The Economic-mathematical Nature of the HGN Model Concept as a Tool for Measuring Performance of Enterprises[1]

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Abstract

The article talks about the newly-conceived HGN model based on ratio indicators. The main characteristic of the model is a synthetic indicator based on "refining" chosen financial efficiency indicators by separating out impacts measured by using chosen efficiency decreasing indicators. We identify and present a way to determine the minimum limits of the synthetic indicator characterizing the performance of a non-financial enterprise. We apply both the classical and tolerance approach to sensitivity analysis in a linear optimization model. We demonstrate the performance measurement possibilities provided by the gradual improvement of the HGN model by designing two versions of the model.

Keywords: *non-financial profitable enterprise performance, HGN model, efficiency indicators, efficiency decreasing indicators, ratio outliers, linear optimization model, classical and tolerance approach to sensitivity analysis*

JEL Classification: C53, G33

Introduction

The authors of the HGN model – Hyránek, Grell, Nagy – have been developing it since 2014. The model belongs to the approaches to performance modelling by means of traditional financial ratios. The key feature and the ultimate defining

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indicator of the HGN model is a synthetic indicator based on "refining" chosen financial performance indicators by separating out impacts measured by using chosen indicators that decrease efficiency. Thus, the "refined" efficiency (*net efficiency*) expresses financial performance. Based on determining the optimum synthetic indicator interval, we identify the low threshold of the minimum performance of a non-financial profitable business.

The HGN model works with financial ratios which are defined as the ratio of the corresponding absolute variables having the nature of inputs and outputs in an enterprise's transformation process. The financial ratios are then expressed as follows:

$$\sum_{i=1}^{n} \frac{\text{output}_i}{\text{input}_i} - \sum_{j=1}^{m} \frac{\text{input}_j}{\text{output}_j} \, , \tag{1}$$

where the *output/input* ratio expresses the efficiency indicators and the *input/output* ratio represents the indicators that decrease efficiency (Tables 1 and 2).

T a b l e 1

Overview and Method of Calculating the Ratios in the HGN1 Model

Efficiency indicators ↑			Efficiency decreasing indicators ↓		
x_1	x_2	x_3	y_1	y_2	y_3
Return on equity	Cash flow to sales ratio	Turnover of assets	Binding of short-term receivables	Debt capital repayment time	Operating cost
[coefficient]	[coefficient]	[coefficient]	[coefficient]	[years]	[coefficient]
$\frac{\text{net profit}}{\text{own capital}}$	$\frac{\text{net profit+ writeoffs}}{\text{sales}}$	$\frac{\text{sales}}{\text{assets}}$	$\frac{\text{short-term receivables}}{\text{sales}}$	$\frac{\text{liabilities}}{\text{net profit+ writeoffs}}$	$\frac{\text{economic activity costs}}{\text{sales}}$

Source: Own work.

T a b l e 2

Overview and Method of Calculating the Ratios in the HGN2 Model

Efficiency indicators ↑			Efficiency decreasing indicators ↓		
x_1	x_2	x_3	y_1	y_2	y_3
Return on equity	Share of value added on sales	Turnover of assets	Time of collecting short-term receivables	Long-term debt capital repayment time	Time of payment of short-term liabilities
[coefficient]	[coefficient]	[coefficient]	[day]	[years]	[day]
$\frac{\text{net profit}}{\text{own capital}}$	$\frac{\text{value added}}{\text{sales}}$	$\frac{\text{sales}}{\text{assets}}$	$\frac{\text{short-term receivables}}{\text{sales}}$	$\frac{\text{long-term liabilities}}{\text{net profit+ writeoffs}}$	$\frac{\text{long-term liabilities}}{\text{sales}}$

Source: Own work.

We are investigating a case where the number of selected efficiency indicators and of the efficiency decreasing indicators is the same ($n = m$). In general, an enterprise's effort is to *maximize* the efficiency indicators and *minimize* the efficiency decreasing indicators. In the HGN model, we take both of these requirements into account by formulating a linear optimization model by maximizing the difference between the sum of the chosen financial efficiency indicators and the efficiency decreasing indicators. Thus, in other words, we maximize the net efficiency.

