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THE SEKED: ON ANCIENT EGYPTIAN MATHEMATICS AND ASTRONOMY

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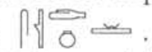
In this article the ancient Egyptian terms expressing the slope of a pyramid and voyage of the sun god across the sky are discussed in context of ancient Egyptian mathematics, astronomy and religious iconography.

The ancient Egyptians developed and used a special mathematical method for calculation of the slope of a pyramid. In hieroglyphic texts the slope appears in this form . This word is transliterated as *śḳd*[1] and conventionally called seked. The word is a noun of masculine gender. As can be seen in its hieroglyphic transcription, the first three signs give the phonetic consonant skeleton of the word. They are the uniliteral hieroglyph  (*ś*), followed by the biliteral sign  (*ḳd*) accompanied by the phonetic complement  (*d*). The biliteral sign *ḳd* of uncertain meaning belongs to category "Aa" of the Sign-list named "unclassified".[2] Nevertheless, in Gardiner's Sign-list, this sign is described as an instrument used by ancient Egyptian builders.[3] The word ends with the sign representing the papyrus scroll used as determinative.

The term *śḳd* appears in the mathematical papyri for solving the specific problems concerning the pyramids.[4] As for its modern definition, Gillings says:[5]

[1] R. O. Faulkner, A Concise Dictionary of Middle Egyptian, Oxford 1996, p. 250; R. Hannig, Großes Handwörterbuch Ägyptisch-Deutsch, Mainz 1995, p. 772; A. Erman, H. Grapow, Wörterbuch der aegyptischen Sprache, Vol. IV, Leipzig 1930, p. 309.

[2] A. H. Gardiner, Egyptian Grammar, Oxford 1982[3], p. 543.

[3] A. H. Gardiner, Egyptian Grammar, p. 543, sign Aa-28 and note 1. Perhaps, this sign may reflect the shape of an instrument originally used in some way in process of marking and checking the precise slope of a wall, or a pyramid core built up from stone blocks.

[4] R. J. Gillings, Mathematics in the time of the pharaohs, New York 1982, pp. 185ff.

[5] R. J. Gillings, Mathematics, p. 212.

The seked of a right pyramid is the inclination of any one of the four triangular faces to the horizontal plane of its base, and is measured as so many horizontal units per one vertical unit rise. It is thus a measure equivalent to our modern cotangent of the angle of slope. In general, the seked of a pyramid is a kind of fraction, given as so many palms horizontally for each cubit vertically, where 7 palms equal one cubit. The Egyptian word "seked" is thus related to our modern word "gradient".

As one can see, the Egyptians used *śḳd* to express the angle. In fact, they did not work with the system of degrees known to us, but instead they expressed an inclination as a kind of special fraction.

Slope as such was a favourite pattern very often used by the architects in ancient Egypt. There are number of buildings of various functions and sizes such as tombs, temples, enclosure walls, etc., built up from different stones, or mudbricks having inclined outer walls of these structures. The slope in Egyptian architecture can be archaeologically attested in each period of the long history of dynastic Egypt.[6]

One of the important written sources for the study of ancient Egyptian mathematics is the so-called Rhind Mathematical Papyrus (RMP). This papyrus bears the name of Alexander Henry Rhind (1833-1863), a British excavator, who bought it in Luxor. The author of the RMP is a certain Ahmose who wrote the papyrus during the Hyksos period in the Year 33 of the king Auserre (Apophis). Important information coming from Ahmose is that he was copying an earlier work written down in the reign of the king Nimaatra (Amenemhat III). The reign of this king is dated to the second half of the 19th centrury BC. The RMP is now in the British Museum catalogued as BM 10057 and BM 10058. The RMP includes various arithmetical problems containing their description and solution. The RMP also consists of several problems concerning the geometry of the pyramids where their slope, height and base are calculated. Problem 56 of the RMP belongs to this category. It can exemplify method of such calculation. It is presented here as mentioned in Gillings' book.[7]

Example of reckoning a pyramid.
Height 250, base 360 cubits.
What is its seked?
Find 1/2 of 360, 180.
Divide 180 by 250, 1/2 1/5 1/50 cubit.
Now a cubit is 7 palms.
Then multiply 7 by 1/2 1/5 1/50.

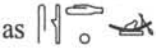
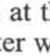
1	7		
\ 1/2	\ 3	1/2	
\ 1/5	\ 1	1/3	1/15
\ 1/50	\	1/10	1/25

Totals 1/2 1/5 1/50 5 1/25 palms. This is its seked.

[6] D. Arnold, Building in Egypt. Pharaonic stone masonry, Oxford 1991, pp. 12f.

[7] R. J. Gillings, Mathematics, pp. 185f.

