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PROBLEM SESSION

The 5th Polish, Slovak, and Czech Conference on Number Theory
Terchová, June 14–18, 2004

The Problem Session, chaired by Andrzej Schinzel, took place on June 18, 2004. The following problems were proposed.

1. A. Rotkiewicz: Let p be an odd prime and a and b distinct positive integers not divisible by p. Do there exist infinitely many Carmichael numbers n divisible by p such that $n \mid a^{n-1} - b^{n-1}$?

2. J. Krempa: Let F be a field. F is said to be *square independent* if there exists an infinite sequence $s_1, s_2, \dots$ of elements of F such that for any n the quadratic form $\sum_{i=1}^{n} s_i X_i^2$ does not represent zero nontrivially.

Question: Suppose that for any $n \geqslant 1$ there exists a quadratic form in n variables over F which does not represent zero nontrivially. Is then F square independent?

Comment. This problem was stated in an equivalent form by H. Gross in his book *Quadratic Forms in Infinite Dimensional Vector Spaces*, Birkhäuser, 1979. It has been solved in the negative by K. J. Becher and D. Hoffmann in a forthcoming paper *On isotropy of quadratic spaces in finite and infinite dimension*.

3. A. Paszkiewicz: Does there exist a solution n of the congruence $84^{2n-1} \equiv 82^{2n-1} \pmod{2n}$ of the form $n = pq$, where p, q are odd primes?

4. A. Paszkiewicz: Does there exist for every $n \geqslant 4$ an irreducible polynomial of degree n over the 2-element field with exactly 5 nonzero coefficients?

5. K. Szymiczek: Let R be a domain and let I be an invertible ideal in R. Classify up to similarity all nonsingular bilinear forms on the ideal I.

Collected and edited by K. Szymiczek.

6. M. Wołowiec-Musiał: Let K be an algebraically closed field and f be a homogeneous polynomial of degree d in n variables over K. Suppose f has a unique representation of the form

$$f = \ell_1^d + \cdots + \ell_r^d,$$

where ℓ_i are non-zero, projectively different linear forms in n variables, $r \in \mathbb{N}$. (We say that a representation is unique, if the linear forms ℓ_i are unique up to order and multiples of a dth root of unity).

Question. Is such a unique representation the shortest representation of f as a sum of dth powers of linear forms?

Comment. A. Schinzel proved that the answer is yes when $d = 3$ (see his paper in this volume).

7. A. Schinzel: Is $1 + \sqrt{17}$ the Mahler measure of an algebraic number?

8. A. Schinzel: Is the number of solutions of the congruence

$$a_1X_1 + \cdots + a_kX_k \equiv 0 \pmod{n}$$

in the box $0 \leqslant X_i \leqslant b_i \quad (1 \leqslant i \leqslant k)$ at least

$$2^{1-n}\prod_{i=1}^{k}(b_i + 1)\,?$$

9. A. Schinzel: Let $a_i \in \mathbb{C} \quad (i = 1, 2, 3)$ be distinct, $f, g \in \mathbb{C}[X]$,

$$f, g\colon \mathbb{C} \cup \{\infty\} \to \mathbb{C} \cup \{\infty\}\,.$$

Does $f^{-1}(a_i) = g^{-1}(a_i) \quad (i = 1, 2, 3)$ imply $f = g$?

Comment. According to a message from the author in March 2005, a counterexample has been constructed by D. W. Masser.

10. M. Radziejewski: Let G be a finite abelian group, written additively, $U \subsetneq G$. A sequence in U is a formal product $S = \prod_{g\in U} g^{\alpha_g}$. Its sum is defined as

$$\sigma(S) = \sum_{g\in U} \alpha_g g\,,$$

its cross number (cf. [1]) as

$$\mathrm{k}(S) = \sum_{g\in U} \frac{\alpha_g}{\operatorname{ord} g}\,.$$

We call a sequence S minimal if it has no proper non-unit divisor with the same sum as S. What can be said about the set $U^\star$ of elements $a \in G$ such that all minimal sequences in U with sum $-a$ have the same cross number? In particular, when $\emptyset \neq U^\star \subseteq U$ is possible?

Remarks.

(1) U is a half-factorial if and only if $0 \in U^\star$. In that case we have $U \subseteq U^\star$.

(2) The statement that "$U^\star \not\subseteq U$ for at least one half-factorial U of maximum cardinality in G" is equivalent to a conjecture studied by Wolfgang A. Schmid and the author [2].

(3) For $|G| \leqslant 50$ it was verified that $U^\star \not\subseteq U$ for all half-factorial $U \subsetneq G$.

(4) For $|G| \leqslant 95$ and U a maximal subset satisfying the (C_0) condition of J. Śliwa [3] it was checked that either $U^\star = \emptyset$ or $U^\star \not\subseteq U$ (unpublished computational results).

(5) $\emptyset \neq U^\star \subseteq U$ is possible, e.g., if $G = H_1 \oplus H_2$, $H_1 \cong C_2$, $H_2 \cong C_2^2$, $U = H_1 \cup H_2$, then $U^\star = H_1 \setminus \{0\}$.

REFERENCES

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[2] RADZIEJEWSKI, M.—SCHMID, W. A.: *On the asymptotic behavior of some counting functions*, Colloq. Math. **102** (2005), 181–195.

[3] ŚLIWA, J.: *Remarks on factorizations in algebraic number fields*, Colloq. Math. **46** (1982), 123–130.

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