The measurement of financial performance using ratio indicators may be expressed as the ratio of inputs and outputs according to the formula (1) as follows:

• *output/input* – indicators of productivity, efficiency, but also some indicators of profitability,

• *input/output* – efficiency decreasing indicators and cost indicators,

• *input/input* – some indicators of property and financial structure and some cost indicators,

• *output/output* – some indicators of profitability.

1. The Current State of Solved Problems in Slovakia and Abroad

Currently, there are numerous methods and approaches to analysing the financial performance of an enterprise. From the point of view of the main business activity goal, we can divide these into two large groups, namely the approaches that prefer to maximize the profits of an enterprise (a performance analysis using traditional profitability indicators – return on equity, return on assets, return on total capital, etc.) and the approaches that prefer the growth of the market value of an enterprise for its owners. These include indicators such as return on net assets (RONA) and cash return on gross assets (CROGA), EVA indicator and its modifications, Madden's (1998) indicator cash flow return on investment (CF ROI), etc. These performance assessment criteria measure the success of a business activity by its economic gain. Most of these have dynamic nature (e.g. CF ROI) and also take into account the average cost of acquiring and binding the business's own external capital and interest-bearing debt capital (WACC). Model-M may also be used to assess the financial stability of enterprises. The model was created using a scoring function based on analysing and processing the data provided by Slovak companies according to Harumová and Janisová (2014).

According to Lehn and Makhija (1996), the EVA indicator, in contrast to traditional performance indicators, seeks to measure the value; it identifies whether an enterprise generates or destroys value by deducting capital expenditures from the proceeds generated from the invested capital. Zéghal and Maaloul (2010) dealt with analysing the role of the value added indicator as an indicator

of generating value, and with its impact on the economic and financial performance of an enterprise. Their results show that the indicator has a positive impact on economic and financial performance.

Currently, traditional financial ratios are still being used in modelling performance and creating synthetic indicators, the undeniable advantage of which is their simple application, as they are built on data from enterprises' financial statements. It is more sophisticated to apply a group of methods that prefer the growth of the enterprise's market value. These methods require a conversion of multiple data from a financial statement. E.g. indicators such as net assets, gross assets, NOPAT (net operating profit after tax) are not available in financial statements and there is currently no uniform methodology for their calculation. Another problem is the identification of capital costs. There is no uniform procedure how to e.g. determine the cost of equity. The disadvantage of calculating the costs of debt capital using the indicator "interests/interest-bearing debt capital" is that it has a very static nature. A certain shortcoming of the methods preferring the market value growth is their focus on returns and capital costs only, while eliminating the factors determining financial stability and long-term ability to pay. It is these shortcomings that the authors of the HGN model are trying to eliminate to the maximum extent and emphasize the unfavourable debt situation of an enterprise.

It has been fifty years since the publication of the first predictive models (Beaver, 1966; Altman, 1968; Deakin, 1972; Blum, 1974), and since then the applied methodology has undergone significant developments. Since the late 1960s, multidimensional discriminant analysis has begun to be used in order to predict financial development of enterprises. On its basis, Altman (1968) also created his famous model, later on Altman, Haldeman and Narayanan (1977) developed the ZETA model. Since the 1980s, logistic regression has supplemented and gradually substituted the multidimensional discriminant analysis (Ohlson, 1980; Zavgren, 1985; Lau, 1987; Keasey and McGuinness, 1990). It became the most commonly applied prognostic method in developed countries until the late 1990s. The methodology of predictive financial analysis is constantly evolving, as evidenced by the fact that significant steps have been taken in the area of mathematical and statistical methods over the recent years. In recent years, application of a relatively non-standard method called data envelopment analysis has gained momentum. Xu and Wang (2009) were the first to apply this approach to predicting bankruptcy. Feruś (2010) used the data envelopment analysis to create a predictive model for construction enterprises.

