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CONCEPT OF LINEAR FUZZY COALITIONAL GAME

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ABSTRACT. There exist two models of coalitional games — those with and without side-payments. Both can be fuzzified and the way of fuzzification of the former may differ from that of the latter. The linear coalitional games are games which can be described by both models and each of these modifications can be fuzzified in its specific way. While in the deterministic case both approaches are equivalent, the fuzzy models may differ according to different methods of fuzzification. The differences illustrate more general methodological aspects of both approaches to the fuzzification of coalitional games. In this paper we briefly formulate the basic concepts and show the differences connected with the core of coalitional game.

1. Introductory concepts — coalitional games

The deterministic game theory recognizes two basic types of coalitional games — those with and without side-payments (see [5, 7, 6]). Let us very briefly recall the main concepts connected with them. Here, we limit our attention to the so called superadditive games. This limitation essentially simplifies the formalism without significant deformation of the main methodological ideas.

In both models I denotes the finite nonempty set of *players*, each set $K \subset I$ is a *coalition* and any real-valued vector $\boldsymbol{x} = (x_i)_{i \in I} \in \mathbb{R}^I$ is called *imputation*.

1.1 Games with side-payments

A *coalitional game with side-payments* is defined as an ordered pair (I, v) where $v \colon 2^I \to \mathbb{R}$. We suppose that for the empty coalition $v(\emptyset) = 0$ and that function

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v (often called characteristic function) is superadditive, that is, for any pair of disjoint coalitions K, $L \subset I$

$$v(K \cup L) \geq v(K) + v(L)\,. \tag{1}$$

The set

$$C = \Big\{ \boldsymbol{x} \in \mathbb{R}^I \colon \sum_{i \in I} x_i \leq v(I)\,, \quad \text{and for every } K \subset I\,, \ \sum_{i \in K} x_i \geq v(K) \Big\} \tag{2}$$

of imputations is called the *core* of the game (I, v). The core is considered one of the basic solution concepts of the coalitional game.

1.2 Games without side-payments

A *coalitional game without side-payments* is defined as a pair $(I, \mathcal{V})$ where $\mathcal{V}$ is a mapping assigning to each coalition $K \subset I$ a set $\mathcal{V}(K) \subset \mathbb{R}^I$ having the following properties:

$$\mathcal{V}(K) \ \text{ is closed}\,, \tag{3}$$

$$\text{if } \ \boldsymbol{x} \in \mathcal{V}(K)\,,\ \boldsymbol{y} \in \mathbb{R}^I,\ \forall\, i \in K \colon y_i \leq x_i\,, \ \text{ then } \ \boldsymbol{y} \in \mathcal{V}(K)\,, \tag{4}$$

$$\mathcal{V}(K) \neq \emptyset\,, \tag{5}$$

$$\mathcal{V}(K) = \mathbb{R}^I \iff K = \emptyset\,. \tag{6}$$

Mapping $\mathcal{V}$ is called general characteristic function. We suppose, further, that $(I, \mathcal{V})$ is *superadditive*, i.e., for every pair of disjoint coalitions K, $L \subset I$,

$$\mathcal{V}(K \cup L) \supset \mathcal{V}(K) \cap \mathcal{V}(L)\,. \tag{7}$$

If $K \subset I$ and $\boldsymbol{x}$, $\boldsymbol{y} \in \mathbb{R}^I$ then we say that $\boldsymbol{x}$ *dominates* $\boldsymbol{y}$ *via* K and write $\boldsymbol{x}\ \mathrm{dom}_K\ \boldsymbol{y}$ iff

$$x_i \geq y_i \quad \text{for all} \quad i \in K\,, \qquad x_j > y_j \quad \text{for some} \quad j \in K\,.$$

For each $K \subset I$ we define set $\mathcal{V}^*(K) \subset \mathbb{R}^I$ as follows:

$$\mathcal{V}^*(K) = \big\{ \boldsymbol{y} \in \mathbb{R}^I \colon \ \text{there does not exist } \ \boldsymbol{x} \in \mathcal{V}(K) \ \text{ such that } \ \boldsymbol{x}\ \mathrm{dom}_K\ \boldsymbol{y} \big\}\,. \tag{8}$$

