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APPLICATION OF SURFACE AND VOLUME GEOMETRY ANALYSIS IN GEOSCIENCES

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There are many tasks in geosciences where geometry analysis is required to better understand a given phenomenon or process. Surface geometry analysis was originally developed in geomorphology where geometrical properties of relief significantly influence geomorphological processes (e.g. water erosion, landslides, etc.). From the geometrical point of view, this analysis is 2-dimensional because 2 independent variables (x, y) are used. The phenomenon is modelled using a bivariate function $z = f(x, y)$. However in geological applications, 3-dimensional cases are more obvious. Therefore, a trivariate function $w = f(x, y, z)$ is used. The most important surface geomentry analysis parameters expressed using partial derivatives of bivariate function $z = f(x, y)$ are (Krcho 1990), (Mitášová & Hofierka 1992): slope, aspect, profile curvature, tangential curvature.

Derived parameters are, for example, flowlines, their lenght and density interpreted often as an upslope contributing area and often used in water erosion modelling. These parameters are important in landscape processes where relief is the main factor in the spatial differentiation of these processes. There are many examples of using them in geomorphology, landscape ecology, pedology, etc. (Moore et al. 1991). In volume geometry analysis, parameters analogical to 2-D parameters have been derived in (Hofierka et al. 1993):

- magnitude of gradient $|gradw| = \sqrt{f_x^2+f_y^2+f_z^2}$
- $\partial|\text{gradw}|/\partial n = \dfrac{f_x^2 f_{xx}+2f_{xz}f_xf_z+2f_{xy}f_xf_y+2f_{yz}f_yf_z+f_y^2f_{yy}+f_z^2f_{zz}}{f_x^2+f_y^2+f_z^2}$
- Gaussian curvature $G = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}^{-1}$
- mean curvature $H = \frac{1}{3}\left(A+B+C\right) \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}^{-1}$

$$A = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \quad B = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ h_{21} & h_{22} & h_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \quad C = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

where: $g_{11} = 1+f_x^2 \quad g_{22} = 1+f_y^2 \quad g_{33} = 1+f_z^2$

$$g_{21} = g_{12} = f_xf_y \quad g_{31} = g_{13} = f_xf_z \quad g_{32} = g_{23} = f_yf_z$$

$$h_{11}=\frac{f_{xx}}{\sqrt{1+f_x^2+f_y^2+f_z^2}} \quad h_{22}=\frac{f_{yy}}{\sqrt{1+f_x^2+f_y^2+f_z^2}} \quad h_{33}=\frac{f_{zz}}{\sqrt{1+f_x^2+f_y^2+f_z^2}}$$

$$h_{21} = h_{12} = \frac{f_{xy}}{\sqrt{1+f_x^2+f_y^2+f_z^2}} \quad h_{31} = h_{13} = \frac{f_{xz}}{\sqrt{1+f_x^2+f_y^2+f_z^2}}$$

$$h_{23} = h_{32} = \frac{f_{yz}}{\sqrt{1+f_x^2+f_y^2+f_z^2}}$$

- 3-D flowlines, their length and density.

Magnitude of gradient, curvatures, $\partial|gradw|/\partial n$, can be used as topological parameters characterizing the shape of investigated geological solids. Their interpretation and usefulness are in the 3-D case more complicated because of the lack of driving factors like relief. These parameters are more described in (Zlocha et al. 1993). 3-D flowlines, their lenght and density have clear interpretation especially in hydrogeology. For example, places of high densities should indicate zones of matter accumulation. The best way of computing 2-D and 3-D morphometrical parameters is using an appropriate interpolation function (e.g. regularized spline with tension derived in (Mitášová & Mitáš 1992)). Large datasets (tens of thousands points) can be computed in a real time only

using high performance computers.

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