Each model for measuring performance and prediction is different, it uses different mathematical apparatus, it works with different indicators; however, the models also have some common features. In an era of rapidly changing economic

environment, the standard methods for measuring financial performance and assessing financial health are less adequate. Most authors are focused on enhancing the predictive ability of original models by responding appropriately to the existing changed economic environment. Even the authors of the present HGN model attempt to take into account the current economic conditions of companies in terms of debt and insolvency.

The applied newly-conceived model (HGN) is a part of the approaches to performance modelling by means of financial ratios and we create also the so-called correction coefficients c_i^x, c_i^y (2) that give effect to the impact of the efficiency indicators and the efficiency decreasing indicators. Currently, we distinguish two versions of the newly-created model. In the case of $c_i^x = c_i^y = 1$, we applied the "version 1". If c_i^x, $c_i^y > 0$, we work with the „version 2", while the correction coefficients are calculated separately from the current database of the efficiency indicators and the efficiency decreasing indicators. The current state of the art for each version of the model and its further development may be characterized as a focus on indebtedness and ability to repay debt.

2. Research Objectives

The objective of the research is to create a financial performance model using mathematical-statistical apparatus with the possibility of applying it to profit--making non-financial enterprises. In the paper, we present the results of the research in the gradual improvement and development of the newly-conceived HGN model for measuring the performance of an enterprise for the needs of financial decision-making. We follow the basic approaches developed in the monograph (Hyránek, Grell and Nagy, 2014) and we further discuss the issue published in the paper (Hyránek, Grell and Nagy, 2015).

The ultimate indicator of the HGN model is a synthetic indicator based on "refining" chosen financial efficiency indicators by separating out impacts measured by using chosen efficiency decreasing indicators. For further presentation, the HGN model determined by the formula (1) is written in the form:

$$\sum_{i=1}^{3} c_i^x x_i - \sum_{i=1}^{3} c_i^y y_i \qquad (2)$$

where

x_i – are the efficiency indicators,

y_i – the efficiency decreasing indicators,

c_i^x, c_i – the correction coefficients that give effect to the impact of the efficiency indicators and the efficiency decreasing indicators (discussed in more detail in par. 3.2).

When formulating both versions of the HGN (HGN1 and HGN2) model we deal with selection of 3 efficiency indicators and 3 efficiency decreasing indicators (Tables 1 and 2).

For the purposes of defining the limits of the synthetic indicator, we shall use the mathematical apparatus of the linear optimization model of an enterprise formulated as follows:

$$\max z(x) = \sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} a_{ij} x_j \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} b_i \quad i = 1, \ 2, \ \ldots, \ m \qquad (3)$$

$$x_j \geq 0 \quad j = 1, \ 2, \ \ldots, \ n$$

where

c_j – the coefficients of the objective function, $j = 1, 2, ..., n$ corresponding to the correction coefficients in the formula (2);

a_{ij} – the coefficients of the limit systems, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, acquire the value of 1 or they are determined by means of a special calculation;

b_i – the coefficients of the right side, $i = 1, 2, ..., m$ determined based on statistical characteristics *of five figures*, after the exclusion of outliers and extreme data (discussed in more detail in part 3.3);

x_j – decision variables, $j = 1, 2, ..., n$ representing the efficiency indicators and the efficiency decreasing indicators.

In the model, we create conditions that need to be respected when calculating the maximum *synthetic indicator z(x)*. These conditions (through c_j, a_{ij}, b_i) express the essential relations and behaviour of a real enterprise and determine the set of permissible solutions of the optimization model. The optimal solution expresses the values of the efficiency indicators, of the efficiency decreasing indicators and of the synthetic indicator of an ideal enterprise to which the values of a real enterprise should converge. The final step of the calculations is the execution of a post-optimization analysis and the determination of the minimum synthetic indicator limit. We determine the synthetic indicator when changing a chosen element from the right side of the linear programming task, so that it does not change the basis of the optimal solution. We analyse the impact of changes to the right side b_i vector components which express the lower and upper limits of the efficiency indicators and the efficiency decreasing indicators, of the sums of the efficiency indicators, the sums of the efficiency decreasing indicators and we assess the stability of the solution. The sensitivity analysis is carried out using a classical and tolerance approach.