Set $\mathcal{V}^*(K)$ is called *superoptium* of K. Obviously,

$$\mathcal{V}^*(K) = \big\{ \boldsymbol{y} \in \mathbb{R}^I \colon \forall\, \boldsymbol{x} \in \mathcal{V}(K), \ \text{ either } \ \exists\, i \in K,\ y_i > x_i \ \text{ or } \ \forall\, i \in K\,,\ x_i = y_i \big\}\,.$$

The core $\boldsymbol{C}$ of $(I, \mathcal{V})$ is defined as maximal of the sets $\boldsymbol{C}' \subset \mathbb{R}^I$ for which

$$\boldsymbol{C}' \subset \mathcal{V}(I) \qquad \text{and} \qquad \forall\, K \subset I \colon \boldsymbol{C}' \subset \mathcal{V}^*(K)\,,$$

that is

$$\boldsymbol{C} = \mathcal{V}(I) \cap \Big(\bigcap_{K \subset I} \mathcal{V}^*(K) \Big)\,. \tag{9}$$

2. Linear coalitional game

Let (I, v) be a coalitional game with side-payment and $K \subset I$ be a coalition. Let us define $\mathcal{V}_v(K)$ by

$$\mathcal{V}_v(K) = \left\{ \boldsymbol{x} \in \mathbb{R}^I \colon \sum_{i \in K} x_i \leq v(K) \right\}. \tag{10}$$

It can be easily verified that $(I, \mathcal{V}_v)$ is a coalitional game without side-payments. It means that there exists a subclass of the class of coalitional games without side-payments which are defined by games with side-payments by means of (10).

A coalitional game without side-payments $(I, \mathcal{V})$ where the sets $\mathcal{V}(K)$ fulfill (10) for some game (I, v) will be called *linear coalitional game*. Due to the facts mentioned above we know that, for any linear coalitional game $(I, \mathcal{V})$, there exists exactly one coalitional game (I, v) with side-payments connected with $(I, \mathcal{V})$ by means of (10). In the deterministic case, both games $(I, \mathcal{V})$ and (I, v) for a given set of players I represent identical cooperative situation. It is also not difficult to verify that (I, v) is superadditive in the sense of (1) if and only if $(I, \mathcal{V})$ is superadditive in the sense of (7) and that both cores, C of (I, v) and $\boldsymbol{\mathcal{C}}$ of $(I, \mathcal{V})$, are identical.

3. Fuzzification of linear coalitional games

The fuzzification of coalitional games which will be considered in this paper is based on the idea that the expected pay-offs (i.e., the values $v(K)$ and sets $\mathcal{V}(K)$) are more or less vague in real situations, and they might be substituted by their fuzzy counterparts. As the fuzzification methods used for $v(K)$ and $\mathcal{V}(K)$ differ from each other (see [2, 3, 5]) the linear coalitional games can effectively illustrate the differences between both approaches.

3.1 Briefly about fuzzy quantities

A fuzzy subset a of the real line $\mathbb{R}$ with the membership function $\mu_a : \mathbb{R} \to [0, 1]$ such that

$$\exists\, x_a \in \mathbb{R} : \mu_a(x_a) = 1\,, \tag{11}$$

$$\exists\, x_1, x_2 \in \mathbb{R} : x_1 < x_2\,, \quad \forall\, x \notin [x_1, x_2] \colon \mu_a(x) = 0 \tag{12}$$

is called *fuzzy quantity*. Each number x_a in (11) is called modal value of a. If a, b are fuzzy quantities then the *sum* $a \oplus b$ and *opposite quantity* $-a$ are also fuzzy quantities with the membership functions

$$\mu_{-a}(x) = \mu_a(-x), \quad x \in \mathbb{R}\,, \tag{13}$$

$$\mu_{a \oplus b}(x) = \sup_{y \in \mathbb{R}} \Big(\min\big(\mu_a(y),\, \mu_b(x - y)\big) \Big)\,. \tag{14}$$

