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ON QUASI-OSCILLATION FOR SYMMETRICAL QUASI-CONTINUITY

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ABSTRACT. A classical oscillation (quasi-oscillation) characterizes the set of all points at which a function with values in a metric space is continuous (quasi-continuous). These conditions were introduced in [J. Borsík: *Oscillation for quasicontinuity*, Tatra Mt. Math. Publ. **14**, (1998), 117–125], [J. Borsík: *On quasioscillation*, Tatra Mt. Math. Publ. **2**, (1993), 25–36], [J. Ewert: *Superpositions of oscillations and quasi-oscillations*, Acta Math. Hungar. bf 101, (2003), 13–19], [P. Kostyrko: *Some properties of oscillation*, Math. Slovaca, **30**, (1980), 157–162]. For the functions of two variables the symmetrical quasi-continuity (with respect to x and y) can be considered. In this paper we define a quasi-oscillation for symmetrically quasi-continuous functions. We will give properties of this oscillation and a characterization of the symmetrical quasi-continuity with respect to x (to y). Furthermore, we will study the convergence of a net of quasi-oscillations. Moreover, relationships between sets of points of continuity, symmetrical quasi-continuity, and quasi-continuity of the function are considered. For real function of two variables the Baire type (with respect to x and y) functions are introduced.

Let $(X, \mathscr{T})$ be a topological space.

Definition 1. ([5]) A function $f\colon X \to \mathbb{R}$ is called *upper* (*lower*) *quasi-continuous at a point* $x_o \in X$ if for every element $a \in \mathbb{R}$ such that $f(x_o) < a$, ($f(x_o) > a$), and for every neighbourhood U of x_o there exists a nonempty open subset G of U with the condition $f(x) < a$, ($f(x) > a$), for every $x \in G$.

We say that f is *upper* (*lower*) *quasi-continuous* if it has this property at every point $x_o \in X$.

For the function $f\colon X\times Y \to \mathbb{R}$ of two variables we can consider the upper and lower quasi-continuity, but also we can define the symmetrical quasi-continuity with respect to x (to y).

Let X and Y be topological spaces.

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Definition 2. ([5, 6]) A function $f\colon X \times Y \to \mathbb{R}$ is said to be *upper* (*lower*) *symmetrically quasi-continuous at a point* $(x_o, y_o) \in X \times Y$ *with respect to* x if for every neighbourhood $U \times V$ of the point (x_o, y_o) and for every $a \in \mathbb{R}$ such that $f(x_o, y_o) < a$, $(f(x_o, y_o) > a)$, there is a neighbourhood G of x_o contained in U and there exists a nonempty open subset H of V such that $f(x, y) < a$, $(f(x, y) > a)$, for all $(x, y) \in G \times H$.

A definition of the upper (lower) symmetrically quasi-continuous with respect to y is analogous.

If f is upper (lower) symmetrically quasi-continuous with respect to x (with respect to y) at every point $(x_o, y_o) \in X \times Y$, then it is said to be *upper* (*lower*) *symmetrically quasi-continuous with respect to* x (*with respect to* y).

Moreover, if f is upper (lower) symmetrically quasi-continuous with respect to x and y simultaneously we will say that f is *upper* (*lower*) *symmetrically quasi-continuous*.

Definition 3. ([5]) Let (Z, d) be a metric space. A function $f\colon X \times Y \to Z$ is called *symmetrically quasi-continuous at a point* $(x_o, y_o) \in X \times Y$ *with respect to* x if for every $\varepsilon > 0$ and every neighbourhood $U \times V$ of the point (x_o, y_o) there is a neighbourhood G of x_o contained in U and there exists a nonempty open subset H of V such that $d\big(f(x, y), f(x_o, y_o)\big) < \varepsilon$ for each $(x, y) \in G \times H$.

If f is symmetrically quasi-continuous with respect to x at any point of its domain, we say that f is *symmetrically quasi-continuous with respect to* x. A similar definition may be given for a function to be symmetrically quasi-continuous with respect to y.