3. Results

3.1. Drafting the HGN1 Version

The basis for the calculations for modelling purposes was the database of financial statements of 260 non-financial enterprises in the Slovak Republic for the period from 2010 to 2012. By applying selected financial ratios in loss-making enterprises, the value of the synthetic indicator was deformed and therefore it was necessary to exclude these from the database. Loss-making enterprises represented only 10% of the total sample. Their influence on the used mathematical apparatus caused the opposite effect in the synthetic indicator. In addition to evaluating the performance of loss-making enterprises, the evaluation of results in absolute indicators is sufficient.

The database contained 55 absolute financial indicators, out of which 47 ratios were compiled. For the performance modelling purposes, we consider the database to be a representative sample, as the upper and lower quartiles of the selected financial indicators of the database are close to the upper and lower quartiles of the financial indicators of all Slovak enterprises in the Slovak Republic.

Out of 47 ratios, we created two sets of indicators (Table 1), the first set contains three efficiency indicators (x_i) and the other set contains three efficiency decreasing indicators (y_i). We interconnect these indicators or rather synthesize them into one comprehensive performance indicator. The aim is to objectively reflect the financial situation of an enterprise, its performance and to enable comparative assessment of the financial results of enterprises. In the first phase, we verified six financial ratios for the HGN1 version as shown in Table 1.

The synthetic indicator (*SI*) is to be influenced by the values of six indicators. It is defined as follows:

$$SU = \sum_{i=1}^{3} x_i - \sum_{i=1}^{3} y_i \qquad (4)$$

3.2. Drafting the HGN2 Version

By means of gradual assessment of the results from the application of the synthetic indicator computations, we arrived to the conclusion that there is a need to substitute some ratios in the HGN model in order to achieve a better demonstrative ability of the original indicators of the model, while maintaining the emphasis on a company's debt situation. By modifying the selection of the ratios (Table 2), we gradually formulated a modified version of the model named

HGN2 by the formula (2). In this version of the model, we also implant correction coefficients c_i^x, c_i^y which give effect to the impact of the efficiency indicators and the efficiency decreasing indicators on the value of the synthetic indicator:

$$SU = \sum_{i=1}^{3} c_i^x x_i - \sum_{i=1}^{3} c_i^y y_i \qquad (5)$$

For the creation of the HGN2 version, the authors processed a database of the 101 largest Slovak manufacturing, business and service enterprises. The size determination criterion was the turnover in the 2014 accounting period. The database contains 9 absolute and 6 relative financial indicators determined and calculated from the financial statements for the years 2011 to 2015. The model indicators are expressed in various units of measurement. The first two efficiency indicators are in percentage, the third one is a coefficient. The first efficiency decreasing indicator is expressed in days. The second indicator is created on a cash-flow basis and provides information about the number of years needed to cover the company's long-term liabilities. The third indicator is also expressed in days. Due to the fact that the model works with simple mathematical relations (addition and subtraction), we adjusted the indicators with different units of measurement by means of correction coefficients. In order to determine them, we proceeded from the so-called average median of the relevant indicators from the entire database. The average median is the weighted arithmetic average of the respective indicator for the $2011 - 2015$ periods, the weight being the ratio of the number of profitable enterprises in the respective year to the total number of profitable enterprises in all the years.