This means that the slope of the triangular faces of this pyramid is 5 1/25 palms horizontally for every rise of one cubit in height.

The word *śḳd* translated as "slope" is morphologically very similar to another Egyptian term written in slightly different way. It is a verb written as  or  and transliterated as *śḳdy*. Its meaning is "sail", "travel", "row".[8] The verb is characterized as causative belonging to *3ae inf* (*tertiae infirmae verb*).[9] Ancient Egyptians had many terms how to express a movement[10] and one of them is also the aforementioned verb *śḳdy*. The comparison of the written forms of the noun *śḳd* and the verb *śḳdy* reveals the clear difference. It concerns the ending of the word, that is, the sign with the function of a determinative placed at the end of each of both. The former ends with the papyrus scroll and the latter with a barque. In case of the latter some alternative determinatives occur as ,  which express the movement too. As for the ethymology and interrelation of both discussed words, *śḳd* and *śḳdy*, it is not easy to establish any precise conclusions.[11] As mentioned above, one of determinatives in the verb *śḳdy* expressing the movement has the form of a barque.

The celestial bodies such as the sun, moon, planets and stars, were regarded by the Egyptians as gods inhabiting the sky.[12] The sky itself was perceived and imagined in various ways in ancient Egypt.[13] One of them was an image of a watery ocean encircling the world. The sky gods used their own boats to sail and travel the various regions of the sky. They passed the heaven rising higher and higher over the horizon and culminated at the meridian. The travels in their boats are mentioned in many texts and reliefs or paintings dating from various periods of Egyptian history. One of the earliest iconographical sources attesting this idea dates back as far as the first dynasty.[14] It is a picture engraved into the surface of an ivory comb in a form of the relief decoration. The comb comes from the reign of the king Djet. The picture shows the very ancient falcon god over his barque resting on a pair of wings symbolizing the heavens.

In some Egyptian inscriptions the movement in the sky was also expressed by the verb *śḳdy*. The following citations represent such examples occurring in

[8] R. O. Faulkner, CD, p. 250; Cf. R. Hannig, HWB, p. 771.

[9] R. Hannig, HWB, p. 771; R. O. Faulkner, CD, p. 250; J. P. Allen, Middle Egyptian. An Introduction to the Language and Culture of Hieroglyphs, Cambridge 2000, p. 468.

[10] A. Erman, H. Grapow, Wb VI, 1950, pp. 48, 62, 126; Cf. R. Hannig, Lexica 2, Mainz 1999, pp. 697-736.

[11] W. Vycichl, Dictionnaire étymologique de la lingue copte, Leuven 1983, p. 89.

[12] J. von Beckerath, Astronomie und Astrologie, 511-514, In: W. Helck, E. Otto (eds.), Lexikon der Ägyptologie, Vol. I, Wiesbaden 1975.

[13] E. Hornung, Himmelsvorstellungen, 1215-1218, In: W. Helck, W. Westendorf (eds.), Lexikon der Ägyptologie, Vol. II, Wiesbaden 1977.

[14] V. G. Callender, The Eye of Horus. A History of Ancient Egypt, Melbourne 1993, p. 52, Fig. 3.28; W. Westendorf, Altägyptische. Darstellungen des Sonnenlaufes auf der abschüssigen Himmelsbahn, MÄS, Vol. 10, Berlin 1966, pp. 22–24, Taf. 8/14.

the main religious texts coming from three main periods of Egyptian history: the Old, Middle and New Kingdoms.

Utterance 513 of the Pyramid Texts in Paragraph 1171 reads:[15]

"row (i.e. the king) with the Unwearying stars".

In the Spell 1029 of the Coffin Texts we can read:[16]

"Speech for sailing in the great bark of Ra"

The Spell 136B of the Book of the Dead contains the same text:[17]

"Speech for sailing in the great bark of Ra"

Besides these texts, there are other types of inscriptions describing the movement of the sun. In these inscriptions the verb, hieroglyphic signs of which are transliterated as *śḳdy*, has also been used. These texts known as the hymns to the sun god are also of a religious character and appear in the private tombs in the Theban necropolis.[18] Some citations from these hymns in transliterated form can be presented as follows:[19]

...śḳd.n.k-ḫrt...[20]
...śḳdd.k m-ḫrt...[21]
...[śḳd]d mʿ-jmj-itn.f[22]
śḳdd-Rʿw [...][23]
...śḳdd.k-[jm.f m3ʿ ḫrw] [n-mwt.k Nwt rʿw-nb][24]
...śḳdd jmj-jtn.f[25]

