T)}{T-t}.$$

Zero-coupon yield curve, also called term structure of interest rates, is then formed by interest rates with different maturities. Short rate (or instantaneous interest rate) is the interest rate for infinitesimally short time. It can be seen as the beginning of the yield curve: $r(t) = \lim_{T\to t^+} R(t,T)$. For a more detailed introduction to short rate modelling see, e.g., [1], [4].

In short rate models, the short rate is modelled by a stochastic differential equation. In particular, in Vasicek model [7], it is modelled by a mean-reverting

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Ornstein-Uhlenbeck process

$$dr = \kappa(\theta - r)dt + \sigma dw,$$

where κ, θ, σ are positive parameters and w is a Wiener process. It can be shown that after the specification of the so called market price of risk, the bond price $P(\tau, r)$ with maturity τ, when the current level of the short rate is r, is a solution to the parabolic partial differential equation. In the Vasicek model, it is customary to consider the constant market price of risk λ. Then, the bond price P satisfies

$$-\frac{\partial P}{\partial \tau} + \big(\kappa(\theta - r) - \lambda\sigma\big)\frac{\partial P}{\partial r} + \frac{\sigma^2}{2}\frac{\partial^2 P}{\partial r^2} - rP = 0 \qquad (1)$$

for all r and $\tau > 0$ and the initial condition $P(0, r) = 1$ for all r. This equation has an explicit solution, which can be written as

$$\ln P(\tau, r) = \frac{1 - e^{-\kappa\tau}}{\kappa}(R_\infty - r) - R_\infty \tau - \frac{\sigma^2}{4\kappa^3}(1 - e^{-\kappa\tau})^2, \qquad (2)$$

where $R_\infty = \frac{\kappa\theta - \lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2}$ (see [7]). In Figure 1 we show a simulated behaviour of the short rate (depicting also its equilibrium value θ) and term structures for several values of the short rate for the parameters equal to $\kappa = 5.00$, $\theta = 0.02, \sigma = 0.02, \lambda = -0.5$.

Note that, although the model has four parameters, short rate parameters κ, θ, σ and market price of risk λ, parameters θ and λ enter the partial differential equation (1) and hence also its solution (2) only through the term $\kappa\theta - \lambda\sigma$. Subsequently, it is possible to find formula for the bond price with three parameters. It is customary to do so by defining $\alpha = \kappa\theta - \lambda\sigma$, $\beta = -\kappa$. Parameters α, β are called risk neutral parameters, because they are related to an alternative formulation of the model in the so called risk neutral measure. For more details about risk neutral methodology see, e.g., [4].

Our aim is to use observable market term structures to calibrate the model, i.e., infer the values of the parameters using a certain criterion. One approach to calibration of the short rate models is based on minimizing the errors of the theoretical yields compared to the yields observed on the market. This approach was used for example in [6], [5]. Let us denote by R_{ij} the yield observed on the ith day for jth maturity and by $R(\tau_j, r_i)$ the yields computed using the Vasicek formula with the jth maturity τ_j and the short rate r_i realized on the ith day. Using the weighted mean square error (the weight given to the ith day and the jth maturity is w_{ij}), we minimize the function

$$F = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}\big(R(\tau_j, r_i) - R_{ij}\big)^2, \qquad (3)$$

where n is number of days and m is number of maturities which are observed on each of the days.

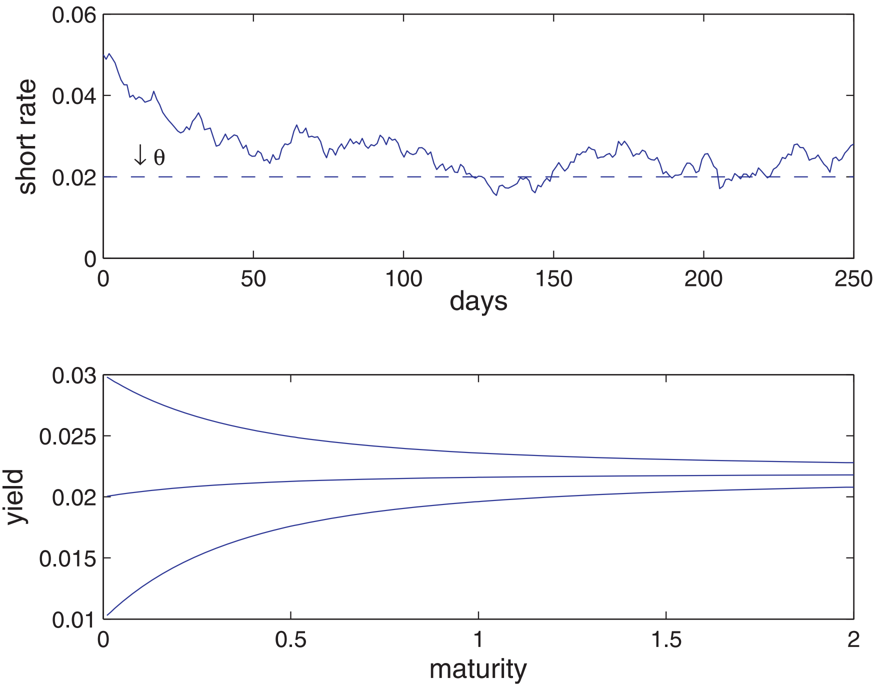

FIGURE 1. Simulated behaviour of the short rate (above) and examples of term structures (below) for the parameters $\kappa = 5.00$, $\theta = 0.02$, $\sigma = 0.02$, $\lambda = -0.5$.