The correction coefficients c_i^x, c_i^y in formula (2) that give effect to the impact of the efficiency indicators and the efficiency decreasing indicators on the value of the synthetic indicator are calculated from the weighted arithmetic average of the median for the years 2011 to 2015 according to the equation:

$$X_{kjt} = P_{kjt} / / \overline{Me}_{5jt} \qquad (6)$$

where

X_{kjt} – the financial ratio (t) of the enterprise (k) recalculated by the average median of the industry (j);

P_{kjt} – the financial ratio (t) of the enterprise (k) in the industry (j);

$\overline{Me}_{5jt}$ – the weighted arithmetic average of the median of the ratio (t) in the industry (j) for a period of 5 years.

If we write the equation (6) in the form:

$$X_{kjt} = P_{kjt} * (1/\overline{Me}_{5jt}) \qquad (7)$$

then the formula $1/\overline{Me}_{5jt}$ represents the correction coefficients that give effect to the impact of the efficiency indicators and the efficiency decreasing indicators on the value of the synthetic indicator and we name them c_i^x, c_i^y, where $i = 1, 2, 3$. The financial results of an enterprise and of the corresponding industry are related to a certain extent. Unexpected impacts of the external environment (such as the adverse effects of the global financial and economic crisis) affect not only the given business entity, but they affect the results of all enterprises within the industry to almost the same extent.

3.3. Ratio Outliers in the HGN Model Versions

We defined a database consisting of six ratio indicators characterized by means of five numbers (a five-number summary) (Terek, 2013; Barnett and Lewis, 1994) for both versions of the model. It is a characterization of the distribution with the highest value (*MAX*), the upper quartile (*UQ*), the median (*Me*), the lower quartile (*LQ*) and the lowest value (*MIN*). Such characterization provides an overall view of the statistical distribution. In all the researched data files and, therefore, in the enterprises' database, there are data that differ significantly from the other data, indicating the existence of a source of error. We call these data outliers and define it as the data that seem to be inconsistent with the other data in a dataset and their economic origin is in the customer-supplier relationship setting of enterprises. It can only cause complications and incorrect direction of the analysis to take the outliers into consideration. Based on statistical characteristics using five numbers (Terek, 2013; Barnett and Lewis, 1994) we have the following options in defining the outliers in both versions of the model:

a) the outliers are not taken into account/ignored,
b) the outliers are eliminated (and so are the extreme values),
c) only the extreme values are eliminated.

The further analyses combine the options b) and c).

Determining the outliers means to evaluate the integrity of a data set. We use a method based on the *interquartile range $R_Q = UQ - LQ$*.

A value is an outlier when (Terek, 2013; Barnett and Lewis, 1994):
- it is $\geq UQ + 1.5\, R_Q$,
- it is $\leq LQ - 1.5\, R_Q$.

In specific analyses, the decision on which data shall be labelled as outliers depends on the analyst's consideration. It is also common to distinguish the so-called far outliers, i.e. the values that are further from the quartiles than $3\, R_Q s$.

An outlier is within the interval ($UQ + 1.5\, R_Q$, $UQ + 3\, R_Q$) or within the interval ($LQ - 3\, R_Q$, $LQ - 1.5\, R_Q$). *An extreme value* is within the interval ($UQ + 3\, R_Q$, ∞) or within the interval ($-\infty$, $LQ - 3\, R_Q$).

3.4. Minimum Limits of the Synthetic Indicator

The application of the model versions (HGN1, HGN2) in the environment of economic entities, in this case, of profitable non-financial enterprises, also requires determining the nature of the synthetic indicator in terms of performance. It logically follows from the method of calculation and the content of each indicator that the best enterprise should be the one with the highest value of the synthetic indicator. An analysis of this issue requires the use of an adequate mathematical apparatus. At this stage of work, we applied a linear programming apparatus.[2] In order to determine the optimal synthetic indicator intervals, we use a post-optimization analysis in linear programming tasks with both classical and tolerance approaches.

