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COVARIANCE ANALYSIS OF HIERARCHICAL CLASSIFICATION IN SPLIT PLOT DESIGN WITH RESPECT TO THE CONTROL

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ABSTRACT. The paper deals with the analysis of covariance in the split-plot design where the so-called control treatment is allocated to the subplot. The factor placed on the small plots is subject to two stage classification, hence the hierarchical classification arises. The linear model for such experiment with the partition according to the test levels and control levels is presented. The estimators of model parameters and the tests for hypothesis are given. The consideration are illustrated by a practical example.

1. Introduction

An experiment carried out according to the balanced split-plot design, where one so-called control treatment on the subplot occured, was considered. The factor allocated on the small plots is subject to two stage classification hence the third factor arises inside. The main problem concerns the comparison of few subplot treatment means with the control. Additional factor and appropriate interaction effects extend the classical linear model for split-plot design. The effects of blocks are treated as fixed effects. Random errors are the same as these in the classical model, the reason being that the experiment has been arranged according to scheme of the split-plot design with two factors. The aim of this paper is a presentation of covariance analysis for a new form of model of such experiment.

Here is the regression variable $\mathbf{Y}$ considered with respect to the concomitant variable $\mathbf{Z}$ for factor A on the whole plot and for factor B on the subplot separately. Hence, there are two regression coefficients in the model.

Let us consider the linear split-plot model

$$\mathbf{Y} = \mathbf{X}_w \boldsymbol{\theta}_w + \mathbf{X}_z \boldsymbol{\eta} + \mathbf{X}_{\mathbf{e}} \mathrm{e} ,$$

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where $\mathbf{Y}$ is the main variable, $\mathbf{X}_w\theta_w$ is the component referring to the analysis of variance model, $\mathbf{X}_w$ is the matrix of the form

$$\mathbf{X_w} = \left[\mathbf{X_M} \vdots \mathbf{X_R} \vdots \mathbf{X_A} \vdots \mathbf{X_B} \vdots \mathbf{X_{C(B)}} \vdots \mathbf{X_{AB}} \vdots \mathbf{X_{AC(B)}}\right],$$

$\boldsymbol{\theta}_\mathbf{w} = \left[\boldsymbol{\mu}', \boldsymbol{\rho}', \boldsymbol{\alpha}', \boldsymbol{\beta}', \boldsymbol{\gamma}', \boldsymbol{\alpha\beta}', \boldsymbol{\alpha\gamma}'\right]'$ is the vector of fixed effects.

Hence, the number of observations n is equal to $ra(bc+1)$, and $\mathbf{y}$ is the $(n \times 1)$ vector of observations. Further

$$\mathbf{X}_M = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{1}_{bc} & \mathbf{0}_{bc} \\ 0 & 1 \end{bmatrix}$$

is the matrix for vector of the general means $\boldsymbol{\mu} = [\mu_1, \mu_0]'$,

$$\mathbf{X}_R = \mathbf{I}_r \otimes \mathbf{1}_{a(bc+1)}$$

is the matrix for blocs R, $\boldsymbol{\rho}$ is the $(r \times 1)$ vector of fixed effects of blocs,

$$\mathbf{X}_A = \mathbf{1}_r \otimes \mathbf{I}_a \otimes \mathbf{1}_{bc+1}$$

is the matrix for factor A, $\boldsymbol{\alpha} = \left[\alpha_1, \alpha_2, \ldots \alpha_a\right]'$ is the $(a \times 1)$ vector of fixed effects of factor A,

$$\mathbf{X}_B = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_c & \mathbf{0}_{bc} \\ \mathbf{0}'_b & 1 \end{bmatrix}$$

is the matrix for factor B, $\boldsymbol{\beta} = \left[\beta_1, \ldots, \beta_b, \beta_0\right]'$ is the $\left[(b+1) \times 1\right]$ vector of fixed effects of factor B,

$$\mathbf{X}_{C(B)} = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{I}_{bc} \\ \mathbf{0}'_{bc} \end{bmatrix}$$

