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A CONTRIBUTION TO THE POSSIBILITIES TO EXTENT THE GRAPHICAL METHODS OF MULTIFACTORAL ANALYSES IN GEOGRAPHY

L'article esquisse les possibilités de la représentation graphique et de l'analyse des processus formés par la relation de quatre indices numériques. Il explique le procédé de la représentation de ces rapports dans un système orthogonal de quatre coordonnées, défini à la circonférence du carré S et à son intérieur, avec les axes *a*, *b*, *c*, *d* identiques avec ses côtés. L'application graphique de ces relations mène à la figure Q_i (Fig. No 2). Les considérations finales sont basées sur quatre signes d'identification de la figure résultante (de sa position dans le cadre du système de coordonnées, de la contenance de sa surface, de la forme et du caractère du périmètre). On allègue aussi le sens général et le caractère de chaque des signes d'identification. Pour l'avenir, l'auteur envisage une application de la méthode présentée à un exemple concret.

Most phenomena studied within geographical investigation is the product of several co-operating factors. This fact modifies the interest of geography in the methodical procedures that permit to ascertain, analyse and express correlation of several components simulaneously. Since the most common possibilities to express the correlation of several factors are affored by exact formulas, models, eventually graphical records, the endeavour after enriching the scales of the methods of multifactoral analyses provoked an extension of quantifying and mathematical tendencies in geography.

Especially the use of matrices and determinants promises wide possibilities for the multifactoral analyses within the geographical investigation. The application of them renders possible to extend the correlative analyses theoreticaly to an optional, finite number of components determining the investigated process. A more common use of these methods in geography, however, for the present has been obstructed by the difficulties connected with the numerical expression of geographical processes, which can be overcome only by solving the questions concerning the hierarchization, bonification, eventually codification of values of the individual landscape components.

Alongside of the purely numerical analyses of the correlation of several factors geography uses also the methods of graphical representation of correlative connections. To the traditional graphical representation of correlation of two factors in bi-coordinate system, within the geographical analyses the possibility of studying processes determined by the correlation of three components in the equilateral triangle has been added, altrough under definite more rigorous demands to the nature of indices represented.

The complexity and structural multiplicity in the most phenomena appearing as the object of geographical study can be, however, under exceedingly favourable circumstances only interpreted by the correlation of two or three indices, eventually modified to the presuppositions required in the equilateral triangle representation (i. e. to create a triad of indices excluding each other by content, but their sum creates at the same time a constant value). Owing to this fact he significance of graphical analyses in studying correlations of several factors in geography has relatively decreased in recent time, especially when these possibilities seemed to be substantially narrowerly confined than the possibilities of numerical analyses. Such a view was apparently confirmed even by the perfunctory glance at analytic geometry, which operates in graphical records especially with bi- and tri-dimensional coordinate systems. Some confinements, to whom without greater difficulties almost all the values of indices used in various geographical researches (for instance, that they are basically almost everytimes finite and real numbers) can be subordinated, permit, however, to express the view that the possibilities graphically studying the correlations of the individual, numerically expressed characteristics of geographical processes do not end in tri-dimensional coordinate system.

In evidence of this view we present a theoretical outline of methodical procedure and the possibilities of graphical representation and analysis of the process formed by the correlation of our numerical indices. On purpose we have chosen the form of a theoretical record for this contribution so that the common size of using the considerations done should be documented. This aim has to a certain measure confined the extent of considerations in describing the significance of the individual identifying marks, whose eloquence is dependent upon concrete conditions determined by the nature of represented indices. We shall, therefore, realize these considerations on the example of a concrete problem in the near future, when we will further elaborate the properties of identifying marks that arise out of the concrete situation.

THE GENERAL PROCEDURE IN REPRESENTING THE CORRELATIONS OF FOUR INDICES

The central idea in the graphical representation and in the analysis of structural relations within a process determined by four indices is the introduction of four-coordinate orthogonal system defined in the perimeter and inside the square S, with the coordinate axes a, b, c, d identical with the sides of the square. A generalized graphical representation of the system S (a, b, c, d) looks as follows (Fig. 1.).