Recall that to compute the Vasicek yields, the value of the short rate is necessary. However, the short rate, defined as the beginning of the term structure of interest rates, is only a theoretical variable, not observed on the market. In practice, it can be approximated by a yield with short maturity, such as overnight in [5], [6], [8] or 1-month yields in [2], [3], etc. Using 1-month (or some other) yields is, however, not consistent with the interpretation of the short rate as limit of the yields, as maturity approaches zero. Note that in the papers [2], [3] this problem did not arise, since they considered only one time serie as an approximation of the short rate, not the whole term structure. In [5], [6], [8], when dealing with term structures, overnight was taken to approximate the short rate. However, even using the overnight, which is closest to the short rate regarding the time, is questionable. The overnight rate, observed on the market, can be influenced by speculations. Hence we consider the short rate as an unobservable variable and estimate it from the term structures together with the parameters of the model.

The paper is organized as follows: In the following section we present the procedure for calibrating model parameters and the evolution of the short rate. In Section 3 we simulate data and test the proposed procedure. Finally, in Section 4, we apply the procedure data to the real market data. We end the paper with some concluding remarks.

2. Calibration procedure

According to the considerations above, the objective function (3) will be minimized with respect to the model parameters α, β, σ^2, as well as the time serie of the short rate $\mathbf{r} = (r_1, \dots, r_n)'$.

The key observation is noting that the logarithm of the bond price in the Vasicek model (2) is a linear function of the parameters α and σ^2 and the short rate r:

$$\ln P(\tau, r) = c_0(\tau) r + c_1(\tau)\alpha + c_2(\tau)\sigma^2,$$

where

$$c_0 = \frac{1 - e^{\beta\tau}}{\beta}, \quad c_1 = \frac{1}{\beta}\left[\frac{1 - e^{\beta\tau}}{\beta} + \tau\right], \quad c_2 = \frac{1}{2\beta^2}\left[\frac{1 - e^{\beta\tau}}{\beta} + \tau + \frac{(1 - e^{\beta\tau})^2}{2\beta}\right].$$

Hence the objective function (3)

$$\begin{aligned} F(\alpha, \beta, \sigma^2, \mathbf{r}) &= \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}\big(R(\tau_j, r_i) - R_{ij}\big)^2 \\ &= \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{w_{ij}}{\tau_j^2}\big(\ln P(\tau_j, r_i) + \tau_j R_{ij}\big)^2 \qquad (4) \end{aligned}$$

is quadratic in α, σ^2 and the components of $\mathbf{r}$. The optimal values for the given value of β are then easily obtained from the first order conditions, which form a system of $n+2$ linear equations:

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix},$$

where

$$\boldsymbol{A} = \begin{bmatrix} \sum_{i,j} \frac{w_{i,j}}{\tau_j^2} c_1^2 & \sum_{i,j} \frac{w_{i,j}}{\tau_j^2} c_1 c_2 \\ \sum_{i,j} \frac{w_{i,j}}{\tau_j^2} c_1 c_2 & \sum_{i,j} \frac{w_{i,j}}{\tau_j^2} c_2^2 \end{bmatrix},$$

$$\boldsymbol{B} = \boldsymbol{C'} = \begin{bmatrix} \sum_j \frac{w_{1,j}}{\tau_j^2} c_1 c_0 & \sum_j \frac{w_{2,j}}{\tau_j^2} c_1 c_0 & \dots & \sum_j \frac{w_{n,j}}{\tau_j^2} c_1 c_0 \\ \sum_j \frac{w_{1,j}}{\tau_j^2} c_2 c_0 & \sum_j \frac{w_{2,j}}{\tau_j^2} c_2 c_0 & \dots & \sum_j \frac{w_{n,j}}{\tau_j^2} c_2 c_0 \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \sum_j \frac{w_{1,j}}{\tau_j^2} c_0^2 & 0 & \cdots & 0 \\ 0 & \sum_j \frac{w_{2,j}}{\tau_j^2} c_0^2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_j \frac{w_{n,j}}{\tau_j^2} c_0^2 \end{bmatrix},$$

$$\boldsymbol{x'} = \left[\alpha, \sigma^2\right], \qquad \boldsymbol{y'} = [r_1, r_2, \cdots, r_n],$$

$$\boldsymbol{u'} = \left[-\sum_{i,j} \frac{w_{i,j}}{\tau_j} R_{i,j} c_1, \; -\sum_{i,j} \frac{w_{i,j}}{\tau_j} R_{i,j} c_2 \right],$$

$$\boldsymbol{v'} = \left[-\sum_{j} \frac{w_{1,j}}{\tau_j} R_{1,j} c_0\,, \; -\sum_{j} \frac{w_{2,j}}{\tau_j} R_{2,j} c_0\,, \; \cdots, \; -\sum_{j} \frac{w_{n,j}}{\tau_j} R_{n,j} c_0 \right].$$

Because of the special structure of the linear system, it is possible to reduce its dimensionality. The block $\boldsymbol{D}$ is diagonal and hence it is easy to find its inverse. Consequently, we are able to express the vector $\boldsymbol{y}$ in the following way:

$$\boldsymbol{Cx} + \boldsymbol{Dy} = \boldsymbol{v} \quad \Rightarrow \quad \boldsymbol{y} = \boldsymbol{D}^{-1}(\boldsymbol{v} - \boldsymbol{Cx}).$$

From the equation $\boldsymbol{Ax} + \boldsymbol{By} = \boldsymbol{u}$ we obtain

$$(\boldsymbol{A} - \boldsymbol{BD}^{-1}\boldsymbol{C})\boldsymbol{x} = \boldsymbol{u} - \boldsymbol{BD}^{-1}\boldsymbol{v},$$

which is a system of two linear equations.

