Synchronization of Capacitated Vehicle Routing Problem among Periods[1]

*Juraj PEKÁR – Ivan BREZINA – Zuzana ČIČKOVÁ**

Abstract

The routing problems are one of the most important problems in the field of logistic with great practical applicability. This article deals with synchronized distribution during time periods. It is based on optimization using classical capacitated vehicle routing problem while providing different customers' demands among time periods. The goal is, except the minimization of the total distance, to achieve the stability of solution among the time periods, which contributes to simplification of transportation planning. Although using of the presented model can lead to a partial increase of distribution cost, on the other hand it ensures more transparent routes and eliminates the need for daily optimization. Therefore it brings a positive effect for distributors, drivers and customers that may result in greater economic benefits than the benefit from the optimization on a daily basis.

Keywords: *capacitated vehicle routing problem, multi-periods vehicle routing problem, synchronization of distribution*

JEL Classification: C61, C90, L91

Introduction

Transport services constitute an important and irreplaceable part of the tactical and operational decision-making in many companies. Transport costs represent a significant portion of total company costs (the share of logistics costs on the total costs is within the range 10 – 25% (Brezina, Čičková and Reiff, 2009) as well as its quality significantly affect customer satisfaction.

* Juraj PEKÁR – Ivan BREZINA – Zuzana ČIČKOVÁ, University of Economics in Bratislava, Faculty of Economic Informatics, Dolnozemská cesta 1, 852 35 Bratislava 5, Slovak Republic; e-mail: pekar@euba.sk; brezina@ euba.sk; cickova@euba.sk.

[1] This paper is supported by the Grant Agency of Slovak Republic – VEGA, grant No. 1/0245/15 *Transportation planning focused on greenhouse gases emission reduction*.

The problem of material distribution planning generally remains an attractive theme. Its topicality is mainly determined by the ongoing economic and ecological processes that primarily interfere with tactical and operational management of distribution and forwarding agencies. Constantly changing fuel prices, increasing tolls for lorries, still closer supervision of overloading of trucks (every third controlled vehicle is overloaded causing road damage and affecting safety), reducing the permitted emission levels, these are just some of the reasons why transport companies try to minimize transportation cost. Expenses can be decreased by minimizing mileage, optimizing utilization of the vehicle and in that way also improving environmental performance (possible 7 – 10% reduction in fuel consumption in the case of road transport over long distances brings very important economic and environmental added value).

The optimization of material distribution and dispatching routes typically involves reduction of company's transport (and related) expenses. Saving direct kilometric costs, however, may not be the only benefit. Other benefits include a lower number of vehicles in use, shorter distribution times, and better customer service which together produce positive effects that can play an important role in the competitiveness of the company. Related costs reduction can be supported by using of mathematical models of various routing problems that constitute not only popular math problem, but in its different modifications also offer economically interesting practical applicability. The management of physical distribution of commodities is interesting not only for its practical relevance, but also for theoretical research, because a lot of related problems belong to the NP-hard problems. The economic impact of optimization of these problems can lead to considerable savings in logistic costs and in that way assist in increasing the competitive advantage of many companies.

This article focuses on the synchronization of capacitated vehicle routing problem among periods, which was designed upon request of company with regional scope that implements regular daily delivery. Distribution routes were often changed depending on the customers' demands. Nowadays many distribution and forward agencies are forced by increased complexity of daily traffic to prefer some stability of distribution routes over everyday route optimization. In the case of a relatively stable demand with low standard deviation (e.g. distribution of medicinal products, food and bakery products etc.) may be interesting to create a compromise between everyday optimized distribution and a "synchronized" distribution. This "synchronized" distribution is based on the expected daily deliveries, but providing such a shortest identical daily route meeting all customers' demands.

In the following, the authors present the problem of stabilization of distribution, as well as model approach and its application. Mathematical model is based on solving everyday isolated dispatching routes. The paper is focused on designing the model for solving the problem of stable routes specification. Empirical analysis is aimed on determining daily consistent routes of distribution.