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15 K. Sethe, Altaegyptischen Pyramidentexte, Vol. 2, Leipzig 1910, p. 154; R. O. Faulkner, The Ancient Egyptian Pyramid Texts, Oxford 1969, p. 189
16 A. de Buck, The Egyptian Coffin Texts, Vol. VII, Chicago 1961, pp. 257-261.
17 Transcribed from E. Naville, Das aegyptische Totenbuch der XVIII. Bis XX. Dynastie, Vol. I (Texte und Vignetten), Berlin 1886, Taf. CXLIX; Cf. T. G. Allen, The Book of the Dead, Chicago 1974, pp. 110, 112.
18 J. Assmann, Sonnenhymnen in thebanischen Gräbern, Theben I, Mainz 1983.
19 Cited according to Assmann's transliteration.
20 J. Assmann, STG, p. 14 (Text 16/5).
21 J. Assmann, STG, p. 203 (Text 156/4).
22 J. Assmann, STG, p. 216 (Text 158c/8).
23 J. Assmann, STG, p. 224 (Text 163/9).
24 J. Assmann, STG, p. 274f (Text 196/5, 6).
25 J. Assmann, STG, p. 320f (Text 228/XI/2).

In addition, the planets can also be included in the category of celestial bodies, the movement of which is expressed by the discussed verb. In some Egyptian inscriptions, the name of Mars is accompanied with additional texts, one of which is worth mentioning here. It is *śḳdd.f m ḫtḫt* "he who travels backwards".[26] This example occurs on the astronomical ceiling of the Ramesseum, the mortuary temple of Ramesse II, as well as in the royal tomb of Seti I in the Valley of the Kings.[27] In the Ramesseum, Mars is depicted as falcon-headed god with a star above the head standing in a barque, however, in the accompanying text the term *śḳdd.f* is not determined with the sign for the barque. On the contrary, in the tomb of Seti I, this example shows that in the verb phrase *śḳdd.f* the determinative of the barque is added in front of suffix *.f.* In the picture from the Valley of the Kings only the upper part of the body with a star above the head is preserved in the scene of the Mars falcon-god. However, Lepsius' picture taken in the 1840s shows that the iconography of the Mars god was originally comparable with that from the Ramesseum.[28]

Now, let us turn our attention to the sun as the most dominant celestial body. Each position of the sun in the sky is characterized by the specific height above the horizon at a specific time. In modern terminology, the height above the horizon reminds us of the altitude.[29] So, the whole path of the sun in the sky is the summary of all positions on and above the horizon between the sunrise and sunset. Since the sun had been the most important deity in ancient Egyptian religion, the symbol of the sun and its voyage in the sky very often formed an important motif in the relief or painted decoration of the tombs and temples in all historical periods. The images and symbols of the sun god can be found, for example, in the decoration of the Old Kingdom temples, on the uppermost parts of such monuments as the Middle Kingdom pyramids and the New Kingdom obelisks,[30] stelae and so on. In these scenes, the sun is usually depicted in the form of the winged sun disk or as the anthropomorphic god, sometimes accompanied by the king, while the voyage of the sun can be expressed in the form of the barque. According to ancient Egyptian iconography the representations of the wings or barque expressed the movement of the sun in the sky. Therefore, the picture of the wings or barque of the sun god moving across the sky can be un-

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[26] O. Neugebauer, R. Parker, Egyptian Astronomical Texts, Vol. III, London 1964, p. 179; Moreover, this epithet may imply an allusion to the retrograde movement of Mars in the starry background of the sky. Cf. E. C. Krupp, Echoes of the ancient skies, New York 2003, p. 70.

[27] M. Clagett, Ancient Egyptian Science, Vol. II (Calendars, Clocks, and Astronomy), Philadelphia 1995, Figs. III.2 and III.65a,b,c.

[28] M. Clagett, AES II, Fig. III.65c.

[29] It is the angular distance between the celestial body and the point on the horizon measured vertically above the horizon. Together with azimuth measured eastwards along the horizon from the north point it belongs to the horizontal coordinates of the celestial coordinate system used to define the position of any celestial body in the sky.

[30] D. Arnold, Lexikon der ägyptischen Baukunst, Zürich 1994, p. 205f.

derstood, as the dynamic aspect of the sun. Besides the other celestial bodies, the sun was also used to measure the time between the sunrise and sunset. Sundials were later developed on the the basis of this knowledge. One of the conditions for doing this was appreciation of the relationship between a vertical pole and its shadow cast on a flat surface.