is the matrix for factor C inside B, $\boldsymbol{\gamma} = \left[\gamma_{1(1)}, \gamma_{2(1)}, \ldots, \gamma_{c(1)}, \ldots, \gamma_{1(b)}, \gamma_{2(b)}, \ldots, \gamma_{c(b)}\right]'$ is the $(bc \times 1)$ vector of fixed effects of factor C,

$$\mathbf{X}_{AB} = \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_c & \mathbf{0}_{bc} \\ \mathbf{0}'_b & 1 \end{bmatrix}$$

is the matrix for interaction AB, $\boldsymbol{\alpha\beta} = \left[\alpha\beta_{11}, \ldots, \alpha\beta_{1b}, \alpha\beta_{10}, \alpha\beta_{21}, \ldots, \alpha\beta_{ab}, \alpha\beta_{a0}\right]'$ is the $\left[a(b+1) \times 1\right]$ vector of interaction effects between A and B and, besides, control effects for B inside A,

$$\mathbf{X}_{AC(B)} = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{I}_{bc} \\ \mathbf{0}'_{bc} \end{bmatrix}$$

is the matrix for interaction $AC(B)$, $\boldsymbol{\alpha\gamma}$ is the $\left[a(bc+1) \times 1\right]$ vector of interaction effects of $AC(B)$, Z is a concomitant variable,

$$\mathbf{X}_z \boldsymbol{\eta}$$

is the component referring to the concomitant variable $\mathbf{Z}$,

$$\mathbf{X_z} = \left[\mathbf{P}(\mathbf{X_1}) - \mathbf{P}(\mathbf{X_R}) \colon \mathbf{I_N} - \mathbf{P}(\mathbf{X_1})\right]\mathbf{Z}\,,$$

$\boldsymbol{\eta} = \left[\eta_1 : \eta_2\right]'$ is the vector of regression coefficients, $\mathbf{X_e} = \left[\mathbf{X_1} \colon \mathbf{I_N}\right]$, $X_1 = \left[\mathbf{I_{ra}} \otimes \mathbf{1_{bc+1}}\right]$-component referring to the experimental error, $\mathbf{e} = \left[\mathbf{e}_1', \mathbf{e}_2'\right]'$-two types of experimental error, namely: the whole plot error and the split-plot error. Moreover, the notation given below is used:

- $\otimes$ denotes Kronecker product of matrices,
- $\mathbf{I}_n$ is the $n \times n$ identity matrix,
- $\mathbf{1}_n$ is the $n \times 1$ vector of ones,
- $\mathbf{0}_n$ is the $n \times 1$ vector of nulls,
- $\mathbf{E}_n$ is the $n \times n$ matrix of ones,
- $\mathbf{E}_{n\times m}$ is the $n \times m$ matrix of ones,
- $\mathbf{P}$ denotes projection operators.

2. MS Estimators

Restrictions and assumptions:
Random effects of $\mathbf{e_1} = (e_{ij})'$, $\mathbf{e}_2 = (e_{ijk})'$ are normally, independently distributed random error terms

$$\mathbf{e}_{ij} \approx N\left(0; \sigma_1^2\right), \qquad \mathbf{e}_{ijk} \approx N\left(0; \sigma_2^2\right),$$

respectively. Finding the general inverse of matrix for normal equations we take some restrictions on models parameters to obtain a unique general inverse. For fixed effects it is assumed that its appropriate sums are equal to zero,

$$\sum_{i=1}^{r} \rho_i = 0\,, \quad \sum_{j=1}^{a} \alpha_j = 0\,, \quad \sum_{k=1}^{b} \beta_k = 0\,,\ \beta_0 = 0\,, \quad \forall k \neq 0 \ \sum_{l=1}^{c} \gamma_{l(k)} = 0\,,$$

$$\forall k \neq 0 \ \sum_{j=1}^{a} (\alpha\beta)_{jk} = 0\,, \quad \forall j \ \sum_{k=1}^{b} \left[c(\alpha\beta)_{jk} + (\alpha\beta)_{j0}\right] = 0\,,$$

$$\forall k \neq 0 \wedge \forall j \ \sum_{l=1}^{c} (\alpha\gamma)_{jl(k)} = 0\,, \quad \forall l \ \sum_{k=1, j=1}^{b,a} (\alpha\gamma)_{jl(k)} = 0\,.$$

We have assumed that the fixed effect β_0 is equal to zero for the reason that the factor B is not applied in the control plots.