Let X, Y be topological spaces and (Z, d) be a metric space. For a function $f\colon X \times Y \to Z$ we shall study the function $q_x(f, \cdot)\colon X \times Y \to \mathbb{R} \cup \{\infty\}$ given by the formula

$$q_x\big(f, (x_o, y_o)\big) = \sup_{U \times V} \inf_{G \times H} \operatorname{diam} f\Big((G \times H) \cup \big\{(x_o, y_o)\big\}\Big),$$

where the supremum is taken over all neighbourhoods $U \times V$ of (x_o, y_o) and infimum runs the set of all $G \times H$ such that G is a neighbourhood of x_o, $G \subset U$, H is a nonempty open subset of V.

Theorem 1. *Let $f\colon X \times Y \to Z$ be a function. The function $q_x(f, \cdot)\colon X \times Y \to [0, \infty]$ is upper symmetrically quasi-continuous with respect to x.*

Proof. Assume that $U \times V$ is a neighbourhood of (x_o, y_o), a is a real number such that $q_x\big(f, (x_o, y_o)\big) < a$. Then $q_x\big(f, (x_o, y_o)\big) < a - \varepsilon < a$ for some $\varepsilon > 0$. From the definition of $q_x\big(f, (x_o, y_o)\big)$ there exists a subset $G_o \times H_o$ of $U \times V$ such that G_o is a neighbourhood of x_o, H_o is a nonempty open set in Y, and

$$\operatorname{diam} f\Big((G_o \times H_o) \cup \big\{(x_o, y_o)\big\}\Big) < a - \varepsilon .$$

Let us take $(x_1, y_1) \in G_o \times H_o$ and let $U_1 \times V_1$ be an arbitrary neighbourhood of (x_1, y_1), where V_1 is open. We denote $G_1 = U_1 \cap G_o$, $H_1 = V_1 \cap H_o$. Then $(G_1 \times H_1) \cup \{(x_1, y_1)\} \subset G_o \times H_o$ and hence

$$\operatorname{diam} f\Big((G_1 \times H_1) \cup \{(x_1, y_1)\}\Big) < a - \varepsilon .$$

In consequence,

$$\inf_{G \times H} \operatorname{diam} f\Big((G \times H) \cup \{(x_1, y_1)\}\Big) < a - \varepsilon$$

and

$$\sup_{C \times D} \inf_{G \times H} \operatorname{diam} f\Big((G \times H) \cup \{(x_1, y_1)\}\Big) \leqslant a - \varepsilon < a ,$$

where the supremum is taken over all neighbourhoods $C \times D$ of (x_1, y_1) and infimum runs the set of all $G \times H$ such that G is a neighbourhood of x_1, $G \subset C$, H is a nonempty open subset of D.

We proved that

$$q_x(f, (x_1, y_1)) < a$$

for each $(x_1, y_1) \in G_o \times H_o$. It means that the function $q_x(f, \cdot)$ is upper symmetrically quasi-continuous with respect to x at the point (x_o, y_o). $\square$

Now, we will give a characterization of the symmetrical quasi-continuity with respect to x. Namely:

Theorem 2. *A function f is symmetrically quasi-continuous with respect to x at a point $(x_o, y_o) \in X \times Y$ if and only if $q_x(f, (x_o, y_o)) = 0$.*

Theorem 3. *Let $f, g \colon X \times Y \to Z$ be locally bounded functions. For every point $(x_o, y_o) \in X \times Y$ and for every neighbourhood W of this point the inequality*

$$\big|q_x(f, (x_o, y_o)) - q_x(g, (x_o, y_o))\big| \leqslant 2 \sup\big\{d\big(f(x, y), g(x, y)\big) : (x, y) \in W\big\}$$

holds.