Its orientation, i. e. the choice of direction, according to which the values on the individual axial scales will rise, can be chosen in accordance with the correlation of the indices represented on opposite axes.[1]

In the coordinate system S (a, b, c, d) any correlation of four numerical indices can be represented, the values of which range within the region of positive real numbers and are different from zero. The result of graphical representation of this correlation is the figure drawn by the coordinates whose parametres are determined by the

[1] Totally there are 16 possibilities to choose its orientation. In determining the orientation of the individual axes of the four-coordinate system let us designate the orientation of the axis a according to the clockwise direction by the symbol $\overrightarrow{a}$ and that of the counterclockwise direction by the symbol $\overleftarrow{a}$. Let an analogical designation hold to determine the orientation of the axes b, c, d. The symbol S (a, b, c, d) designates the standard form of four-coordinate system without regard to the orientation of its axes.

numerical values of the represented indices. The basis for considerations of the nature of the studied problem is the position, area, shape and way of delimitation of this resulting figure.

Let us base on the supposition that we are graphically to represent and analyse a certain process P on the basis of its difference in n cases, the P being characterized by four numerical data (a, b, c, d) in each of these n cases. Thus have the n-nomial group of data in a shape (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), ..., (a_n, b_n, c_n, d_n) where (a_i, b_i, c_i, d_i) for $i = 1, 2, \ldots, n$ are real numbers greater than zero. Then the general principles and methodical procedure in representing the correlation between four numerical indices can be summarized to following seven items:

I. The group of data should be divided into 4 numerical series A, B, C, D in the shape $A = a_1, a_2, \ldots, a_n$; $B = b_1, b_2, \ldots, b_n$; $C = c_1, c_2, \ldots, c_n$; $D = d_1, d_2, \ldots, d_n$.

II. The four-coordinate orthogonal system S (a, b, c, d) should be defined on the perimeter and inside the square S with the side s, with coordinate axes a, b, c, d, identical with its sides.

III. To each of the numerical series A, B, C, D one of the axes of the coordinate system S (a, b, c, d) should be assigned, on which the numerical values of members of one and the same series should be represented.

IV. On the basis of the logical value of the correlation of the numerical series represented on oposite axes the orientation of coordinate system should be chosen, i. e. the direction according to which the scale of values will rise should be determined.

V. The unit measure j should be defined, which has the property that

$$s = k . j \tag{1}$$

holds, where k is a whole positive number.

Under these suppositions, there is a numerical series of whole positive numbers r, which can be expressed in the shape

$$\sum_{r=1}^{k} r . j, \tag{2}$$

the values of members of this sequence expressing the division of the side s into k equal portions.

VI. On all four sides of the square S in accordance with the chosen orientation, the values of each r from the sequence (2) should be plotted, thus an even division of each of the axes of the coordinate system S (a, b, c, d) into k portions will be reached. This division is a base for the forming of numerical scale on each of the axes. The concrete values of the scale on the scale on the individual axes are to be determined on the basis of the view and nature of the research being realized, with regard for the

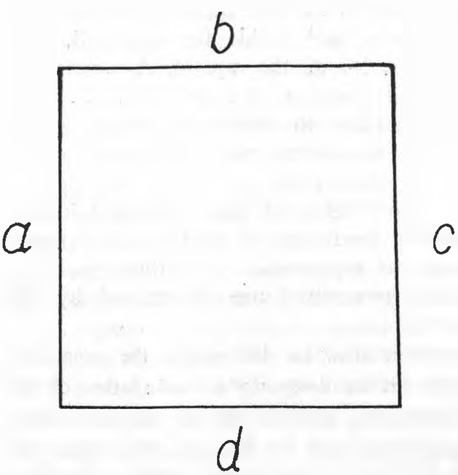

Fig. 1.

values of members of the numerical series A, B, C, D so that all the values from these series may be represented within the framework of the definition of coordinate system.