With respect to the concomitant variable $\mathbf{Z}$ it is necessary to assume that: expected value $E(\mathbf{Y})$ is included in the direct sum of spaces $R(\mathbf{X})$ and $R(\mathbf{Z})$ spanned on the columns of the matrix $\mathbf{X}$ and vector $\mathbf{Z}$, respectively,

$$E(\mathbf{Y}) \subset R(\mathbf{X}) \oplus R(\mathbf{Z}).$$

The solution of the normal equations

$$\mathbf{X}'\mathbf{V}^{-1}\boldsymbol{\theta} = \mathbf{X}'\mathbf{V}^{-1}\mathbf{y}.$$

(Searle [7]) for this model, where covariance matrix of vector $\mathbf{y}$ takes the form

$$\mathbf{V} = (bc+1)(\sigma_1^2+\sigma_2^2)\mathbf{P}(\mathbf{X_1}) + \sigma_2^2\left[\mathbf{I_N} - \mathbf{P}(\mathbf{X_1})\right],$$

gives (under the taken assumptions) the following estimators of the parameters (Kuna-Broniowska I. and Przybysz T. [3]):

$$\hat{\boldsymbol{\mu}} = \left\{\frac{1}{ra}\mathbf{1}'_{ra} \otimes \begin{bmatrix} \frac{1}{bc}\mathbf{1}'_{bc} & 0 \\ \mathbf{0}'_{bc} & 1 \end{bmatrix}\right\}\mathbf{y},$$

$$\hat{\boldsymbol{\rho}} = \left[\frac{1}{a(bc+1)}\left(\mathbf{I}_r - \frac{1}{r}\mathbf{E}_r\right) \otimes \frac{1}{a(bc+1)}\mathbf{1}'_{a(bc+1)}\right](\mathbf{y} - \mathbf{z}\eta_1),$$

$$\hat{\boldsymbol{\alpha}} = \left\{\frac{1}{r}\mathbf{1}'_r \otimes \left[\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right] \otimes \frac{1}{bc+1}\mathbf{1}'_{bc+1}\right\}(\mathbf{y} - \mathbf{z}\eta_1),$$

$$\hat{\boldsymbol{\beta}} = \left\{\frac{1}{ra}\mathbf{1}'_{ra} \otimes \left[\mathbf{I}_b \otimes \frac{1}{c}\mathbf{1}'_c - \frac{1}{bc}\mathbf{E}_{b\times bc} \vdots \mathbf{0}_b\right]\right\}(\mathbf{y} - \mathbf{z}\eta_2),$$

$$\hat{\boldsymbol{\gamma}} = \left\{\frac{1}{ra}\mathbf{1}'_{ra} \otimes \left[\mathbf{I}_{bc} - \mathbf{I}_b \otimes \frac{1}{c}\mathbf{E}_c \vdots \mathbf{0}_{bc}\right]\right\}(\mathbf{y} - \mathbf{z}\eta_2),$$

$$\boldsymbol{\alpha}\hat{\boldsymbol{\beta}} = \left\{\frac{1}{r}\mathbf{1}'_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes \left(\begin{bmatrix} \mathbf{I}_b \otimes \frac{1}{c}\mathbf{1}'_c & \mathbf{0}_b \\ \mathbf{0}'_{bc} & 1 \end{bmatrix} - \frac{1}{bc+1}\mathbf{E}_{(b+1)\times(bc+1)}\right)\right\}$$
$$(\mathbf{y} - \mathbf{z}\eta_2),$$

$$\boldsymbol{\alpha}\hat{\boldsymbol{\gamma}} = \left\{\frac{1}{r}\mathbf{1}'_r \otimes \left(\mathbf{I}_a - \frac{1}{a}\mathbf{E}_a\right) \otimes \left[\mathbf{I}_{bc} - \mathbf{I}_b \otimes \frac{1}{c}\mathbf{E}_c \vdots \mathbf{0}_{bc}\right]\right\}(\mathbf{y} - \mathbf{z}\eta_2),$$