Classical Approach

We observe the calculated changes Δb (calculations are made by the software product QMwin) in the components of the b_i vector (other limits remain unchanged, $\Delta b_{j\neq i} = 0$). We examine whether these changes are permissible in terms of the optimal basis of the original task and what new solution corresponds to them. We set the permissible interval of the b_i component changes so that, under other unchanged conditions, the basis for the optimal solution of the linear programming task is maintained. Although the base of the optimal solution does not change, the basic variables and the value of the objective function (obtained in the optimal solution) change with the mentioned changes, thus we shall get a new optimal solution. We calculate the optimal solution in the resulting Simplex tableau according to the equation (8):

$$x = B^{-1}b \tag{8}$$

where
    x – the vector of the basic components of the optimal solution,
    B^{-1} – the inverse matrix of the optimal base,
    b – the original vector of the right side.

Any change in the components of the right-side vector is reflected in the solution and the objective function value, resulting from the correlation of the equation (9):

$$B^{-1}(b + \Delta b) = B^{-1}b + B^{-1}\Delta b \geq 0$$
$$x + B^{-1}\Delta b \geq 0 \tag{9}$$

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[2] Currently, the authors of the model are elaborating different types of matrix calculations (based on the appropriate layout of input and output indicators), using regression analysis and data envelopment analysis in order to specify the relevant values of the synthetic indicator in relation to business performance.

From the equation (9), we calculate the lower (l) and upper (u) limits of the change $\Delta b_i \in \langle l, u \rangle$. We calculate the lower ($L = b_i + l$) and upper ($U = b_i + u$) limits for the changed right side. We designate the appropriate solution and objective function values x_D, x_H and z_D, z_H. The intervals for the synthetic indicator are as follows: synthetic indicator $\in \langle z_D, z_H \rangle$.

Tolerance Approach

Tolerance approach to sensitivity analysis in linear programming (unlike the classical sensitivity analysis) deals with changes in multiple (not only one) coefficients of the objective function, right side or the matrix of technological coefficients. These changes are considered to be simultaneous and independent. The tolerance approach provides a percentage of maximum tolerance within which all or only some values of the listed coefficients may move simultaneously and independently of the original values, while the original set of basic variables in the optimal solution remains unchanged (Brezina, 1990). Tolerance sensitivity analysis is considered in the case of changes to the right-side elements. Due to the vastness of the theoretical background, we only mention the main relations that are sufficient for practical application. The change in the components of the right-side vector is noted down by the formula (10):

$$b_i + \beta_i b_i \tag{10}$$

We assume that this change in terms of the optimum basis of the original task is permissible if the absolute value of each parameter β_i does not exceed the non-negative number p: $|\beta_i| \leq p$ i.e. each number β_i meets the condition $-p \leq \beta_i \leq p$. Such number p is called *the permissible tolerance* for the right side change.

One of the goals of the tolerance approach is to define the maximum tolerance p^* for changes to the right side elements, where p is the tolerance allowed if $p \leq p^*$. The expression $p^*.100\%$ is called the maximum percentage tolerance.

The following applies for the maximum tolerance for changes to the right-side coefficients:

$$p^* = \begin{cases} \min_{k=1,\dots,m} \left[\dfrac{B_k^{-1}b}{\sum_{j=1}^{n}\left|B_{kj}^{-1}b_j\right|} \right] & \text{if } \sum_{j=1}^{n}\left|B_{kj}^{-1}b_j\right| \neq 0 \\ +\infty & \text{or else.} \end{cases} \tag{11}$$

The data needed to calculate the equation (11) are available in the original and optimal Simplex tableau. The numerator in the equation (11) is the optimal

solution to the original task. If any denominator in the equation (11) equals zero, then the corresponding value is $+\infty$. If $p^* = 0$, the optimal solution is degenerate. However, there are situations in which the values of some of the right-side limits are known to be accurate and there is no reason for them to change. As a result, we get a lower value for the denominator in the equation (11) and hence a higher value p^* (since in that case the corresponding b_i equals zero, $b_i = 0$). This conditions an important feature that we get a higher maximum tolerance for the remaining limits with some accurately specified right-side coefficients.