For the purpose of describing the way of representing the elements of the numerical series A, B, C, D in the four-coordinate orthogonal system our introductory presuppositions have to be extended by the orientation of the system S (a, b, c, d) and by the division of its axis into appropriate segments in the sense of the considerations of the item VI. Let us, thus, suppose the orientation of the coordinate system in a shape S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$, the axes of which have a marked scale with the values designated on the axis a as $a, 2a, \ldots, ka$, on the axis b as $b, 2b, \ldots, kb$, on the axis c as $c, 2c, \ldots, kc$, and on the axis d as $d, 2d, \ldots, kd$, holding that all the values from the numerical series A, B, C, D can be measured by this scale within the definition S, $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$, that is for a_h, which has the maximal numerical value of all the $a_i \in A$ $(i = 1, 2, \ldots, n)$, the relation

$$a_h \leq k \,.\, a \tag{3}$$

holds, and analogically for maximal numerical values from the series B, C, D

$$b_h \leqq k \,.\, b, \quad c_h \leqq k \,.\, c, \quad d_h \leq k \,.\, d \tag{4}$$

hold, and then:

VII. For the purpose of representing the correlation of the data given by any of the fours (a_i, b_i, c_i, d_i) it is sufficient to find and into the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to draw the values x_i, y_i, z_i, q_i, determined by the relations

$$x_i = \frac{a_i}{a}; \; y_i = \frac{b_i}{b}; \; z_i = \frac{c_i}{c}; \; q_i = \frac{d_i}{d} \tag{5}$$

These values represent the numerical expression of the coordinates on the individual axes, namely in the multiples of the basic values a, b, c, d, since

$$a_i = x_i a; \quad b_i = y_i b; \quad c_i = z_i c; \quad d_i = q_i d \tag{6}$$

results from the relations (5). At the same time they are the parametres of the figure O_i represented in S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ by the relation (a_i, b_i, c_i, d_i).

Let us define this figure as a geometrical configuration, which arises among the coordinates a_i, b_i, c_i, d_i, where

- the coordinate a_i is a perpendicular line segment led across all the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to the axis a at the point X_i;
- the coordinate b_i is a perpendicular line segment led across all the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to the axis b at the point Y_i;
- the coordinate c_i is a perpendicular line segment led across all the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to the axis c at the point Z_i;
- the coordinate d_i is a perpendicular line segment led across all the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to the axis d at the point Q_i.

The graphical record completing the results of the presuppositions and considerations resulting from the points I to VII looks as follows (Fig. 2.).

The figure o_i is the final graphical product of the analysis (a_i, b_i, c_i, d_i) in the coordinate system $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$. On the basis of its position, dimensions of area, shape and nature of delimitation the statements about the structure of the process P in the individual cases can be enunciated. By the representation and judgement of all n-fours of data suppositions for the complex analysis of the structural construction of the process P will be created, thus even for a definite typification of the individual cases (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), . . ., (a_n, b_n, c_n, d_n) on the basis of various affinities of the resulting representation.

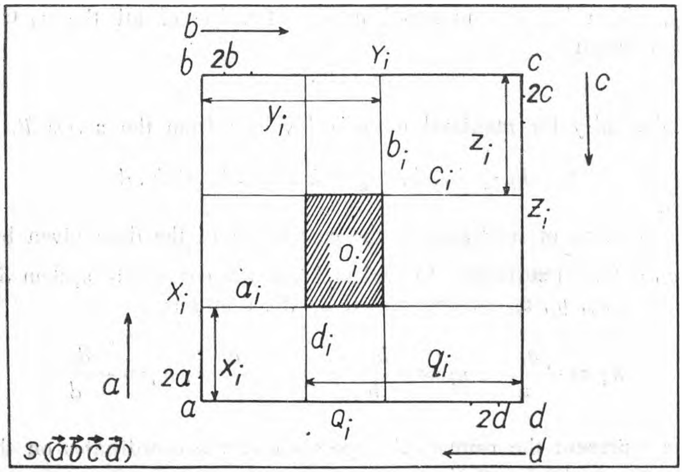

Fig. 2.

THE IMPORTANCE AND NATURE OF IDENTIFYING MARKS

A. Position of the resulting figure

The most important and most eloquent identifying mark in the described graphical representation of mutual dependences among four data is the position of the resulting figure within the coordinate system. It is unambiguously dependent upon the value of each of the data, i. e. each four of positive real numbers determines within anywise oriented we choose the four-coordinate system $S\ (a, b, c, d)$ the only figure[2] and vice versa each figure in this system can be unambiguously determined by the only four of real numbers.