$$\hat{\boldsymbol{\eta}}_1 = (\mathbf{z}'\mathbf{P_1}\mathbf{z})^{-1}\mathbf{z}'\mathbf{P_1}\mathbf{y}, \qquad \hat{\boldsymbol{\eta}}_2 = (\mathbf{z}'P_{\mathbf{2}}\mathbf{z})^{-1}\mathbf{z}'\mathbf{P_2}\mathbf{y},$$

where: $\mathbf{P_1} = \mathbf{P}(X_1) - \mathbf{P_M}, \mathbf{P_1} = \mathbf{P}(X_2) - \mathbf{P_M}$.

According to the taken model, the estimators of the all elements of the vectors parameters $\boldsymbol{\rho}, \boldsymbol{\alpha}$ and $\boldsymbol{\alpha\beta}$ are based on all observations. The estimators of the elements of the vectors parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}$ and $\boldsymbol{\alpha\gamma}$ are based on the observations with omission of the values from control plots.

3. Analysis of covariance

In the split-plot model with control level we are interested in comparing the test levels of the subplot treatment B with control level of B and comparing the levels of the whole plot treatment A on the background of the contrast: test levels of B-control level of B. Using the matrix notation in analysis of covariance we decompose the N dimensional data space R_N into orthogonal sub-spaces corresponding to the particular sources of variation. This decomposition is well known for traditional split-plot design (Mikos H. and Niedokos E. [5]; Kuczyński M. [2]). With respect to the additional control treatment we have to separate the additional sub-spaces, which will be obtained by further partition some of the sub-spaces. The finding these spaces and corresponding projection operators needs the writing some expressions from model as the following sums:

– expression $\mathbf{X}_M\mu$ for vector means into $\mathbf{X}_{M_1}\mu_1$ and $\mathbf{X}_{M_0}\mu_0$,

$$\mathbf{X}_M\boldsymbol{\mu} = \mathbf{X}_{M_1}\mu_1 + \mathbf{X}_{M_0}\mu_0 = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{1}_{bc} \\ 0 \end{bmatrix} \mu_1 + \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{0}_{bc} \\ 1 \end{bmatrix} \mu_0 ,$$

– expression $\mathbf{X}_B\beta$ for factor on the subplot into two components $\mathbf{X}_{B_1}\boldsymbol{\beta}_1$ and $\mathbf{X}_{B_0}\boldsymbol{\beta}_0$

$$\mathbf{X}_B\boldsymbol{\beta} = \mathbf{X}_{B_1}\boldsymbol{\beta}_1 + \mathbf{X}_{B_0}\boldsymbol{\beta}_0 = \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_c \\ \mathbf{0}'_b \end{bmatrix} \boldsymbol{\beta}_1 + \mathbf{1}_{ra} \otimes \begin{bmatrix} \mathbf{0}_{bc} \\ 1 \end{bmatrix} \boldsymbol{\beta}_0 ,$$

and

– expression $\mathbf{X}_{AB}\boldsymbol{\alpha\beta}$ for interaction into $\mathbf{X}_{AB_1}\boldsymbol{\alpha\beta}_1$ and $\mathbf{X}_{AB_0}\boldsymbol{\alpha\beta}_0$, respectively,

$$\mathbf{X}_{AB}\boldsymbol{\alpha\beta} = \mathbf{X}_{B_1}\boldsymbol{\alpha\beta}_1 + \mathbf{X}_{B_0}\boldsymbol{\alpha\beta}_0 = \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{I}_b \otimes \mathbf{1}_c \\ \mathbf{0}'_b \end{bmatrix} \boldsymbol{\alpha\beta}_1 + \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{0}_{bc} \\ 1 \end{bmatrix} \boldsymbol{\alpha\beta}_0 .$$