Proof. In the proof we will follow the same procedure as in the proof of Lemma 3 in [2]. Assume that

$$q_x(f, (x_o, y_o)) - q_x(g, (x_o, y_o)) > 2 \sup\big\{d\big(f(x, y), g(x, y)\big) : (x, y) \in W\big\} = a$$

for some $(x_o, y_o) \in W$. Then there is α such that

$$q_x(f, (x_o, y_o)) > \alpha > q_x(g, (x_o, y_o)) + a .$$

Now there is a neighbourhood $U \times V$ of (x_o, y_o) such that $\inf \operatorname{diam} f\big((G \times H) \cup \{(x_o, y_o)\}\big) > \alpha$. Further there are nonempty open sets G and H such that

$x_o \in G \subset U$, $H \subset V$, $G \times H \subset W$ and $\operatorname{diam} g\big((G\times H)\cup\{(x_o,y_o)\}\big) < \alpha - a$. Since $\operatorname{diam} f\big((G\times H)\cup\{(x_o,y_o)\}\big) > \alpha$ there are $(x_1,y_1),(x_2,y_2) \in (G\times H)\cup\{(x_o,y_o)\}$ such that $d\big(f(x_1,y_1), f(x_2,y_2)\big) > \alpha$. We obtain

$$\begin{aligned} &d\big(g(x_1,y_1), g(x_2,y_2)\big) + a < \alpha < d\big(f(x_1,y_1), f(x_2,y_2)\big) \\ &\quad\leqslant d\big(f(x_1,y_1), g(x_1,y_1)\big) + d\big(g(x_1,y_1), g(x_2,y_2)\big) + d\big(g(x_2,y_2), f(x_2,y_2)\big) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\leqslant d\big(g(x_1,y_1), g(x_2,y_2)\big) + a\,, \end{aligned}$$

a contradiction. □

We shall study the convergence of a sequence of symmetrical quasi-oscillations, i.e., uniform convergence and convergence in the sense of Predoi.

Definition 4. ([7]) Let X be a topological space, (Z,d)—a metric space. We say that a sequence $\{f_n : n \in \mathbb{N}\}$ of functions $f_n : X \to Z$, for $n \in \mathbb{N}$, is *quasi-uniformly convergent to* $f : X \to Z$ if for every point $x \in X$ and every $\varepsilon > 0$ there exists $n_o \in \mathbb{N}$ such that for every $n \geqslant n_o$ there is a neighbourhood U of x_o such that the inequality $d\big(f_n(x), f(x)\big) < \varepsilon$ holds for every $x \in U$.

Theorem 4. *Let $f_n, f : X \times Y \to Z$ be locally bounded functions for $n \in \mathbb{N}$. If a sequence $\{f_n : n \in \mathbb{N}\}$ is quasi-uniformly convergent to f, then the sequence $\{q_x(f_n,\cdot) : n \in \mathbb{N}\}$ is quasi-uniformly convergent to the function $q_x(f,\cdot)$.*

Proof. Let $\varepsilon > 0$ and $(x_o,y_o) \in X \times Y$. From the quasi-uniform convergence of a sequence $\{f_n : n \in \mathbb{N}\}$ to the function f it follows that there exists $n_o \in \mathbb{N}$ such that for every $n \geqslant n_o$ we can choose such a neighbourhood $W = W(n)$ of the point (x_o,y_o) with $d\big(f_n(x,y), f(x,y)\big) < \frac{\varepsilon}{4}$ for every $(x,y) \in W$. For arbitrary $n \geqslant n_o$ we obtain $\sup\limits_{(x,y)\in W(n)} d\big(f_n(x,y), f(x,y)\big) \leqslant \frac{\varepsilon}{4} < \frac{\varepsilon}{2}$. Let us take $(x,y) \in W(n)$. According to Theorem 3 we have

$$\big|q_x\big(f_n,(x,y)\big) - q_x\big(f,(x,y)\big)\big| \leqslant 2 \sup_{(x',y')\in W(n)} d\big(f_n(x',y'), f(x',y')\big) < \varepsilon\,.$$

As a result we obtain that for each $(x_o,y_o) \in X \times Y$ and for a positive number ε there is $n_o \in \mathbb{N}$ such that for every $n \geqslant n_o$ we will find a neighbourhood W of (x_o,y_o) with a property

$$\big|q_x\big(f_n,(x,y)\big) - q_x\big(f,(x,y)\big)\big| < \varepsilon$$

for every $(x,y) \in W$. This means that the sequence $\big\{q_x(f_n,\cdot) : n \in \mathbb{N}\big\}$ is quasi-uniformly convergent to $q_x(f,\cdot)$ in the sense of Predoi. □