The ordering relation $\succsim$ between fuzzy quantities is a fuzzy relation which is, for particular a and b, valid with the possibility

$$\nu_{\succsim}(a,b) = \sup_{\substack{x,y\in\mathbb{R}\\ x\geq y}} \Big(\min(\mu_a(x),\, \mu_b(y))\Big)\,. \tag{15}$$

If a is a fuzzy quantity and $r \in \mathbb{R}$ then analogously to (15)

$$\nu_{\succsim}(a,r) = \sup\big(\mu_a(x)\colon x \geq r\big)\,, \quad \nu_{\succsim}(r,a) = \sup\big(\mu_a(x)\colon r \geq x\big)\,. \tag{16}$$

The properties of the above operations and relations are summarized, e.g., in [1] or [4].

3.2 Fuzzy coalitional games with side-payments

Let (I,v) be a coalitional game with side-payments and let for every $K \subset I$ a fuzzy quantity $w(K)$ with the membership function μ_K be such that

$$\mu_K\big(v(K)\big) = 1\,, \tag{17}$$

$$\mu_K(x) \text{ is nondecreasing for } x < v(K)\,,$$

$$\mu_K(x) \text{ is nonincreasing for } x > v(K)\,, \tag{18}$$

$$\text{for empty coalition, } \mu_{\emptyset}(0) = 1\,,\ \mu_{\emptyset}(x) = 0 \text{ for } x \neq 0\,. \tag{19}$$

The pair (I,w) is called a *fuzzy extension* of (I,v).

As the expected pay-offs of coalitions in (I,w) are vague (fuzzy) quantities then also the property of superadditivity will be a fuzzy property. Let us denote by $G(I,v)$ the class of all fuzzy extensions of (I,v). The superadditivity of its elements will be characterized by a fuzzy subset of $G(I,v)$ with the membership function $\nu_{\text{super}}(\cdot)\colon G(I,v) \to [0,1]$ whose values are defined by the following procedure. Let (I,w) be an arbitrary fuzzy extension of (I,v), and let $K,L \subset I$ be its disjoint coalitions. According to (15),

$$\nu_{\succsim}\big(w(K\cup L)\,,\, w(K)\oplus w(L)\big)$$

denotes the possibility with which the fuzzy relation

$$w(K\cup L) \succsim w(K)\oplus w(L) \tag{20}$$

analogous to (1), holds. It is natural to define the values of ν_{super} by

$$\nu_{\text{super}}(I,w) = \min\big[\nu_{\succsim}\big(w(K\cup L)\,,\, w(K)\oplus w(L)\big)\colon K,\, L\subset I\,,\, K\cap L = \emptyset\big]\,. \tag{21}$$

Now we define the *fuzzy core* C_F of (I,w) as a fuzzy subset of $\mathbb{R}^I$ with the membership function $\gamma_C\colon \mathbb{R}^I \to [0,1]$ such that for any $\boldsymbol{x} = (x_i)_{i\in I}$

$$\gamma_C(\boldsymbol{x}) = \min\left[\nu_{\succsim}\Big(\sum_{i\in I} x_i,\, w(K)\Big), \min\bigg(\nu_{\succsim}\Big(w(K), \sum_{i\in K} x_i\Big)\colon K\subset I\bigg)\right]\,. \tag{22}$$