In this way we are able to find the optimal values of the parameters α and σ, and the short rate vector $\mathbf{r}$ for the given value of β. Then, finding the optimal β is a one-dimensional optimization problem.

3. Application to simulated data

In the previous section we have proposed calibration procedure, which estimates model parameters $\alpha, \sigma^2, \mathbf{r}$ using closed formulae from Section 2 based on the first order conditions for minimizing the quadratic function for given parameter β. Given the optimal parameters $\alpha, \sigma^2, \mathbf{r}$ for each β, it is easy to find the optimal value of the parameter β, since it is a one dimensional optimization problem.

The accuracy of the estimation, when tested on simulated data, is very good and there seem to be no numerical problems. We show one illustrative example here.

Using the real measure parameters from the introduction (i.e., $\kappa = 5.00$, $\theta = 0.02, \sigma = 0.02, \lambda = -0.5$) we simulate the time serie of the daily short rate values for 252 days (i.e., one year) and for each day we compute the yield curves with 12 maturities: 1 month, 2 months, ..., 12 months. We use these yields as the input for the proposed calibration procedure. Following [5] and [6], we use the weights equal to the square of the corresponding maturity, i.e., $w_{ij} = \tau_j^2$.

Our values of real measure parameters and the market price of risk imply the following risk neutral parameters: $\alpha = 0.11$, $\beta = -5.00$, $\sigma = 0.02$. Recall that the calibration reduces to one-dimensional optimization, where the optimal value of β is found. Figure 2 shows the dependence of the objective function on β using a simulated set of data described above. Finding the optimal β and corresponding values of α and σ, we obtain the following estimates of the parameters:

$$\alpha = 0.1099999999979, \quad \beta = -5.000000000000018, \quad \sigma = 0.01999999943821.$$

As we can see, the parameters are almost exactly estimated. Also real and estimated short rates almost coincide. Figure 3 shows their difference, which is of the order 10^{-16}. Figure 4 shows some of the fitted term structures.

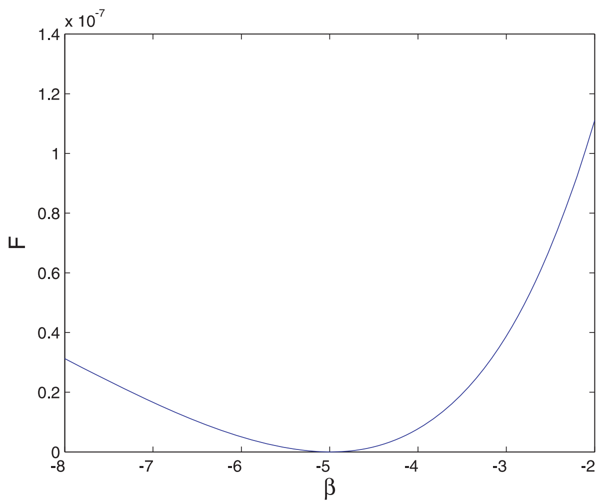

FIGURE 2. Dependence of the objective function F on parameter β using simulated data.

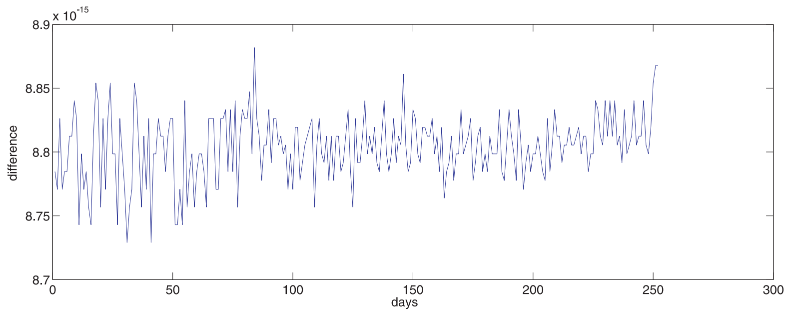

FIGURE 3. Difference between the real and the estimated short rate using simulated data.

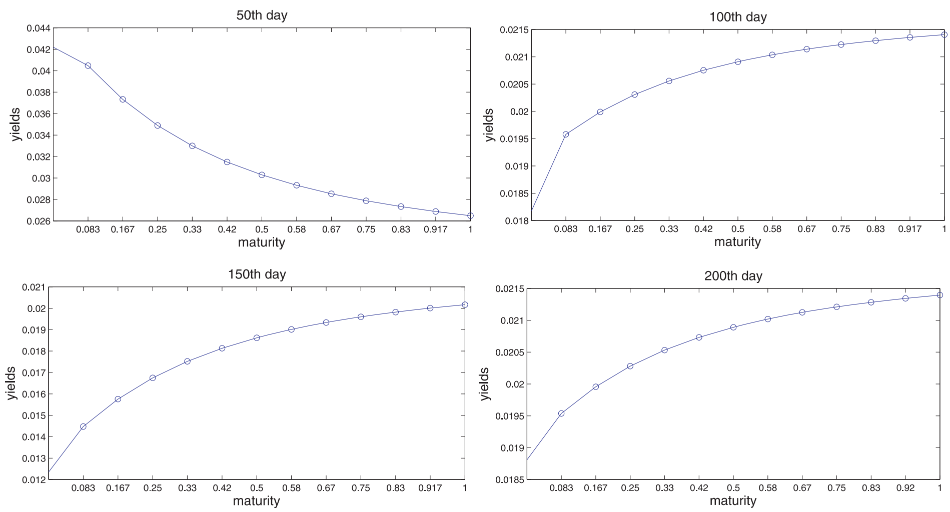

FIGURE 4. Examples of the fitted term structures using simulated data.