The gnomon vertically fixed on the ground offers very good example of what can be called "the solar geometry".[31] In this case we have something to do with the right-angled triangle (the right angle is at the foot of the gnomon). Such a triangle consists of the longest side, that is, the distance between the tops of the gnomon and its shadow, and two shorter sides that is the length of the shadow of gnomon cast on the ground and the height of gnomon itself. From these two shorter sides the former is changeable in time while the latter is constant. The shadow periodically changes its length as the sun moves slowly across the sky. It shortens from the sunrise untill the culmination of the sun at midday and lengthens after midday untill the sunset. This everyday periodicity gives the shadow the same length twice a day. The length of the gnomon equals its height above the ground and is still constant. It can be said that the height of gnomon and the length of its shadow determine the shape of the virtual right-angled triangle. Its shape depending on the length of the horizontal shadow changes in time too. The fact that the unchangeable constant length of the gnomon is vertical and the changeable length of the shadow is horizontal should be empasized especially in context of the aforementioned term *śḳd.* The peculiar similarity in the principle cannot be overlooked here. We can remember that the ratio expressed between the vertical cubit and the horizontal distance is important for calculating the slope as such. As to the former, it is the length of 7 palms or 1 cubit, the basic linear measure used by the Egyptian architects (ca. 52,5 cm). In the RMP 56, 58 and 59 the slope of the pyramid is determined by the mathematical calculation knowing the height and the base of the pyramid.

In fact, in the case of the gnomon and its shadow, it is the slope of the sunray that is to be calculated. Otherwise, the constant height of gnomon and the changeable horizontal line of its shadow provide the method of calculating the slope (seked) of the sunrays, and at the same time the height of the sun above the horizon as well. The slope of the sunray is given by the position of the sun in the sky (its height above the horizon) and it can be revealed by the length of the shadow on the horizontal ground to the vertical gnomon. The higher is the sun above the horizon, the shorter is the shadow of the gnomon and *vice versa*. Moreover, the length of the shadow gives here, at the same time, the zenith distance of the sun. The longer is the shadow on the ground, the greater is the zenith distance of the sun. In modern words, this position of the sun is the angular one, however, expressed in the Egyptian way it is a kind of fraction or ratio expressed by the constant length of the gnomon to the linear horizontal length of

[31] D. Magdolen, The solar origin of the "sacred triangle" in ancient Egypt?, Studien zur altägyptischen Kultur, Vol. 28, Hamburg 2000, pp. 207–217.

its shadow. For example, the position of the sun at the height 45 degrees above the horizon equals the length of the shadow of gnomon of the same length as is its vertical height itself (ratio 1:1). In this case one could say that the seked of the sun equals 1. From the geometrical point of view, it is the case of the isosceles right-angled triangle with its two shorter sides of the same length. If the sun, however, is higher (a) or lower (b), then the following relations are valid:

a.) the seked < 1 cubit, (height > 45 degrees)
b.) the seked > 1 cubit, (height < 45 degrees)

Theoretically, these formulae could also be valid for the other celestial bodies observed in the sky. However, among the celestial bodies with the exception of the sun, only the light of the moon in the nightly sky is strong enough to cause that the objects can cast the shadows on the ground. The principle of what we call "solar geometry" described above plays the fundamental role in construction of sundials.

As mentioned above, both words of almost the same hieroglyphic form, however, with the different meanings: "slope" and "sail", have the same consonantal skeleton, i.e. *ś-ḳ-d.* The difference between them is morphologically expressed by determinatives standing at the end of each word. In case of the noun "slope" it is a hieroglyphic sign for the papyrus scroll. The verb "sail" ends with the sign among the others for a barque. We saw that the latter term appears in connection with the sun as well as other celestial bodies expressing their movement. The term meaning "slope" is connected with calculation of the slope of the pyramid, or pyramidion, the uppermost part of the pyramids and obelisks. Both monuments are regarded as solar symbols related to the sun god. So, *śḳd* meaning "slope" and *śḳdy* meaning "sail" have got one common factor, – the sun. The aforementioned "solar geometry" provides a good and simple basis for finding the principles of the seked calculation. There are, in fact, no inscriptions or other records saying how the seked was developed by the Egyptians. In the country where every field must have been measured out again and again after the annual flood it is quite possible that it might have been derived from the horizontal rather then the vertical geometry.[32] In fact, absence of this evidence provides us to look inside and analyse the intellectual skills and cognitive ability of the people living thousands of years ago in the Nile Valley. In doing so we may uncover the principles of mathematical knowledge of the ancient Egyptians, which are sometimes misunderstood and considered as less developed and unimportant from the point of view of history of science.

[32] In fact, it is the trigonometric function cotangent. Cf. G. Robins, Mathematics, Astronomy, and Calendars in Pharaonic Egypt, p. 1810f, In: J. M. Sasson (ed.), Civilizations of the Ancient Near East, Peabody (Massachustetts) 2000, vols. 3 and 4., p. 1810f.