THEOREM 5. *Let $f_n, f\colon X \times Y \to Z$ be locally bounded functions for $n \in \mathbb{N}$ and let a sequence $\{f_n\colon n \in \mathbb{N}\}$ be uniformly convergent to f. Then $\{q_x(f_n,\cdot)\colon n \in \mathbb{N}\}$ is uniformly convergent to $q_x(f,\cdot)$.*

By an analogous way we can define the function $q^y(f,\cdot)$ with respect to variable y as follows:

$$q^y\big(f,(x_o,y_o)\big) = \sup_{U\times V}\inf_{G\times H} \operatorname{diam} f\Big((G\times H)\cup\big\{(x_o,y_o)\big\}\Big)$$

for every $(x_o,y_o)\in X\times Y$, where the supremum is taken over all neighbourhoods of the point (x_o,y_o) and infimum is taken over all $G\times H\subset U\times V$ such that G is a nonempty open set and H is a neighbourhood of y_o. The function $q^y(f,\cdot)$ satisfies similar properties as $q_x(f,\cdot)$—the symmetrical quasi-oscillation with respect to x.

THEOREM 6. *The function $q^y(f,\cdot)\colon X\times Y\to[0,\infty]$ is upper symmetrically quasi-continuous with respect to y.*

THEOREM 7. *A function f is symmetrically quasi-continuous with respect to y at a point $(x_o,y_o)\in X\times Y$ if and only if $q^y\big(f,(x_o,y_o)\big)=0$.*

Now, let us define:

$$q\big(f,(x_o,y_o)\big) = \sup_{U\times V}\inf_{G\times H} \operatorname{diam} f\Big((G\times H)\cup\big\{(x_o,y_o)\big\}\Big)$$

for every $(x_o,y_o)\in X\times Y$, where $U\times V$ runs the set of all neighbourhoods of (x_o,y_o) and $G\times H$ is a nonempty open subset of $U\times V$ ([1]).

Then we obtain the equivalent condition for the quasi-continuity:

THEOREM 8. *A function $f\colon X\times Y\to Z$ is quasi-continuous at a point $(x_o,y_o)\in X\times Y$ if and only if $q\big(f,(x_o,y_o)\big)=0$.*

The oscillation of the function f is traditionally denoted by $\omega(f,\cdot)$. Then we remark that

$$\begin{aligned} q\big(f,(x_o,y_o)\big) &\leqslant \min\Big\{q_x\big(f,(x_o,y_o)\big),\, q^y\big(f,(x_o,y_o)\big)\Big\}\\ &\leqslant \max\Big\{q_x\big(f,(x_o,y_o)\big),\, q^y\big(f,(x_o,y_o)\big)\Big\} \leqslant \omega(f,\cdot)\,. \end{aligned} \tag{1}$$

As usual, by $C(f)$ we understand the set of all points at which f is continuous, by $E(f)$—the set of all points of quasi-continuity of the function f. By symbol $E_{s,x}(f)$ $(E^{s,y}(f))$ we denote the set of all points at which f is symmetrically quasi-continuous with respect to x (to y).

Then the above conditions imply the following inclusions:

Corollary.

$$C(f) \subset E_{s,x}(f) \cap E^{s,y}(f) \subset E_{s,x}(f) \cup E^{s,y}(f) \subset E(f).$$

Now, we will consider a real locally bounded function f of two variables defined on a product topological spaces X and Y. Let

$$M_x(f,(x_o,y_o)) = \sup_{U\times V} \inf_{G\times H} \sup\Big\{f(x,y)\colon (x,y) \in (G\times H) \cup \big\{(x_o,y_o)\big\}\Big\}$$

for every $(x_o,y_o) \in X\times Y$, where the supremum is taken over all neighbourhoods $U\times V$ of (x_o,y_o) and infimum runs the set of all $G\times H$ such that G is a neighbourhood of x_o, $G \subset U$, H is a nonempty open subset of V.

Directly from this definition we obtain

Corollary. $f(x_o,y_o) \leqslant M_x\big(f,(x_o,y_o)\big)$ *for every* $(x_o,y_o) \in X\times Y$.

Theorem 9. *A function* $M_x(f,\cdot\,)\colon X\times Y \to \mathbb{R}$ *is upper symmetrically quasi-continuous with respect to* x.