The basic analysis of the process P is realized on the basis of different position of the resulting figure. The location of the figure expresses the importance of the individual

[2] If we suppose that the figure is determined not only by the parametres of coordinates (by the position), the size of area and shape, but also by the nature of its delimitation, i. e. it differs also by which of the coordinates creates ist upper, eventually lower and at the same time its left and right limitations. (More details about it are spoken in the paragraph *D. Way of delimitation of the resulting figure.*)

indices in forming its structure, thus it allows to obtain an idea of the relations determining the resulting figure in the graphical outline.

The division of the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ into quadrants and subquadrants with a schematic expression of values of the individual coordinates in the various parts of the coordinate system affords the basic view of the spatial arrangement of values and of mutual relation of the coordinates within such oriented coordinate system (Fig. 3).

Essentially wider considerations about significance of a different position can be realized in an analysis of a concrete example, where the possibilities to identify structural dissimilarities within a studied process can be better documented.

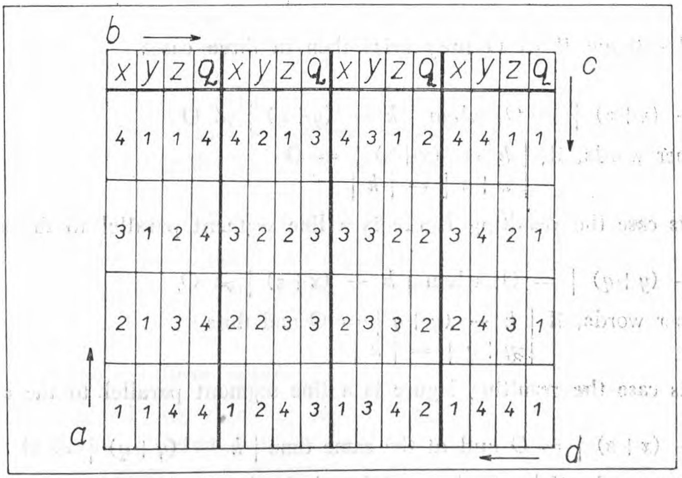

Fig. 3.

B. A r e a of the resulting figure

The size of the area P of the resulting figure is determined by the product of length of its sides. Generally, regardless of the choice of orientation of the coordinate system S (a, b, c, d), defined on the perimeter and inside the square S with a side s it is possible to delimitate the size of the area P as follows:

$$O \leqq P \leqq s^2 \tag{7}$$

which according to the considerations and equation (1) from the point V can be also written as

$$O \leqq P \leqq k^2 . j^2 \tag{8}$$

where k is the expression of value of the side of the square S by means of a whole number and j is a unit measure of such a property that $s = k . j$ holds. On condition that s has a value of a whole number[3] the relation $s = k . j$ gets the shape $s = k . a,$

[3] This supposition does not interfere with the general use of the method, since the size of the square's side may be chosen according to our liking.

thus the size of the area can be substituted into the inequality (7) also as

$$O \leqq P \leqq k^2 \tag{9}$$

The incidence of extreme cases and the formula for expression of the area's size of the resulting figure depends upon the chosen orientation of the coordinate system.

If we continue the considerations of the size and significance of the size of area in the coordinate system S $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ defined on the perimeter and inside the square S with a side $s = k$, then we can express the size of area of any figure chosen, determined by the coordinates (x, y, z, q) in the form

$$P = | k - (x+z) | \, . \, | k - (y+q) | \tag{10}$$

The minimal extreme $P = O$ may arise then in three cases.

If

I. $| k - (x+z) | = O$, when $| k - (y+q) | \neq O$

in other words, if, $| k - (x+z) | = O$
$| x+z | = | k |$

in this case the resulting figure is a line segment parallel to the axes a and c.

II. $| k - (y+q) | = O$, when $| k - (x+z) | \neq O$

in other words, if $| k - (y+q | = O$ and thus
$| y+q | = | k |$

in this case the resulting figure is a line segment parallel to the axes b and d.