In the first part authors present assumptions and formulation of a capacitated vehicle routing problem, which was used as the basis for further modelling and analysis. The second part is focused on model of synchronization of vehicle routing problem among periods. This section contains both the assumptions on which the model is based and also the mathematical formulation of the model. The model enables determining the stable optimal solution of the distribution for a given time horizon. The experiments are contained in the third chapter, which is dedicated to solving real distribution problem in the district of Pezinok.

1. Literature Review

The problem of distribution optimization belongs to that class of math problems, which is in the spotlight since the early 60s of the last century. One of the most important and well-known problems in this fields is capacitated vehicle routing problem (CVRP) (e.g. Desrosiers et al., 1995; Golden, Raghavan and Wasil, 2008). The goal of the problem is finding such lowest cost vehicle(s) routes from origin to the customer that begin and end in the origin so that each customer is visited exactly once with exactly one vehicle while the no vehicle capacity is exceeded and the requirements of all customers are met.

Vehicle routing problem mainly focuses on solving problems of daily operational planning of vehicle routes and enables optimizing the size and composition of vehicle fleet, the location of delivery points, optimizing storage location etc. It is oriented on fixed route optimization providing no substantial change in customers' location.

Practical requirements often bring the necessity implementing delivery on a daily basis. While optimization of distribution for each day separately allows optimizing delivery based on daily requirements of customers, it can lead to daily changes in dispatching routes, which can cause complications when planning a route for the driver. Therefore distribution companies quite often fall back to a certain unification, which not only simplifies the driver routes but also enables expecting identical time of supply, which is beneficial for consumers. Saving from daily optimization of routes versus their unification in a certain time period usually does not exceed the relatively low percentage of unified regular distribution routes.

The problem of routing stability among periods is relatively new and different approaches of various authors can be found in literature. Sorensen (2006) develop a way measuring the difference between two vehicle routing solutions and he also proposed a metaheuristic approach that finds solutions that are close (in the solution space) to a given baseline solution and at the same time have a high quality in the sense that their total distance travelled is small. Ribeiro and Lourenço (2001) found that in real-life problems, "marketing objectives" like the relationship between drivers and the customers they serve, are often more important than cost considerations. Thangiah et al. (2004) found that in the routing of school buses, there is a preference for routes remaining the same throughout an entire semester. Ribeiro and Lourenço (2005) proposed a multi-period inventory routing problem with customer that faces two types of demand: stochastic and deterministic. The objective was planning the deliveries for a week period. Ferdergruen and Simchi-Levi (1995) divided the inventory routing problems into two variants: the single period model and the infinite horizon model. Baita et al. (1998) presented dynamic routing and inventory problems characterized by having a dynamic environment where repeated decisions have to be taken at different times within some time horizon. Murata and Itai (2008) proposed approach enhancing solution similarity in multi-objective vehicle routing problems with different demand periods. Groer, Golden and Wasil (2009) characterized a consistent vehicle routing problem. This is a vehicle routing problem in which the total travel time is minimized, while meeting the capacity and maximum travel time. Furthermore, additional constraints are added to make sure that if customers are visited on more than once, they are always visited by the same driver and at roughly the same time of the day. Pekár, Brezina and Čičková (2014) dealt with synchronization of distribution in two time periods. Difference in the quantity of customers' demands between periods should be covered by routes with minimal (parametrized) number of differences. Except for the minimization of the total distance it enables to achieve some stability of solution in the given periods.