4. Application to real data

In this section we address the following two questions:

- How is the estimated short rate related to the market overnight? Can the short rate be approximated by market overnight or is it necessary to treat it as an unobservable factor which needs to be estimated?
- Is the estimated short rate robust to changing the maturities of the interest rates used for calibration?

4.1. Comparison between estimated short rate and overnight

One of the motivations for estimating the short rate from the market (observable) data are the results from the paper [8], where we considered the convergence model for the Slovak interest rates before adoption of Euro currency in 2009. The first step, when building the convergence model, is specifying the one-factor model for the European rates. We have used Euribor[1] term structures and Eonia[2] as the approximation of the European short rate when calibrating the model. However, this leads to poor fit of the term structure. The difference between the short rate as estimated from the term structures and the market overnight would explain the observed bad quality of the fit. Therefore, we use the proposed methodology for the Euribor rates in 2008. With a similar motivation in mind (Estonia adopted Euro in 2011), we do the same for the Euribor rates in 2010. For a comparison, we use also Estonian interest rates (Talibor[3]) from the same time periods. These data sets are described in Table 1.

Table 1. Data sets for comparing the estimated short rate with market overnight. Every data set is considered separately for the years 2008 and 2010.

Data set	Frequency	Maturities
Euribor	daily	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 months
Talibor	daily	1, 2, 3, 4, 5, 6 months

The results are presented in Figure 5 (Euribor) and Figure 6 (Talibor). We see that although the estimate of the short rate for Euribor in 2008 has a similar behaviour as the market overnight, it is higher and has a smaller volatility. The latter feature is especially pronounced in 2010, when the levels are approximately the same, but they are very different regarding the volatility. In the case of Talibor in 2008, there seems to be a difference between the estimated short rate and the market overnight, which does not vary much in time, while their volatility is similar.

[1]Euribor – European Interbank Offered Rate – is the rate at which euro interbank term deposits are offered by one prime bank to another prime bank; source: `http://www.euribor-ebf.eu/`

[2]Eonia – Euro OverNight Index Average – is the effective overnight reference rate for the euro and is computed from overnight unsecured lending transactions undertaken in the interbank market, source: `http://www.euribor-ebf.eu/`

[3]Talibor – Tallinn Interbank Offered Rate – was based on the interest rates at which banks offered to lend unsecured funds to other banks in the Estonian wholesale money market or interbank market in Estonian kroons, source: `http://www.eestipank.ee/`

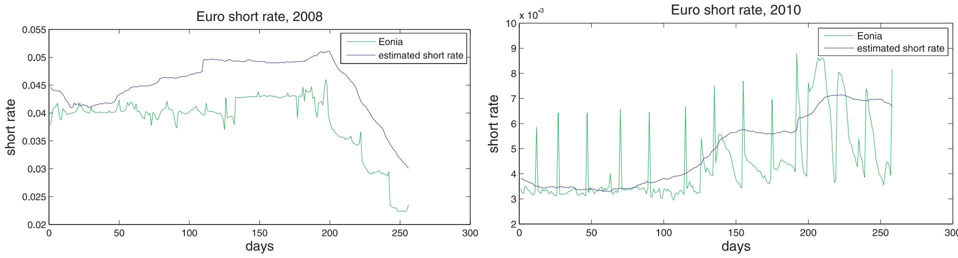

FIGURE 5. Comparison of the estimated short rate and the overnight for Euribor: 2008 (left), 2010 (right).

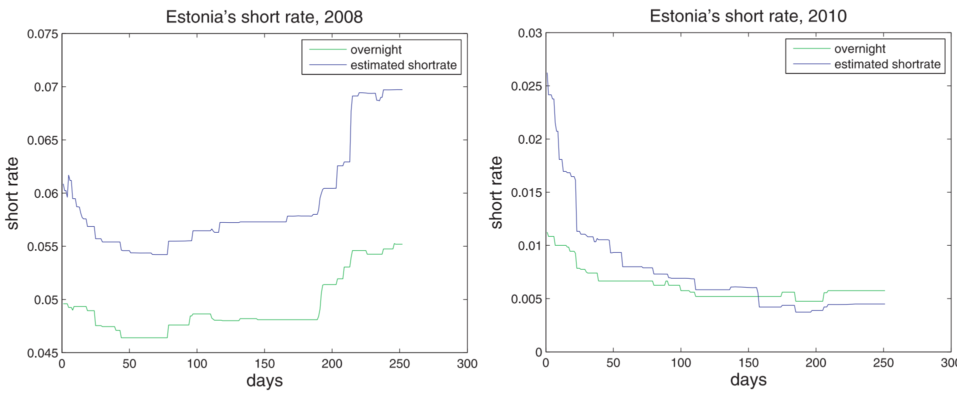

FIGURE 6. Comparison of the estimated short rate and the overnight for Talibor: 2008 (left), 2010 (right).