Index "1" concerns the tested effects and index "0" concerns the control effects. Besides, the matrix X_A can be written as the sum

$$\mathbf{X}_A = \mathbf{X}_{A_1} + \mathbf{X}_{A_2} = \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{1}_{bc} \\ \mathbf{0} \end{bmatrix} + \mathbf{1}_r \otimes \mathbf{I}_a \otimes \begin{bmatrix} \mathbf{0}_{bc} \\ 1 \end{bmatrix} .$$

This decomposition was made with respect to the control effect β_0 of B and control mean μ_0. With respect to the control treatment and two general means, the following additional sub-spaces have been separated:

- $R(\mathbf{X}_{B1})$ for nonzero levels of factor B,
- $R(\mathbf{X}_T)$ for contrast $T = \mu_1 - \mu_0$,
- $R(\mathbf{X}_{AB1})$ for interaction between A and non zero levels of B,
- $R(\mathbf{X}_{T_j})$ for contrasts $T_j = (\alpha\beta)_{j0} - \frac{1}{b}\sum_{k=1}^{b}(\alpha\beta)_{jk}$.

According to this decomposition we obtain 12 orthogonal projection operators onto suitable data subspaces (Przybysz T. [6])

$$\mathbf{P_R}, \mathbf{P_A}, \mathbf{P_1}, \mathbf{P_T}, \mathbf{P_{B_1}}, \mathbf{P_{C(\mathit{B})}}, \mathbf{P_{T_{A_j}B}}, \mathbf{P_{AB_1}}, \mathbf{P_{AC(\mathit{B})}}, \mathbf{P_2}, \mathbf{P_{\eta_1}}, \mathbf{P}_{\eta_2}.$$

It is easy to show that appropriate quadratic forms kind of $\mathbf{y'Py}$ have non-central χ^2 distributions and their quotients have non-central F distributions.

Sums of squares for factors A, B and $C(B)$ from analysis of variance will be changed by replacing the vector of observations $\mathbf{y}$ with the new vectors:

$$\mathbf{y} - \mathbf{z}\eta_1 \quad \text{in } SS_A,$$

$$\mathbf{y} - \mathbf{z}\eta_2 \quad \text{in } SS_B, SS_{C(B)}, SS_{AB} \text{ and in } SS_{AC(B)},$$

when the F-tests for regression parameters η_1 and η_2 decided on the alternatives.

Table 1. Analysis of covariance.

Source of variation	DF	MS	Expected values of V	Hypothesis H_0	Test statistics
R	$r-1$	V_n	$\sigma_2^2 + (bc+1)\sigma_1^2 + \lambda_R$	$\lambda_R = 0$	V_n/V_1
Regression for A	1	V_{η_1}	$\sigma_2^2 + (bc+1)\sigma_1^2 + \lambda_{\eta_1}$	$\lambda_{\eta_1} = 0$	V_{η_1}/V_1
A	$a-1$	V_A	$\sigma_2^2 + (bc+1)\sigma_1^2 + \lambda_A$	$\lambda_A = 0$	V_A/V_1
Error e_1	$(r-1)(a-1)-1$	V_1	$\sigma_2^2 + (bc+1)\sigma_1^2$	–	–
Regression for B	1	$V_{\eta 2}$	$\sigma_2^2 + \lambda_{\eta 2}$	$\lambda_{\eta 2} = 0$	$V_{\eta 2}/V_2$
B	b	V_B	$\sigma_2^2 + \lambda_B$	$\lambda_B = 0$	V_B/V_2
Contrast T	1	V_T	$\sigma_2^2 + \lambda_T$	$\lambda_T = 0$	V_T/V_2
B_1	$b-1$	V_{B_2}	$\sigma_2^2 + \lambda_{B_1}$	$\lambda_{B_2} = 0$	V_{B1}/V_2
C inside B_1	$(c-1)b$	V_C	$\sigma_2^2 + \lambda_C$	$\lambda_C = 0$	V_C/V_2
$A \times B$	$(a-1)b$	V_{AB}	$\sigma_2^2 + \lambda_{AB}$	$\lambda_{AB} = 0$	V_{AB}/V_2
Contrasts T_j	$a-1$	V_{T_j}	$\sigma_2^2 + \lambda_{T_j}$	$\lambda_{T_j} = 0$	V_Z/V_2
$A \times B_1$	$(a-1)(b-1)$	V_{AB1}	$\sigma_2^2 + \lambda_{AB_1}$	$\lambda_{AB_1} = 0$	V_{AB1}/V_2
$A \times C(B)$	$(a-1)(c-1)b$	V_{AC}	$\sigma_2^2 + \lambda_{AC}$	$\lambda_{AC} = 0$	V_{AC}/V_2
Error e_2	$acb(r-1)-1$	V_2	σ_2^2	–	–
Total	$ra(bc+1)-1$				