P r o o f. We will prove that $M_x(f,\cdot\,)$ is upper symmetrically quasi-continuous with respect to x at the arbitrary point $(x_o,y_o) \in X\times Y$. Let us take a neighbourhood $U\times V$ of (x_o,y_o) and a real number a such that $M_x\big(f,(x_o,y_o)\big) < a$. We can choose $\varepsilon > 0$ with the property $M_x\big(f,(x_o,y_o)\big) < a-\varepsilon < a$. Therefore for $U\times V$ of (x_o,y_o) there exists a neighbourhood G of x_o, $G\subset U$ and a nonempty open subset H of V satisfying the inequality $f(x,y) < a-\varepsilon$ for every $(x,y) \in (G\times H)\cup\big\{(x_o,y_o)\big\}$. Let $(x_1,y_1) \in G\times H$. Then $(G\times H)\cup\big\{(x_1,y_1)\big\} = G\times H$. For an arbitrary neighbourhood $U_1\times V_1$ of the point (x_1,y_1) we will consider the sets $G_1 = U_1\cap G$, $H_1 = V_1 \cap H$. These sets are open in X and Y, respectively. Moreover, $x_1 \in G_1$, $H_1 \neq \emptyset$ and $f(x,y) < a-\varepsilon$ for each $(x,y) \in (G_1\times H_1)\cup\big\{(x_1,y_1)\big\}$. As a consequence

$$\sup\Big\{f(x,y)\colon (x,y)\in (G_1\times H_1)\cup\big\{(x_1,y_1)\big\}\Big\} \leqslant a-\varepsilon$$

and

$$\inf_{C\times D} \sup\Big\{f(x,y)\colon (x,y)\in (C\times D)\cup\big\{(x_1,y_1)\big\}\Big\} \leqslant a-\varepsilon\,,$$

where the infimum is taken over all neighbourhoods C of x_1 and nonempty open sets D such that $C\times D \subset U_1\times V_1$. Finally, $M_x\big(f,(x_1,y_1)\big) < a$ for every $(x_1,y_1) \in G\times H$. It means that the function $M_x(f,\cdot\,)$ is upper symmetrically quasi-continuous with respect to x at the point (x_o,y_o) and it completes the proof. □

Theorem 10. *A function $f\colon X \times Y \to \mathbb{R}$ is upper symmetrically quasi-continuous with respect to x at a point $(x_o, y_o) \in X \times Y$ if and only if*

$$M_x\big(f,(x_o,y_o)\big) = f(x_o,y_o)\,.$$

P r o o f. First we assume that f is upper symmetrically quasi-continuous with respect to x at the point (x_o, y_o). Then for a positive number ε and a neighbourhood $U \times V$ of (x_o, y_o) there exist a neighbourhood G_1 of x_o, $G_1 \subset U$ and an open subset H_1 of V such that $f(x,y) < f(x_o,y_o)+\varepsilon$ for every $(x,y) \in G_1 \times H_1$. Obviously $f(x_o,y_o) < f(x_o,y_o)+\varepsilon$. Thus,

$$\sup\Big\{f(x,y)\colon (x,y) \in (G_1 \times H_1) \cup \big\{(x_o,y_o)\big\}\Big\} \leqslant f(x_o,y_o)+\varepsilon\,.$$

It implies an inequality

$$\sup_{U\times V}\inf_{G\times H}\sup\Big\{f(x,y)\colon (x,y) \in (G \times H) \cup \big\{(x_o,y_o)\big\}\Big\} \leqslant f(x_o,y_o)+\varepsilon\,,$$

where the supremum is taken over all neighbourhoods $U \times V$ of (x_o, y_o) and infimum runs the set of all $G \times H$ such that G is a neighbourhood of x_o, $G \subset U$, H is a nonempty open subset of V, and

$$M_x\big(f,(x_o,y_o)\big) \leqslant f(x_o,y_o)\,.$$

From the last corollary we obtain the second inequality

$$f(x_o,y_o) \leqslant M_x\big(f,(x_o,y_o)\big)\,.$$

Finally $M_x\big(f,(x_o,y_o)\big) = f(x_o,y_o)$.