We present also the fitted term structures from 2010: Figure 7 and Table 2 show Euribor term structures; Figure 8 and Table 3 show Talibor term structures. In Figures 7 and 8 we can observe a good fit of term structures compared to Figure 4 in [8], where the short rate was identified with the market values of the overnight rates. This observation is confirmed also by Tables 7 and 8 (differences between exact and estimated yields are 10^{-4}–10^{-5}) in contrast to Table 4 in [8] (differences are about 10^{-1}). To sum it up, we have achieved much higher estimation accuracy using the estimated short rate values in our models.

4.2. Estimated short rates using different sets of maturities

The Canadian interest rates[4] are available for a wide range of maturities up to 30 years, which allows us to test the robustness of the short rate estimates to the choice of maturities used in calibration. We used three sets of parameters:

[4]yield curves for zero-coupon bonds, generated using pricing data for Government of Canada bonds and treasury bills, source: `http://www.bankofcanada.ca`

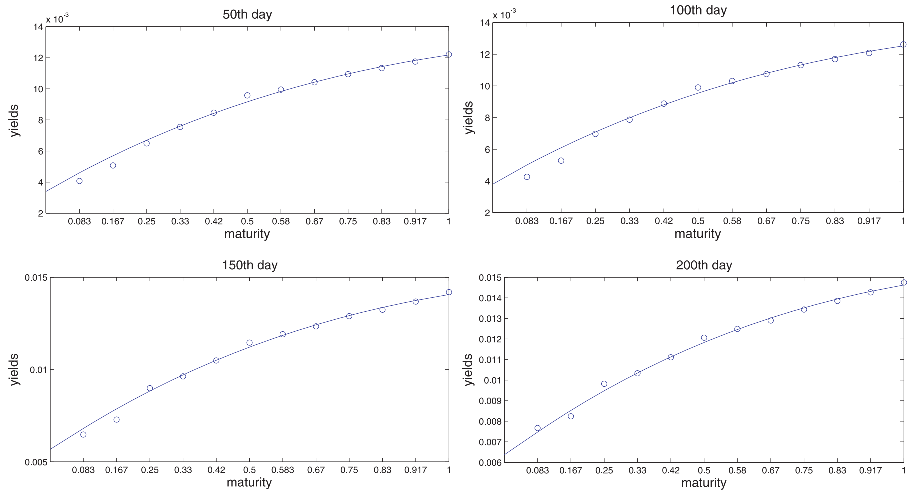

Figure 7. Accuracy of the estimated yield curves. Euribor 2010.

Table 2. Accuracy of the estimated yield curves - absolute values of differences between the real and the estimated rates. Euribor 2010.

Maturity [years]	50th day	100th day	150th day	200th day
0.083	5.30E-04	7.49E-04	3.29E-04	1.96E-04
0.167	6.22E-04	8.16E-04	5.79E-04	2.83E-04
0.25	2.00E-04	1.17E-04	1.56E-04	3.41E-04
0.33	4.73E-05	1.19E-04	8.04E-05	1.55E-05
0.42	3.98E-05	7.38E-05	1.23E-05	1.81E-05
0.5	4.17E-04	3.57E-04	2.45E-04	2.28E-04
0.58	1.19E-04	1.04E-04	5.73E-05	2.96E-05
0.67	7.25E-06	4.67E-05	8.01E-05	1.19E-04
0.75	1.68E-05	1.10E-05	3.10E-05	8.14E-05
0.83	9.34E-05	9.23E-05	1.20E-04	9.14E-05
0.917	8.05E-05	1.04E-04	5.92E-05	4.28E-05
1	2.82E-05	8.94E-05	1.27E-04	1.21E-04

the first one includes equally spaced maturities up to 30 years, the second one consists of shorter maturities up to 2 years and the third one goes up to 10 years. Details are given in Table 4.

We estimate the model separately for each of the years and in Figure 9 we record the different estimates of short rate depending on the input data.

Having in mind the high precision of the method on the simulated data (although we have presented a simulation example only with maturities

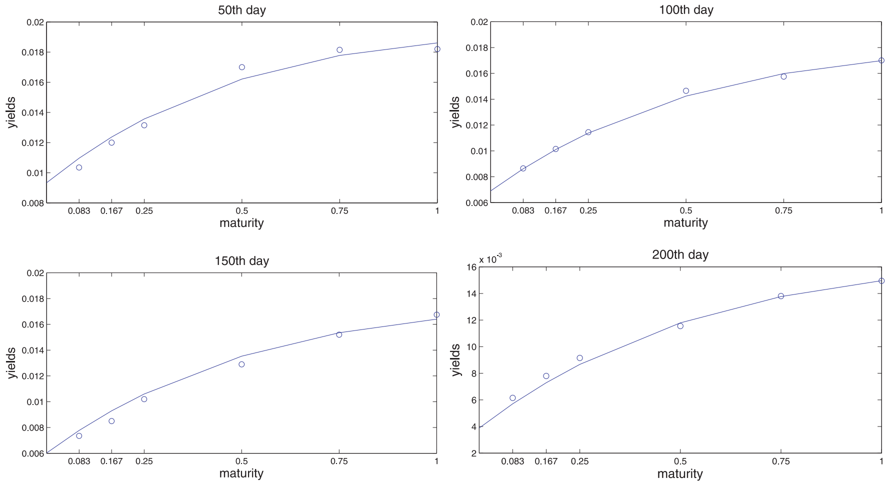

FIGURE 8. Accuracy of the estimated yield curves. Talibor 2010.

TABLE 3. Accuracy of the estimated yield curves - absolute values of differences between the real and the estimated rates. Talibor 2010.