2. Capacitated Vehicle Routing Problem

This section is devoted to the CVRP. The idea of this model is further refined in part 3. The formulation CVRP is based on the graph representation. Consider a graph with n + 1 nodes. Let $N_0 = \{0,1,...,n\}$ be the set of nodes representing the location of customers as well as the depot (origin). The standard CVRP consists in designing the optimal set of routes for a vehicle (vehicles) in order to serve a given set of customers located in a certain nodes of the net representing by subset $N = \{1, 2, ..., n\}$. Each customer has a certain demand (g_i, $i \in N$). The demand need to be met from origin ($\{0\}$). Further on there exists a matrix $\mathbf{D}_{(n+1)x(n+1)}$ that represents the minimum distances between all the pairs of nodes (customers and the depot). The optimal vehicle routes must be designed in such a way that each customer is visited only once by exactly one vehicle, all routes start and end at the origin, and the total demands of all customers on one particular route must not exceed the capacity of the vehicle (g). Consider that demands of customers are met in full and also consider the individual demands do not exceed the capacity of the vehicle, otherwise a quantity equal to the vehicle capacity is realized in self-route and the model includes only the remaining demand.

Based on this assumption, the model can be stated as follows:

$$\min f(\mathbf{X}, \mathbf{u}) = \sum_{i \in N_0} \sum_{\substack{j \in N_0 \\ i \neq j}} d_{ij} x_{ij} \tag{1}$$

$$\sum_{i \in N_0} x_{ij} = 1, \; j \in N, \; i \neq j \tag{2}$$

$$\sum_{j \in N_0} x_{ij} = 1, \; i \in N, \; i \neq j \tag{3}$$

$$u_i + q_j - g(1 - x_{ij}) \leq u_j, \; i \in N_0, \; j \in N, \; i \neq j \tag{4}$$

$$q_i \leq u_i \leq g, \; i \in N \tag{5}$$

$$u_0 = 0 \tag{6}$$

$$x_{ij} \in \{0, 1\}, \; i, j \in N_0, \; i \neq j \tag{7}$$

Mathematical programming formulation of CVRP requires two types of variables: the binary variables x_{ij}, $i, j \in N_0$ where $x_{ij} = 1$ if the edge between node I and node j is used, otherwise $x_{ij} = 0$ (7). Further on the free variables u_i, $i \in N$ based on well-known Miller-Tucker-Zemlin's formulation of the traveling salesman problem (Miller, Tucker and Zemlin, 1960) are applied. These variables enable to calculate the load of vehicle.

Objective function (1) determines the total traveled distance. Equations (2) and (3) ensure that each customer (except the origin) is visited exactly ones. Equations (4) are anti-cyclical conditions that prevent the formation of such subcycles which do not contain an origin. The set of variables u_i, $i \in N$ also ensures the calculation of current load of vehicles in its route to i-th customer (including) and together with equations (5) they ensure that the current load does not exceed the capacity of vehicle, which is set to zero (6) in the origin.

3. Modeling of Synchronized Capacitated Vehicle Routing Problem among Periods

This section describes the original model, including its assumptions and mathematical formulation. The model respects the main objectives of suppliers, drivers and also customers. These objectives can be further characterized as follows: unified distribution decreases costs for the optimization calculations (everyday optimization is not necessary), and also provides more transparent data about the estimated vehicle position (it enables e.g. responding to traffic problems). Such route also enables easier route realization and better orientation in delivery times.

The primary objective of such a model is to build strategic relationships with customers. The model is generally applicable for periodic delivery with small changes in demand in a stable road network. It can be implemented to optimize weekly, respectively monthly distribution. The advantage is that there is no need to formulate new mathematical programming problem every day. When the calculation was realized by external company then the application of synchronized model can save computer time and thus the cost of calculation. Distribution using synchronized route allows the distributors, except systematic building strategic relationships with its customers, also using such routes when that same driver visit the same clients about the same time in each period (eg. daily or weekly). The driver then realizes more comfortable deliveries that do not change in periods. Daily changes of routes could lead to more errors and consequently higher costs. The advantage for customers is the realization of a regular supply at relatively the same time, which especially for the small customers can lead to cost reduction that may be extended by potential extension of working hours (including real time windows into mathematical models leads to the formulation of complex routing models with time windows, e.g. Čičková, Brezina and Pekár, 2013).