III. $| k - (x+z) | = O$ and at the same time $| k - (y+q) | = O$

in other words, if $| x+z | = | k |$ and also $| y+q | = | k |$

in this case the resulting figure is a point.

In all the cases, when $k \neq | x+z |$ and at the same time $k \neq | y+q |$ the area of the resulting figure $P > O$.

For an incidence of the maximal extreme $P = k^2$ the following equality holds: $P = k^2 = | k - (x+z) | \, . \, | k - (y+q) |$, which according to the presuppositions (3) and (4), i. e. that all x, y, z, q are less, at most equal to k, is fulfilled only, if $| k - (x+z) | = k$ and at the same time also $| k - (y+q) | = k$, which can theoretically occur in two cases

I. $| z+z | = O$; $| y+q | = O$

which leads to a variance with the introductory presupposition according to which all x, y, z, q are positive, thus in our case the maximal extreme can occur only, if

II. $| x+z | = 2k$ and $| y+q | = 2k$,

which under the conditions stated in the choice of k can be fulfilled only, if $x = k$, $y = k$, $z = k$, and $q = k$.

Then really it holds as follows

$| k - (x+z) | = | k - (k+k) | = k$

$| k - (y+q) | = | k - (k+k) | = k$, in other words the area

$P = k \, . \, k = k^2$.

The identifying function of the area of the resulting figure results from its mathematical significance and it changes according to the chosen orientation of the coordinate system. In the coordinate system $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$ it is determined by the product of two differences expressing the mutual deviation of the values x and z and the values y and q. Along with enlarging difference within some of these couples even the values expressing the size of the area of the resulting figure rise.

This circumstance can be used for expressing correlative relations in the case, if the values of the scales on the opposite axes is chosen so that their relation may correspond to a definite optimum measure of the mutual correlation of the indices represented. This optimum measure will then be expressed by the figure with a null area (point) and every growing in the size of area will signal the increasing deviation from the optimum dimensions transformed into the coordinate system by the scale chosen on the coordinate axes.

The numerical expression of the size of area of the resulting figure enables in $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$ to put the individual cases into a series according to the relation between the differences of the values x and z and the values y and q. This indication may be an auxiliary criterion in typifying the structures of process represented.

C. Shape of the resulting figure

The size of area of the resulting figure depends upon the length of its sides. Nevertheless even with different parametres of the length of sides, its area may remain the same. For these reasons it is necessary to perceive even the shape of the resulting figure.

Mathematically the shape of the resulting figure can be characterized by the coefficient of the relation of both the sides. The concrete form of such a formula depends upon the choice of orientation of the coordinate system.

In the coordinate system $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$ coefficient t can be expressed by the formula

$$t = \frac{| k - (x+z) |}{| k - (y+q) |} \tag{11}$$

If the resulting figure is of a square's shape, then its sides are equally long, thus $| k - (x+z) | = | k - (y+q) |$ holds good, of which it follows that $t = 1$.

If the resulting figure is of a rectangle's shape with the longer side parallel to the axes a and c, then $| k - (x+z) | > | k - (y+q) |$ holds, in other words $t > 1$.

If the resulting figure is of a rectangle's shape with the longer side parallel to the axes b and d, then $| k - (x+z) | < | k - (y+q) |$ holds, of which it follows that $t < 1$.

Special cases are the circumstances, if $| k - (x+z) | = O$, or if $| k - (y+q) | = O$ or if both the expressions are simultaneously equal to zero. In these cases a line segment or a point represent the resulting figure. For these circumstances let us define the coefficient of shape t as follows:

if $|k - (x+z)| = O$, then let $t = |k - (y+q)|$
if $|k - (y+q)| = O$, then let $t = |k - (x+z)|$
if $|k - (x+z)| = O = |k - (y+q)|$, then let $t = O$.

The coefficient of shape t may thus take the values $O \leqq t \leqq k$.

In the system $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$ the shape of the resulting figure is documented by the ratio of the indices represented on the opposite axes. Provided the values on the scales of the opposite axes would be chosen so that their relation can express a definite optimum measure of the mutual correlation of the oppositely represented indices, the coefficient of shape is an expression of the difference between these indices. If $t = 1$, in other words the resulting figure is a square, then the mutual dissimilarity of these couples of indices regarding the optimum value, represented by the null area, is equal. If $t \neq 1$, the difference within one of the couples of indices represented on the opposite axes is greater, namely in the measure, which is mathematically expressed by the difference $|t - 1|$.