Maturity [years]	50th day	100th day	150th day	200th day
0.083	6.14E-04	3.12E-05	4.27E-04	4.43E-04
0.167	3.66E-04	4.73E-05	7.90E-04	5.08E-04
0.25	4.20E-04	6.57E-05	4.00E-04	4.80E-04
0.5	7.85E-04	4.09E-04	6.32E-04	2.40E-04
0.75	3.72E-04	2.39E-04	1.47E-04	3.30E-05
1	4.12E-04	1.61E-05	3.51E-04	1.21E-05

TABLE 4. Data sets for comparing the estimated short rate using different maturities. Every data set is considered separately for the years 2007, 2008, 2009, 2010 and 2011.

Data set	Frequency	Maturities
Canada 1	daily	0.25, 2.5, 5, 7.5, 12.5, 10, 15, 17.5, 20, 22.5, 25, 27.5, 30 years
Canada 2	daily	0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2 years
Canada 3	daily	1 , 2 , 3 , 4 , 5, 6, 7, 8, 9, 10 years

from 1 month to 12 months, the procedure is very precise also for other choices of maturities), we would expect to obtain almost identical estimates of the short rate behaviour. Hence the differences, such as those observed in Figure 9 would suggest the inadequacy of the Vasicek model.

On the other hand, we are also interested in the impact on accuracy of estimation of yield curves. We present the results from the year 2011 and for each data set we compare real and estimated yield curves for selected days in Figures 10, 11, 12 and Tables 5, 6, 7. In general, the fit can be considered to be good. Note that on the 150th day for data set 2 we observe a term structure shape (firstly decreasing and then increasing) that is not possible to obtain in the Vasicek model, which allows only monotone and humped (firstly increasing and then decreasing) term structures (cf. [7]). These shapes are estimated well; recall that in the construction of the objective function we have put more weight to estimating the longer maturities.

Table 5. Accuracy of the estimated yield curves - absolute values of the differences between real and estimated rates. Canada 2011, set 1.

Maturity [years]	**50th day**	**100th day**	**150th day**	**200th day**
0.25	7.90E-03	4.53E-03	8.12E-05	1.39E-03
2.5	4.38E-03	1.51E-03	3.87E-03	8.14E-04
5	2.13E-03	3.83E-04	3.76E-03	1.12E-03
7.5	1.12E-03	3.95E-04	1.66E-03	6.38E-04
12.5	4.09E-04	7.01E-04	3.89E-04	3.10E-04
10	9.43E-05	3.63E-04	1.97E-04	2.20E-04
15	2.58E-04	1.89E-04	2.41E-04	1.61E-04
17.5	1.19E-04	5.07E-04	1.36E-04	2.04E-05
20	9.45E-05	4.63E-04	3.47E-05	1.49E-04
22.5	1.51E-04	1.92E-04	1.15E-04	2.16E-04
25	9.22E-06	8.63E-05	9.36E-05	1.18E-04
27.5	2.10E-04	2.18E-04	9.77E-05	4.65E-05
30	8.09E-05	1.74E-04	2.98E-04	4.42E-06

Table 6. Accuracy of estimated yield curves - absolute values of differences between the real and estimated rates. Canada 2011, set 2.

Maturity [years]	**50th day**	**100th day**	**150th day**	**200th day**
0.25	1.23E-03	1.10E-03	8.67E-04	3.50E-04
0.5	7.26E-04	7.42E-04	5.57E-04	2.58E-04
0.75	4.24E-04	3.96E-04	1.68E-04	1.71E-04
1	2.10E-04	1.35E-04	8.67E-05	7.44E-05
1.25	6.92E-05	2.19E-05	2.14E-04	1.74E-05
1.5	8.48E-06	7.66E-05	2.26E-04	8.19E-05
1.75	4.30E-05	5.49E-05	1.05E-04	7.80E-05
2	7.77E-05	2.98E-06	2.03E-04	5.84E-05