The mathematical model of synchronized distribution could be formalized as follows. Consider distribution network described in section 1. However, consider the changes in customers' demand in the discrete time periods. Time periods are characterized by a set $T = \{1, 2, ..., t\}$, where t represents number of periods. Then the demand of customers in each time period $k \in T$ can be designated as q_{ki}, $i \in N$, $k \in T$ (provided the individual daily demands do not exceed the capacity of the vehicle, otherwise a quantity equal to the capacity of vehicle is realized in self-route and the model includes only the remaining demands). Because we consider geographically regional distribution it is not necessary to take into account neither the time nor the distance limitations. The cost of transportation is directly reflected by distances.

Individual optimization according to model (1) – (7) in any k-th period $k \in T$ results to t individual routes. Based on ideas given in the introduction of this article it is often desirable to achieve a unified route. The mathematical formulation that enables to synchronize individual routes is given below. The mathematical model deals with two types of variables: the binary variables x_{kij}, which represent the use of the edge between node i and node j, $i, j \in N_0$ in the k-th period $k \in T$ and variables u_{ki} ($k \in T$, $i \in N$) that represent the cumulative demand of vehicle in corresponding period.

The situation can be characterized by following model:

$$\min f\left(\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_t, \mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_t\right) = \sum_{k \in T} \sum_{i \in N_0} \sum_{\substack{j \in N_0 \\ i \neq j}} d_{ij} x_{kij} \tag{8}$$

$$\sum_{i \in N_0} x_{kij} = 1 \qquad k \in T,\ j \in N,\ i \neq j \tag{9}$$

$$\sum_{j \in N_0} x_{kij} = 1 \qquad k \in T,\ j \in N,\ i \neq j \tag{10}$$

$$u_{ki} + q_{kj} - g(1 - x_{kij}) \leq u_{kj},\ k \in T,\ i \in N_0,\ j \in N,\ i \neq j \tag{11}$$

$$q_{ki} \leq u_{ki} \leq g,\ k \in T,\ i \in N \tag{12}$$

$$u_{k1} = 0,\ k \in T \tag{13}$$

$$x_{kij} - x_{k-1ij} = 0,\ k \in T - \{1\},\ i, j \in N_0,\ i \neq j \tag{14}$$

$$x_{kij} \in \{0,1\},\ k \in T,\ i, j \in N_0,\ i \neq j \tag{15}$$

The objective function (8) minimizes the total route travelled in all periods. Equations (9) and (10) ensure that in each period a vehicle leaves each node and the vehicle enters each node except the origin exactly ones. Equations (11) represent sub-tour elimination constraints that simultaneously with the equations (12) and (13) ensure that the vehicle's capacity is met. The variables u_{ki} ($k \in T$, $i \in N$) calculate the load of vehicle. Equations (14) ensure route consistency within periods.

4. Empirical Results

Further on, let us introduce the real-life problem dealing with distribution scheduling in district of Pezinok. The problem can be described by network consisting of origin from where 16 municipalities need to be served, so that $N = \{1, 2, ..., 16\}$,

$N_0 = \{0\} \cup N$ .The origin is situated in Pezinok. The elements of set N represent municipalities (in given order): Slovenský Grob, Viničné, Svätý Jur, Limbach, Vinosady, Šenkvice, Budmerice, Vištuk, Báhoň, Jablonec, Dubová, Píla, Častá, Doľany, Štefanová, Modra. Consider the five day time horizon (workweek). Known customers' demands in each workday (T = {1, 2, 3, 4, 5} are elements of n-dimensional vectors $\mathbf{q}_{k,}$ $k \in T$, n = 16:

- vector of demands for the 1-st period: $\mathbf{q}_1$ = (2 474, 2 024, 5 091, 1 384, 554, 5 715, 2 459, 1 479, 1 996, 767, 936, 150, 1 300, 719, 333, 6 614)^T;
- vector of demands for 2-nd period: $\mathbf{q}_2$ = (2 652, 2 078, 3 065, 1 521, 644, 5 155, 1 581, 1 202, 815, 825, 811, 281, 1 249, 880, 333, 6 737)^T;
- vector of demands for 3-rd period $\mathbf{q}_3$ = (1 962, 1 340, 4 377, 1 642, 718, 4 114, 1 743, 1 985, 1 389, 937, 1 045, 237, 1 489, 1 065, 244, 6 727)^T;
- vector of demands for 4-th period $\mathbf{q}_4$ = (2 574, 1 893, 2 531, 1 920, 1 145, 3 930, 643, 1 321, 882, 746, 1 095, 332, 2 082, 1 078, 437, 7 732)^T;
- vector of demands for 5-th period $\mathbf{q}_5$ = (1 388, 1 401, 4 816, 650, 1 143, 4 106, 3 390, 1 182, 1 312, 222, 749, 126, 3 536, 1 786, 501, 8 800)^T;

Table 1

Matrix of Minimal Distances between all the Pairs of Nodes (km)

D	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	7	4	7	7	3	6	16	11	22	19	11	18	15	18	19	7
1	7	0	3	7	7	10	13	23	18	21	26	18	25	22	25	26	14
2	4	3	0	11	11	7	10	20	15	18	23	15	22	19	22	23	11
3	7	7	11	0	6	10	13	23	18	29	26	18	25	22	25	26	14
4	7	7	11	6	0	10	12	22	18	28	26	17	24	22	25	25	13
5	3	10	7	10	10	0	8	13	11	17	16	8	15	12	15	16	4
6	6	13	10	13	12	8	0	11	6	12	15	9	16	13	16	15	5
7	16	23	20	23	22	13	11	0	6	7	4	10	10	7	7	4	9
8	11	18	15	18	18	11	6	6	0	6	9	9	16	13	13	9	8
9	22	21	18	29	28	17	12	7	6	0	10	15	16	14	14	10	14
10	19	26	23	26	26	16	15	4	9	10	0	14	13	11	11	7	13
11	11	18	15	18	17	8	9	10	9	15	14	0	7	5	8	8	4
12	18	25	22	25	24	15	16	10	16	16	13	7	0	3	6	6	11
13	15	22	19	22	22	12	13	7	13	14	11	5	3	0	3	4	9
14	18	25	22	25	25	15	16	7	13	14	11	8	6	3	0	4	12
15	19	26	23	26	25	16	15	4	9	10	7	8	6	4	4	0	12
16	7	14	11	14	13	4	5	9	8	14	13	4	11	9	12	12	0

Source: <www.vzdialenosti.sk>.

The daily distribution need to be provided by vehicles with the same capacity (g = 9000). The number of vehicles can be adapted as required; therefore it is not necessary to consider limits on the initial number of vehicles. The known matrix of shortest distances **D** between all municipalities is given in Table 1.

Model CVRP (1) – (7) and model (8) – (15) were implemented in software GAMS (solver Cplex 12.2.0.0 was used).

Firstly, individual daily optimization was computed (using model (1) – (7)). Results are given in Table 2.