Now, suppose that the equality $M_x\big(f,(x_o,y_o)\big) = f(x_o,y_o)$ holds. From the last theorem the real function $M_x(f,\cdot\,)$ is upper symmetrically quasi-continuous with respect to x. Let $\varepsilon > 0$ and $U \times V$ be a neighbourhood of (x_o, y_o). There exists a subset $G \times H$ of $U \times V$ such that G is a neighbourhood x_o, H is a nonempty open set and

$$M_x\big(f,(x,y)\big) < M_x\big(f,(x_o,y_o)\big) + \varepsilon$$

for every $(x,y) \in G \times H$. Then we can write

$$f(x,y) \leqslant M_x\big(f,(x,y)\big) < M_x\big(f,(x_o,y_o)\big) + \varepsilon = f(x_o,y_o)+\varepsilon$$

for each $(x,y) \in G \times H$. This is the condition of upper symmetrical quasi-continuity of the function f with respect to x at the point (x_o, y_o). $\square$

In a similar way we can define a function $m_x(f,\cdot\,)\colon X \times Y \to \mathbb{R}$ as follows:

$$m_x\big(f,(x_o,y_o)\big) = \inf_{U\times V}\sup_{G\times H}\inf\Big\{f(x,y)\colon (x,y) \in (G \times H) \cup \big\{(x_o,y_o)\big\}\Big\}$$

for every $(x_o, y_o) \in X \times Y$, where the infimum is taken over all neighbourhoods $U \times V$ of (x_o, y_o), the supremum is taken over all neighbourhoods G of x_o and nonempty open sets H such that $G \times H \subset U \times V$.

Let us observe that

Corollary. *$f(x_o, y_o) \geqslant m_x\big(f, (x_o, y_o)\big)$ for each $(x_o, y_o) \in X \times Y$.*

Theorem 11. *A function $m_x(f, \cdot)\colon X \times Y \to \mathbb{R}$ is lower symmetrically quasi-continuous with respect to x.*

Theorem 12. *A function $f\colon X \times Y \to \mathbb{R}$ is lower symmetrically quasi-continuous with respect to x at a point $(x_o, y_o) \in X \times Y$ if and only if*

$$m_x\big(f, (x_o, y_o)\big) = f(x_o, y_o).$$

By analogy we can consider similar functions $M^y(f, \cdot)$ and $m^y(f, \cdot)$ with respect to variable y.

Now, we will investigate relationships between functions $M_x(f, \cdot)$, $m_x(f, \cdot)$ and quasi-oscillation. At first we denote

$$\widetilde{q_x}\big(f, (x_o, y_o)\big) = M_x\big(f, (x_o, y_o)\big) - m_x\big(f, (x_o, y_o)\big)$$

for every $(x_o, y_o) \in X \times Y$.

Under the symbol $(E_{s,x})^{+}(f)$ we will understand the set of all points at which f is upper symmetrically quasi-continuous with respect to x. Similarly, $(E_{s,x})^{-}(f)$ denotes the set of all points of lower symmetrical quasi-continuity with respect to x.

Corollary. $(E_{s,x})^{+}(f) \cap (E_{s,x})^{-}(f) = \Big\{(x, y) \in X \times Y : \widetilde{q_x}\big(f, (x, y)\big) = 0\Big\}$.

P r o o f. Let us observe that the following equivalences hold:

$$\begin{aligned}
&(x_o, y_o) \in (E_{s,x})^{+}(f) \cap (E_{s,x})^{-}(f) \\
&\iff (x_o, y_o) \in (E_{s,x})^{+}(f) \wedge (x_o, y_o) \in (E_{s,x})^{-}(f) \\
&\iff M_x\big(f, (x_o, y_o)\big) = f(x_o, y_o) \wedge m_x\big(f, (x_o, y_o)\big) = f(x_o, y_o) \\
&\iff \widetilde{q_x}\big(f, (x_o, y_o)\big) = M_x\big(f, (x_o, y_o)\big) - m_x\big(f, (x_o, y_o)\big) = f(x_o, y_o) - f(x_o, y_o) = 0 \\
&\iff \widetilde{q_x}\big(f, (x_o, y_o)\big) = 0.
\end{aligned}$$

□

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