D. *Way of delimitation of the resulting figure*

The area and the shape of the resulting figure represented in the four-coordinate system $S\ (a, b, c, d)$ are an expression of definite relations of the parametres represented. According to their graphical significance, eventually mathematical expression it is possible to enunciate considerations about the products and of differences of the values represented on the opposite axes. For an analysis of the structure of the process studied it is necessary to watch also the absolute values of the individual coordinates, which on a graph are represented by the delimitation of the resulting figure. On the basis of their values it can be judged of the causes determining the concrete parametres of the position, of the area's size and shape of the resulting figure.

According to the numerical value of the parametres (a_i, b_i, c_i, d_i)

- the upper limitation of the figure o_i can be formed by the coordinate given by the value x_i, and then the lower limitation of the figure is formed by the coordinate given by the value z_i, or
- the upper limitation of the figure o_i is formed by the coordinate given by the value z_i, then the lower limitation is determined by the coordinate x_i, or
- the coordinates x_i and z_i are confluent in a line segment and in this case we shall not speak either of the upper or the lower limitation of the figure.

Similar are the variants of possibilities and the definition of delimitation of the left and right boundaries of the figure o_i, in which the coordinates y_i and q_i take their part.

The significance of studying the way of delimitation of the resulting figure rises along with the increase of value $|t - 1|$, that means with an increasing difference between the lengths of the sides of the resulting figure. The causes of the similar extremes are different in differently oriented coordinate systems. In the coordinate system $S\ (\vec{a}, \vec{b}, \vec{c}, \vec{d})$ such a state is, for instance, caused by the maximal, eventually minimal extremes of one or at the same time of both the values of the indices represented on the opposite axes. In such cases it is of an importance to distinguish whether the matter is the maximal extreme, the minimal extreme, namely in one or another coordinate, or even in both the coordinates simultaneously. Each of such cases indicates different relations within the structure of the studied process and each one has an influence on the closing statements concerning the complex analysis of the process *P*.

THE POSSIBILITIES TO USE THE DESCRIBED METHOD WITHIN GEOGRAPHICAL *RESEARCHES*

The resulting product of the numerical methods is a numerical value, or a relation given by a formula. In spite of the fact that such a result can be an exact expression of the structure of studied process, its resulting form does not represent the character, but above all the result of the relations of the individual components determining this process. On the contrary, the graphical record keeps for an observer even the basic intrastructural relations, which are together with the result immediately visually perceivable.

This advantage of graphical methods determines the basic features of their applying in geographical researches. Their use is suitable then primarily in the analyses that not only the result of correlative relations is important in, but where it is necessary to distinguish even the different intrastructural composition, namely, for instance, in laying out different structural types, regionalization, functional analyses and the like.

Regarding such possibilities, an extension of the number of graphically analysable indices from two, eventually three to four stands for an enrichment of the scale of using the graphical methods in multifactoral analyses. Contrary to the analyses in the equilateral triangle the method described has a more general applicability, especially for the reasons that it does not make such limiting demands upon the character and mutual correlation of the represented indices. This method, therefore, can be used nearly in all analyses of processes and phenomena determined, eventually sufficiently characterized by four numerical indices. Since such processes and phenomena occur both in studying the physicogeographical problems and in analyses of economicogeographical conditions, its using in geography is basically general. At the same time the possibility of choosing the orientation of the coordinate system enables to adapt the conditions of study to their mutual correlations of the represented indices, and the choice of scale on the individual axes does in turn to an expression of the required, or optimum relations of the couples of indices.

Closing it is to be mentioned that the contribution summarizes exclusively the theoretical considerations of the most general validity only. Each of the researches realized according to the procedure described will bring new pieces of knowledge of the identifying value of the mentioned identifying marks. Some of these further considerations will be brought also by the author's attempt, under preparation, applying this method in studying the problems of the functions of visited cities, which should be published in the near future.

From the Slovak translated by A. Krajčír

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