3.3 Fuzzy coalitional games without side-payments

Also the coalitional games without side-payments can be fuzzified (see [3]). Let $(I, \mathcal{V})$ be such game and let us define for every $K \subset I$ a fuzzy subset $\mathcal{W}(K)$ of $\mathbb{R}^I$ with the membership function $\kappa_K \colon \mathbb{R}^I \to [0,1]$ such that for every $\boldsymbol{x} \in (x_i)_{i\in I} \in \mathbb{R}^I$

$$\text{if } \boldsymbol{x} \in \mathcal{V}(K) \text{ then } \kappa_K(\boldsymbol{x}) = 1\,, \tag{23}$$

$$\text{the set } \{\boldsymbol{x} \in \mathbb{R}^I \colon \kappa_K(\boldsymbol{x}) = 1\} \text{ is closed}\,, \tag{24}$$

$$\text{if } \boldsymbol{x},\ \boldsymbol{y} \in \mathbb{R}^I,\ \boldsymbol{x} \operatorname{dom}_K \boldsymbol{y}, \text{ then } \kappa_K(\boldsymbol{x}) \le \kappa_K(\boldsymbol{y})\,, \tag{25}$$

$$\text{if } K \neq \emptyset \text{ then there exists } \boldsymbol{x} \in \mathbb{R}^I \text{ such that } \kappa_K(\boldsymbol{x}) = 0\,. \tag{26}$$

Then the pair $(I, \mathcal{W})$ is called *fuzzy extension* of $(I, \mathcal{V})$. Analogously to (7) we say that $(I, \mathcal{W})$ is *superadditive* iff for every pair of disjoint coalitions $K,\ L \subset I$,

$$\mathcal{W}(K \cup L) \supset \mathcal{W}(K) \cap \mathcal{W}(L)\,, \quad \text{i.e.,} \quad \kappa_{K\cup L}(\boldsymbol{x}) \ge \min\bigl(\kappa_K(\boldsymbol{x}),\ \kappa_L(\boldsymbol{x})\bigr)\,, \quad \boldsymbol{x} \in \mathbb{R}^I\,. \tag{27}$$

Note that this superadditivity is not a fuzzy property connected with any uncertainty but it is a deterministic relation.

To define the concept of superoptimum for $(I, \mathcal{W})$ analogous to (8) it is necessary to define a proper relation of domination. If $\boldsymbol{x},\ \boldsymbol{y} \in \mathbb{R}^I,\ K \subset I$, and $\boldsymbol{x} \operatorname{dom}_K \boldsymbol{y}$ according to the definition shown in Subsection 1.2, then we say that $\boldsymbol{x}$ *significantly dominates* $\boldsymbol{y}$ *via* K whenever $\kappa_K(\boldsymbol{x}) \ge \kappa_K(\boldsymbol{y})$. The significant domination is defined as a fuzzy relation which is valid with the possibility

$$\sigma_K(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} \kappa_K(\boldsymbol{x}) & \text{if } \boldsymbol{x} \operatorname{dom}_K \boldsymbol{y} \text{ and } \kappa_K(\boldsymbol{x}) \ge \kappa_K(\boldsymbol{y})\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{28}$$

for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^I, K \subset I$.

The fuzzy superoptimum $\mathcal{W}^*(K)$ of coalition K is the fuzzy subset of $\mathbb{R}^I$ related to imputations which are not significantly dominated by any imputation from $\mathcal{W}(K)$. More precisely, its the membership function is defined on $\mathbb{R}^I$ by

$$\kappa_K^*(\boldsymbol{y}) = 1 - \sup_{\boldsymbol{x}\in\mathbb{R}^I} \sigma_K(\boldsymbol{x}, \boldsymbol{y}) = \inf_{\boldsymbol{x}\in\mathbb{R}^I} \bigl(1 - \sigma_K(\boldsymbol{x}, \boldsymbol{y})\bigr)\,. \tag{29}$$

Having defined the fuzzy superoptimum, we can define the fuzzy core $\boldsymbol{\mathcal{C}}_F$ of $(I, \mathcal{W})$, analogously to (9) as the fuzzy subset of $\mathbb{R}^I$ such that

$$\boldsymbol{\mathcal{C}}_F = \mathcal{W}(I) \cap \Biggl(\bigcap_{K\subset I} \mathcal{W}^*(K)\Biggr)\,. \tag{30}$$