T a b l e 2

CVRP in all Periods

Period/ Route	Sequence of nodes	Number of distributed units	Total distance (km)
1. Period (5 routes)	0-1-2-0-4-3-0-6-8-0-9-10-7-15-14-13-12-11-0-16-5-0		139
Route 1	0-1-2-0	4 498	14
Route 2	0-4-3-0	6 475	20
Route 3	0-6-8-0	7 194	23
Route 4	0-9-10-7-5-14-13-12-11-0	8 660	68
Route 5	0-16-5-0	7 168	14
2. Period (4 routes)	0-2-1-3-0-4-6-0-8-9-7-10-15-14-13-12-11-5-0-16-0		123
Route 1	0-2-1-3-0	7 795	21
Route 2	0-4-6-0	6 676	25
Route 3	0-8-9-7-10-15-14-13-12-11-5-0	8 621	63
Route 4	0-16-0	6 737	14
3. Period (4 routes)	0-1-3-4-0-2-9-8-6-0-11-12-13-14-15-10-7-0-16-5-0		136
Route 1	0-1-3-4-0	7 981	27
Route 2	0-2-9-8-6-0	8 828	40
Route 3	0-11-12-13-14-15-10-7-0	6 760	55
Route 4	0-16-5-0	7 445	14
4. Period (4 routes)	0-2-1-4-3-0-6-0-11-12-13-14-15-7-10-9-8-0-16-5-0		116
Route 1	0-2-1-4-3-0	8 918	27
Route 2	0-6-0	3 930	12
Route 3	0-11-12-13-14-15-7-10-9-8-0	8 616	63
Route 4	0-16-5-0	8 877	14
5. Period (5 routes)	0-2-1-4-3-0-5-0-6-9-7-0-8-10-15-14-13-12-11-0-16-0		143
Route 1	0-2-1-4-3-0	8 255	27
Route 2	0-5-0	1 143	6
Route 3	0-6-9-7-0	8 808	41
Route 4	0-8-10-15-14-13-12-11-0	8 102	55
Route 5	0-16-0	8 800	14

Note: 0 – Pezinok, 1 – Slovenský Grob, 2 – Viničné, 3 – Svätý Jur, 4 – Limbach, 5 – Vinosady, 6 – Šenkvice, 7 – Budmerice, 8 – Vištuk, 9 – Báhoň, 10 – Jablonec, 11 – Dubová, 12 – Píla, 13 – Častá, 14 – Doľany, 15 – Štefanová, 16 – Modra.

Source: Own compilation.

T a b l e 3

Stabilized Route

Route	Sequence	Distance (km)
Stabilized route	0-1-2-0-3-4-0-5-0-13-12-11-6-0-14-15-10-7-9-8-0-16-0	151
Route 1	0-1-2-0	14
Route 2	0-3-4-0	20
Route 3	0-5-0	6
Route 4	0-13-12-11-6-0	40
Route 5	0-14-15-10-7-9-8-0	57
Route 6	0-16-0	14

Source: Own compilation.

The total distance is 657 km. It is calculated as the sum of the individual distribution routes for all the days. Further on, the situation was modeled according to (8) – (15), which leads to realization of identical route for whole periods. The resulting distribution is described in Table 3.

Stabilized route consist of 6 routes: 0-1-2-0-3-4-0-5-0-13-12-11-6-0-14-15-10-7-9-8-0-16-0, the cost of the route: 151 km. The detailed information about realized routes is given below.

1-st route: Sequence of nodes: 0-1-2-0, total distance of 1-st route 14 km, total capacity realized in the 1-st day: 4498 units, total capacity realized in the 2-nd day: 4 730 units, total capacity realized in the 3-rd day: 3 302 units, total capacity realized in the 4-th day: 4 467 units, total capacity realized in the 5-th day: 2 789 units.

2-nd route: Sequence of nodes: 0-3-4-0, total distance of 2-nd route 20 km, total capacity realized in the 1-st day: 6 475 units, total capacity realized in the 2-nd day: 4 586 units, total capacity realized in the 3-rd day: 6 019 units, total capacity realized in the 4-th day: 4 451 units, total capacity realized in the 5-th day: 5 466 units.

3-rd route: Sequence of nodes: 0-5-0, total distance of 3-rd route 6 km, total capacity realized in the 1-st day: 554 units, total capacity realized in the 2-nd day: 644 units, total capacity realized in the 3-rd day: 718 units, total capacity realized in the 4-th day: 1 145 units, total capacity realized in the 5-th day: 1 143 units.

4-th route: Sequence of nodes: 0-13-12-11-6-0, total distance of 4-th route 40 km, total capacity realized in the 1-st day: 8 101 units, total capacity realized in the 2-nd day: 7 496 units, total capacity realized in the 3-rd day: 6 885 units, total capacity realized in the 4-th day: 7 439 units, total capacity realized in the 5-th day: 8 517 units.