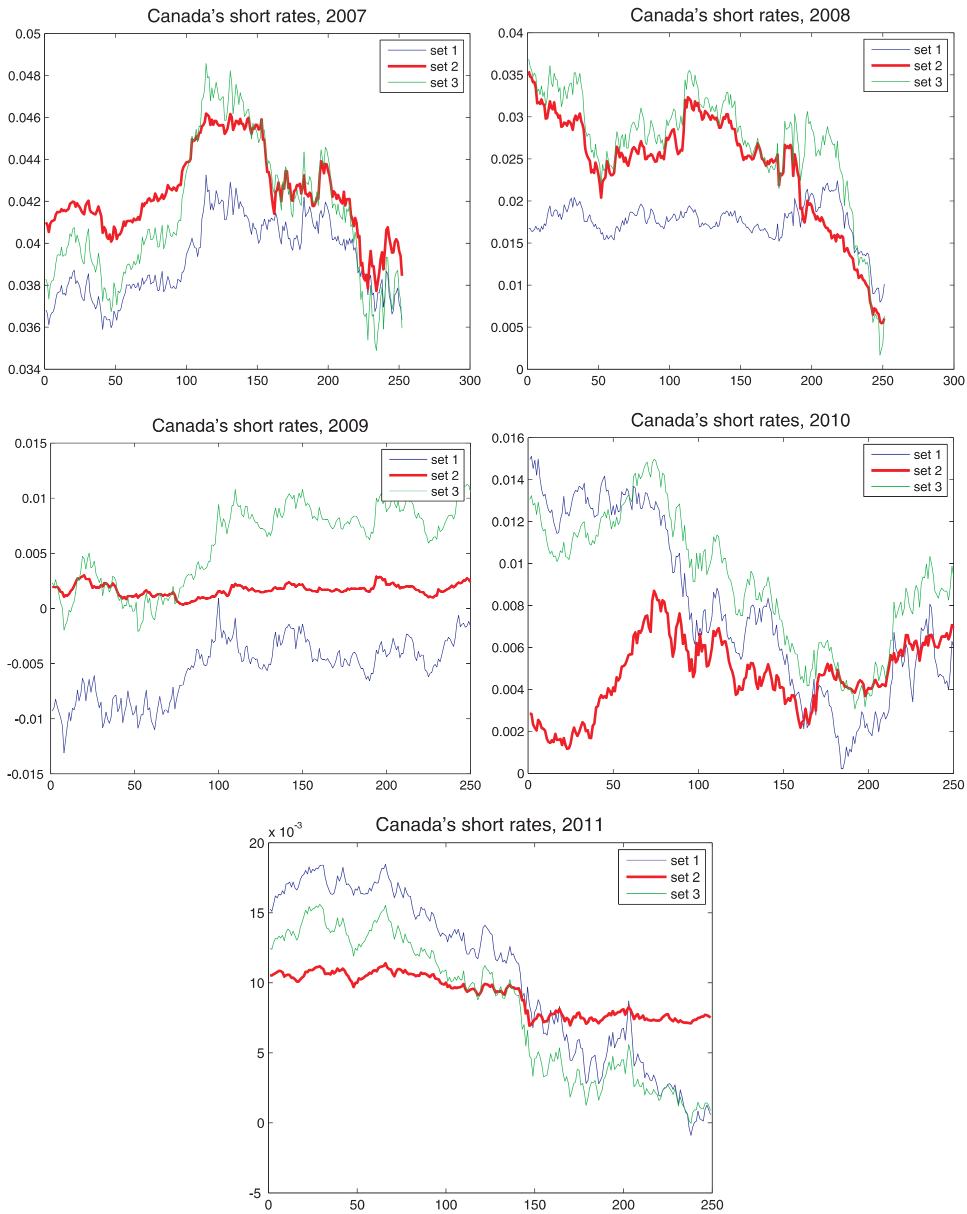

FIGURE 9. Estimated short rate for Canada, estimated separately for each of the years from 2007 to 2011.

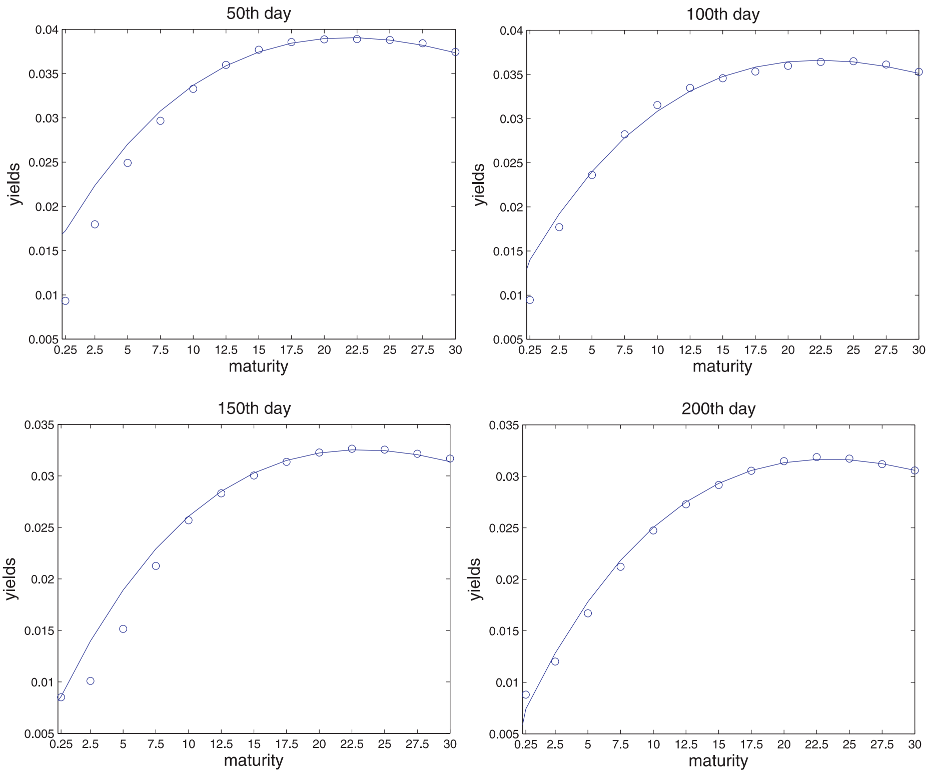

Figure 10. Accuracy of the estimated yield curves. Canada 2011, set 1.

Table 7. Accuracy of estimated yield curves - absolute values of differences between the real and estimated rates. Canada 2011, set 3.