If we denote by $\gamma_{\boldsymbol{\mathcal{C}}}$ the membership function of $\boldsymbol{\mathcal{C}}_F$ then, evidently,

$$\gamma_{\boldsymbol{\mathcal{C}}}(\boldsymbol{x}) = \min\Bigl(\kappa_I(\boldsymbol{x}), \min\bigl(\kappa_K^*(\boldsymbol{x}) \colon K \subset I\bigr)\Bigr)\,, \quad \boldsymbol{x} \in \mathbb{R}^I\,. \tag{31}$$

Some properties of $\boldsymbol{\mathcal{C}}_F$ are introduced and analyzed, e.g., in [2] or [4].

4. Linear fuzzy coalitional game

If $(I, \mathcal{V})$ is a linear coalitional game induced by a game (I, v) with side-payments by means of (10) (cf. Section 2) then we have already seen that their cores are identical and $(I, \mathcal{V})$ is superadditive iff (I, v) is. If we construct fuzzy extension (I, w) of (I, v) and fuzzy extension $(I, \mathcal{W})$ of $(I, \mathcal{V})$ such that for all $K \subset I$ and $\boldsymbol{x} \in \mathbb{R}^I$

$$\kappa_K(\boldsymbol{x}) = \begin{cases} \mu_K(y) & \text{if } \sum\limits_{i \in K} x_i = y > v(K)\,, \\ 1 & \text{if } \sum\limits_{i \in K} x_i = y \leq v(K)\,, \end{cases} \tag{32}$$

then the pair of fuzzy coalitional games (I, w) and $(I, \mathcal{W})$ also preserves the link between two presentations of one fuzzy cooperative situation. A game $(I, \mathcal{W})$ fulfilling (32) for the fuzzy coalitional game with side-payments which is a fuzzy extension of (I, v) can be naturally called *linear fuzzy coalitional game*.

Let us briefly summarize the existing relations between the (deterministic or fuzzy) coalitional games considered in this section. We suppose that there exists a linear coalitional game $(I, \mathcal{V})$ (which is a game without side-payments) derived from a coalitional game with side-payments (I, v) by means of (10). Moreover, we suppose that (I, w) is a fuzzy extension of (I, v) and we derive a pair $(I, \mathcal{W})$ by means of (32). Evidently, $(I, \mathcal{W})$ is a fuzzy extension of $(I, \mathcal{V})$ and it is, analogously to the previous deterministic case, called linear. Each of the games (I, v), $(I, \mathcal{V})$, (I, w), $(I, \mathcal{W})$ formally represents cooperation among a given set of players I (with deterministic or vague knowledge of the pay-offs) using specific "languages" of the respective theory of coalitional games. There exist two methodological approaches to this cooperative situation with vague (i.e., fuzzy) expected pay-offs. One of them deals with the processing of the fuzzy game (I, w) with side-payments and the other one is based on the processing of the fuzzy game $(I, \mathcal{W})$ without side-payments. The concepts shown in Subsection 3.2 and Subsection 3.3 are used for this purpose, respectively. The main results which will be referred in the following subsections concern the relation between deterministic and fuzzy superadditivity and deterministic and fuzzy core in both models. The differences in these results illustrate the methodological and conceptual differences between the procedures of fuzzification of the coalitional games with and without side-payments.

4.1 Superadditivities

Let us start with comparison of the concepts of superadditivity in both types of fuzzy coalitional games. We focus our attention to the fuzzy extensions which in certain sense preserve the set of "certain" pay-offs. Obviously, (32) implies the following statement.

Proposition 1. *Let (I, w) be a fuzzy extension of a coalitional game (I, v) and let for all $K \subset I$, $\mu_K(x) = 1$ if and only if $x = v(K)$. Let $(I, \mathcal{V})$ be connected with (I, v) by means of (10) and let $(I, \mathcal{W})$ be a fuzzy extension of $(I, \mathcal{V})$ fulfilling (32). Then, for all $K \subset I$, $\kappa_K(\boldsymbol{x}) = 1$ if and only if $\boldsymbol{x} \in \mathcal{V}(K)$.*

The next two statements follow from (15), (17), (21) and (13).