The hypothesis

$$H_0 : \tau = \mu_1 - \mu_0 = 0$$

about the equality of the means of the component models should be verified at first. It can be treated as a joined comparison of the tested factors with the control treatment. For that case we obtain an answer to the important question about the influence of experimental factors on a given characteristic. The test function F for this hypothesis is based on a quadratic form $\mathbf{y}'\mathbf{P}_T\mathbf{y}$ with one degree of freedom.

Further the inference concerning the comparison of particular means with the control mean can be carried out through the use of Dunnett's confidence intervals or the F test. The analysis of variance based on the F test for particular hypotheses can be carried out according to the specific needs of the researcher. The sum of squares for the contrast $\mathbf{m}'\mathbf{y}$, in which the special comparisons have been considered, can be calculated by a formula $(\mathbf{m}'\mathbf{y})^2 / \sum_s m_s^2$, where m_s are the components of the vector $\mathbf{m}$ (Joshi D. [1]).

4. The example

Let us consider the influence of 7 levels of fertilization on the 2 variety of potatoes. Hence we have:

- 4 blocks,
- 2 whole plots (2 variety of potatoes),
- 7 subplots (one control level and 3 methods of fertilization: into soil, into leaves and its combination on 2 levels each of them),
- A — variety of potatoes,
- B — fertilization:
- B_0 — without fertilization — control object,
- B_{11} — 1×33.2 kg N/ha — solid form into soil,
- B_{12} — $24,9$ kg N/ha — solid form into soil,
- B_{21} — 4×8.3 kg N/ha — liquid form into leaves,
- B_{22} — 4×1.5l Agrosol-K/ha — liquid form into leaves,
- B_{31} — 2×8.3 kg N/ha + Agrosol-K combination solid and liquid,
- B_{32} — 3×8.3 kg N/ha + Agrosol-K combination solid and liquid,
- Y — dry mass content,
- Z — mass of stalks with tails (concomitant variable).

The hierarchical classification arises for fertilization. Each of considered fertilization methods is applied in 2 different dozes, but the doze changes from method to method.

REFERENCES

[1] JOSHI, D. D.: *Linear Estimation and Design of Experiments*, Willey Eastern Limited, New Delhi, 1987.

[2] KUCZYŃSKI, M.: *Estymacja parametrów i weryfikacja parametrów w analizie kowariancji w układzie z rozszczepionymi jednostkami eksperymentalnymi*, in: VI Coll. Metod. z Agrobiom., PAN, Warszawa, 1976, pp. 102–120.

[3] KUNA-BRONIOWSKA, I.—PRZYBYSZ, T.: *Analysis of hierarhical classification in split-plot design respect to the control*, XXIX Colloquium Biometryczne, 1999, pp. 53–65.

[4] MIKOS, H.: *Modele matematyczne układów rozszczepionych jednostek ze skorelowanymi błądami*, in: IV Coll. Metod. z Agrobiom., 1974, pp. 57–78.

[5] MIKOS, H.—NIEDOKOS, E.: *Estymacja parametrów w układach eksperymentalnych rozszczepionych jednostek*, in: VI Colloquium z Agro-Biometrii, 1976, pp. 29–63.

[6] PRZYBYSZ, T.: *Hierarchical linear model in rotation experiments*, Folia Societatis Scientiarum Lubliniensis. Mat-Fiz-Chem 2 **27** (1985).

[7] SEARLE, S. R.: *Linear Models*, J. Wiley, New York, 1971.

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