5-th route: Sequence of nodes: 0-14-15-10-7-9-8-0, total distance of 5-th route 57 km total capacity realized in the 1-st day: 7 753 units, total capacity realized in the 2-nd day: 5 636 units, total capacity realized in the 3-rd day: 7 363 units, total capacity realized in the 4-th day: 5 107 units, total capacity realized in the 5-th day: 8 393 units.

6-th route: Sequence of nodes: 0-16-0, total distance of 6-th route 14 km, total capacity for the 1-st day: 6 614 units, total capacity realized in the 2-nd day: 6 737 units, total capacity realized in the 3-rd day: 6 727 units, total capacity realized in the 4-th day: 7 732 units, total capacity realized in the 5-th day: 8 800 units.

The total distance for all days is 755 km, which is about 98 km (14.92%) over distance calculated according to CVRP (individual daily optimization). This increase presents the minimal possible cost of route unification. An individual distributor should decide whether he accepted such an increase relative to daily optimization.

Authors also analysed the influence of individual input parameters using generated data set (change of the vehicle's capacity, changes in demand). Clearly, the change in capacity of the vehicle caused a change in routes. In reality, however, it is particularly important to consider changes in demand in different time periods. Testing was performed using generated data with different percentage changes for individual delivery within the analysed period. After subsequent calculations and further analysis of that situation authors concluded that it is generally difficult to determine the boundary changes when it is effective using synchronized delivery, respectively it is more efficient to calculate individual routes for each period. This is due to the fact that the results are influenced not only by changes in demand, but also by a specific matrix shortest distances (the position of the individual customers) and by vehicle capacity. Any further analysis of operating costs and possible scheduling optimization remains an open question.

Conclusion

The analysed distribution problem is oriented on planning of stabilized routes among certain time periods. Although classical daily optimization allows adapting goods distribution to customers' requirements, it can lead to daily changes in dispatching routes. On the other hand it may be desirable to reduce everyday optimization calculation. Stabilized route can also bring more transparent data on the vehicle position (if response to problems in the traffic situation is necessary). Benefit for driver is better orientation in realized routes and also customers can better anticipate the receiving time of their goods. Therefore distribution companies quite often resort to certain unification which allows them to establish stabilized distribution routes and profit from above mentioned conveniences.

The above approach addresses the problem of finding a uniform distribution within a certain time period. Input parameters of presented model are the shortest distances, vehicle capacity and customer demands in each time period. The article on the above parameters analyses the specific situation in the selected region. Generally, it is necessary to specify both the solution using presented model as well as individual optimal solutions in each time period. Then it is necessary to carry out an analysis whether increased cost caused by route synchronization provides adequate effects (reduction of distribution errors, identical times of deliveries) and choose the way of distribution.

Developed model (8) – (15) enables to calculate unified route with minimal increase in the travelled distance (in the presented case study costs increased by 14.92%). On the other hand such unified route provides increased comfort for drivers and customers. Therefore it depends on individual distributor decision whether he accepts this cost increase and gives priority to benefits from unified route relative to daily optimization. Based on the presented case study can be concluded that in the given situation (given the level of demand) it is advisable to use a unified distribution as increased costs do not reflect a significant increase and the effects of a synchronized path create better conditions for delivery and better services for customers (as reported in part 2 of this article).

The article was structured as follows. Concepts of synchronized distribution as well as some relevant works on mathematical modelling are given in introduction. Classical capacitated vehicle routing problem is formulated in the first part of the article. This formulation provides a basis for constructing the model of synchronization of capacitated vehicle routing problem among periods in the second part. The third chapter is devoted to solving real distribution problem based on routes unification in Slovak region. Distribution system was designed based on the model given in part two.