Proposition 2. *If (I, w) is a fuzzy extension of a superadditive game (I, v) then $\nu_{\text{super}}(I, w) = 1$.*

Proposition 3. *If (I, w) is a fuzzy extension of (I, v) and $\mu_K(x) = 1$ iff $x = v(K)$ for all $K \subset I$, then (I, v) is superadditive if and only if $\nu_{\text{super}}(I, w) = 1$.*

The next statement follows from (27) and from Proposition 1.

Proposition 4. *If the assumptions of Proposition 1 are fulfilled, then $(I, \mathcal{V})$ is superadditive whenever $(I, \mathcal{W})$ is.*

The previous propositions can be summarized into the following statement specifying the relation between superadditivities in linear fuzzy coalitional games.

Proposition 5. *Let $(I, \mathcal{V})$ be a linear coalitional game fulfilling (10) for some (I, v), (I, w) be a fuzzy extension of (I, v) such that for all $K \subset I$, $\mu_K(x) = 1$ iff $x = v(K)$, and let $(I, \mathcal{W})$ be a fuzzy extension of $(I, \mathcal{V})$ such that (32) is fulfilled. If $(I, \mathcal{W})$ is superadditive then also $(I, \mathcal{V})$ and (I, v) are superadditive and $\nu_{\text{super}}(I, w) = 1$.*

The previous statement illustrates the different "strength" of superadditivity in fuzzy extensions of (I, v) and $(I, \mathcal{V})$ where the former is based on processing of fuzzy quantities and the latter follows from a direct manipulation with fuzzy sets of imputations.

4.2 Cores

First, let us recall that the core $\boldsymbol{\mathcal{C}}$ of a linear coalitional game is equal to the core C of the corresponding coalitional game with side-payments. The similarities and differences between their fuzzified counterparts C_F and $\boldsymbol{\mathcal{C}}_F$ illustrate some aspects of differences between two approaches to the fuzzification of coalitional games. In the following statements we deal mostly with linear coalitional games fulfilling the assumptions of Proposition 1.

Some of them are based on the results presented in [2] and [3].

Proposition 6. *Let (I, w) be a fuzzy extension of (I, v) and let for all coalitions $K \subset I$, $\mu_K(x) = 1$ iff $x = v(K)$. Then, for each $\boldsymbol{x} \in \mathbb{R}^I$,*

$$\gamma_C(\boldsymbol{x}) = 1 \quad \text{if and only if} \quad \boldsymbol{x} \in C\,.$$

The following proposition shows that the analogous relation for games without side-payments is not as strict as the previous one.

Proposition 7. *Let $(I, \mathcal{W})$ be a fuzzy extension of $(I, \mathcal{V})$ and let for all coalitions $K \subset I$, $\kappa_K(\boldsymbol{x}) = 1$ iff $\boldsymbol{x} \in \mathcal{V}(K)$. Then the inequality $\gamma_{\boldsymbol{\mathcal{C}}}(\boldsymbol{x}) > 0$ is true for all $\boldsymbol{x} \in \boldsymbol{\mathcal{C}}$.*

The previous two statements can be summarized in the following conclusion.

Proposition 8.

Let $(I, \mathcal{V})$ be the linear coalitional game defined by a coalitional game (I, v) with side-payments. Let $(I, \mathcal{W})$ be a fuzzy extension $(I, \mathcal{V})$, and (I, w) be a fuzzy extension of (I, v) such that $\mu_K(x) = 1$ iff $x = v(K)$. If (32) is fulfilled, then

$$0 < \gamma_{\boldsymbol{\mathcal{C}}}(\boldsymbol{x}) \leq \gamma_C(\boldsymbol{x}) = 1$$

for all $\boldsymbol{x}$ from C.

Hence, we can see that also the concepts of core for both components of linear fuzzy coalitional game display different "strength".

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