Maturity [years]	50th day	100th day	150th day	200th day
1	2.68E-03	1.19E-03	1.09E-03	2.50E-03
2	1.55E-03	1.90E-04	5.30E-04	1.37E-03
3	6.81E-04	2.02E-04	1.47E-03	6.15E-04
4	1.67E-04	2.55E-04	1.84E-03	1.92E-04
5	5.87E-05	2.04E-04	1.65E-03	6.98E-06
6	1.05E-04	1.52E-04	1.10E-03	7.77E-05
7	7.23E-05	1.10E-04	4.43E-04	8.79E-05
8	2.83E-05	5.23E-05	1.43E-04	7.46E-05
9	1.44E-05	4.59E-05	5.74E-04	4.82E-05
10	5.13E-05	1.88E-04	8.53E-04	6.40E-07

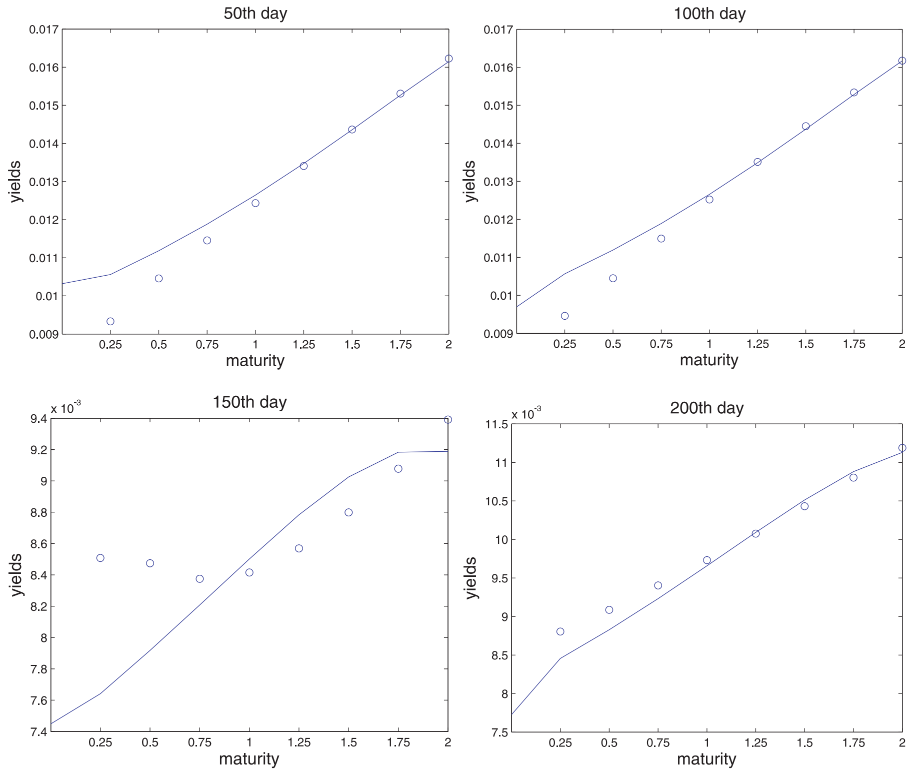

FIGURE 11. Accuracy of estimated yield curves. Canada 2011, set 2.

5. Conclusions

We have proposed and tested a procedure for estimating the short rates together with the parameters of the Vasicek model. Simulations show that the procedure has high precision. When applying it to the real data, we obtain a good fit of the term structures. However, when taking different sets of maturities as inputs to the calibration, we often obtain quite different estimated evolutions of the short rate. Nevertheless, the fit of the term structures is good. We would like to study this phenomenon more deeply, find its financial interpretation and possible explanation. Another problem for future research is to extend the idea of estimating the short rate also to other one-factor and multi-factor interest rate models.

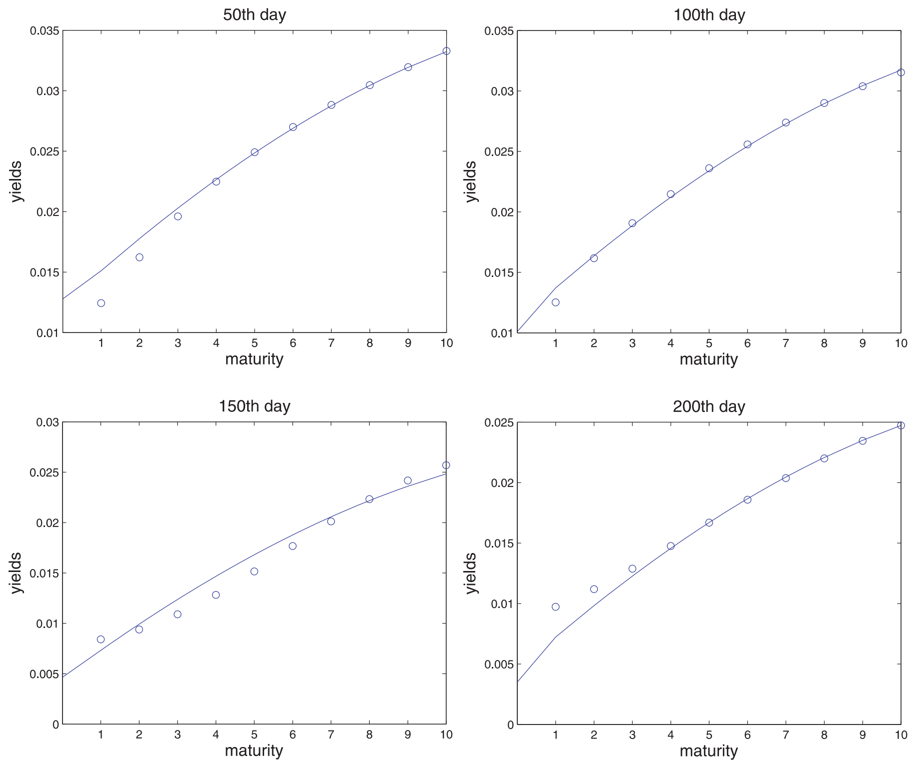

FIGURE 12. Accuracy of estimated yield curves. Canada 2011, set 3.

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