Including operating costs or scheduling optimization represents a further possible extension of presented model and these themes are opened for future research. Due to authors meaning, presented approach offers tool to solve problems of synchronized distribution and can be also applied to a wide variety of real-life problems.

References

BAITA, F. – UKOVICH, W. – PESENTI, R. – FAVARETTO, D. (1998): Dynamic Routing-and-inventory Problems: A Review. Transportation Research A, *32,* No. 8, pp. 585 – 598.

BREZINA, I. – ČIČKOVÁ, Z. – REIFF, M. (2009): Kvantitatívne metódy na podporu logistických procesov. Bratislava: Vydavateľstvo EKONÓM.

ČIČKOVÁ, Z. – BREZINA, I. – PEKÁR, J. (2013): Solving the Real-life Vehicle Routing Problem with Time Windows Using Self Organizing Migrating Algorithm. Ekonomický časopis/ Journal of Economics, *61*, No. 5, pp. 497 – 513.

DESROSIERS, L. – DUMAS, Y. – SOLOMON, M. M. – SOUMIS, F. (1995): Time Constrained Routing and Scheduling. In: BALL, M. O. et al. (eds): Network Routing. [Handbooks in Operations Research and Management Science.] Amsterdam: North-Holland, Vol. 8, pp. 35 – 139.

FEDERGRUEN, A. – SIMCHI-LEVI, D. (1995): Analysis of Vehicle Routing and Inventory-Routing Problems. In: BALL, M. O., MAGNANTI, T. L., MONMA, C. L. and NEMHAUSER, G. L.: Network Routing. Amsterdam: Elsevier Science – North Holland, Vol. 8, pp. 297 – 373.

GOLDEN, B. L. – RAGHAVAN, S. – WASIL, E. A. (2008): The Vehicle Routing Problem: Latest Advances and New Challenges. New York: Springer.

GROER, C. – GOLDEN, B. L. – WASIL, E. (2009): The Consistent Vehicle Routing Problem. Manufacturing & Service Operations Management, *11*, No. 4, pp. 630 – 643.

MILLER, C. E. – TUCKER, A. W. – ZEMLIN, R. A. (1960): Integer Programming Formulation of Traveling Salesman Problems. Journal of the ACM (JACM), *7*, No. 4, pp. 326 – 329.

MURATA, T. – ITAI, R. (2008): Enhancing Solution Similarity in Multi-Objective Vehicle Routing Problems with Different Demand Periods. In: CARIC, T. and GOLD, H.: Vehicle Routing Problem. Rijeka: University Library Rijeka, pp. 99 – 112.

PEKÁR, J. – BREZINA, I. – ČIČKOVÁ, Z. (2014): Synchronization of Vehicle Routing Problem in Two Periods. In: Quantitative Methods in Economics: Multiple Criteria Decision Making XVII. [Proceedings of the International Scientific Conference: 28th – 30th May 2014.] Virt: Vydavateľstvo EKONÓM, pp. 201 – 206.

RIBEIRO, R. – LOURENÇO, H. R. (2005): A New Model and Heuristic for a Multi-period Inventory-routing Problem. [Proceeding of the Decision Sciences Institute International Conference.] Barcelona: IESE, pp. 403 – 414.

RIBEIRO, R. – LOURENÇO, H. R. (2001): A Multi-objective Model for a Multiperiod Distribution Management Problem. In: PINHO de SAUSA, J. (ed.): Book of Extended Abstracts of the 4th Metaheuristics International Conference (MIC'2001). Porto, pp. 97 – 102.

SORENSEN, K. (2006): Route Stability in Vehicle Routing Decisions: A Bi-objective Approach Using Metaheuristics. Central European Journal of Operations Research, *14*, No. 2, pp. 193 – 207.

THANGIAH, S. R. – WILSON, B. – PITLUGA, A. – MENNELL, W. (2004): School Bus Routing in Rural School Districts. In: Computer-Aided Scheduling of Public Transport. [Proceedings of 9th International Conference.] San Diego: CASPT.