In the following text, we compare the results of the classical and tolerance approach to the sensitivity analysis in the HGN1 model version. In both cases it is important to determine the outliers and thereby to set the input conditions c_j, a_{ij}, b_i of the linear programming task (3). As we have already mentioned, the determination of the conditions for defining the outliers is not generally and strictly given. It depends on the character of the given data set and considerations of the analyst who performs the computation. We chose the calculations excluding all and accepting some of the outliers. We also note that the classical sensitivity analysis deals with a change in one element on the right side, the other elements remain unchanged (changes do not occur simultaneously and independently). Tolerance analysis considers changes in several elements of the right side and the changes are taking place simultaneously and independently.
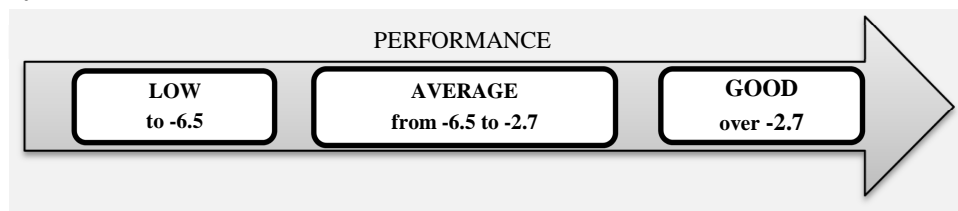
Implementation of the Classical Approach

We conducted the classical approach to sensitivity analysis with the exclusion and acceptance of some of the outliers by means of the equation (9).

Note that the synthetic indicator emphasizes the share of the company's debt problems through the indicator y_2 (repayment time of foreign resources). This results in the fact that the lower the value of the synthetic indicator, the greater the probability of an increase in the company's financial problems.

The Figure 1 shows performance bands according to the classical approach to sensitivity analysis in linear programming tasks.

Figure 1

Performance Bands According to the Classical Approach Depending on the HGN1 Synthetic Indicator Value

Source: Own work.

Implementation of the Tolerance Approach

Calculations are performed according to the equation (11) using the software product QMwin. The matrix B^{-1} is generally made up of vectors that correspond to the original basic variables. We get it in the QMwin program by clicking on the *Step* command repeatedly until the following information appears on the screen: *This is the optimal solution*. We then copy the matrix into an Excel file using the *Edit-Copy-Table* command and make the necessary adjustments and calculations according to (11).

In Tables 3 and 4 we present the results of the tolerance sensitivity analysis with the exclusion or acceptance of some of the outliers in the HGN1 model. We distinguish three types of linear programming tasks which are analysed in more detail in the monograph (Hyránek, Grell and Nagy, 2014). They take into consideration the effects on the optimal SI interval by setting the right-side limits of the linear model. The tolerance approach for the right-side limits provides the threshold percentage at which the right-side coefficients can fluctuate simultaneously and independently while preserving the same basis. If the values of some of the right-side limits are fixed, we obtain lower values for the denominators in the equation (11), since the corresponding $b_i = 0$, and thus larger values of p^*. This conditions an important feature that we get a higher maximum tolerance for the remaining limits for some fixed right-side coefficients. We compare the resulting intervals of both approaches.

Exclusion of the Outliers

T a b l e 3

Classical and Tolerance Approach with Exclusion of the Outliers
Optimal Intervals of the SI

Task type	Approach to sensitivity analysis	
	Classical	Tolerance
I.	<0.7088; 4.2659>	<3.6438; 3.6795> $p^* = 0.5\%$
II.	<0.5189; 1.1775>	<0.7584; 0.7655> $p^* = 0.5\%$
III.	<–6.4547; –0.8423>	<–0.8993; –0.8981> $p^* = 0.1\%$
The resulting interval	<–6.4547; –0.8423> ∨ <0.5189; 4.2659>	<–0.8993; –0.8981> ∨ <0.7584; 0.7655> ∨ ∨ <3.6438; 3.6795>

Source: Own work.

The resulting intervals overlap in both the Tables 3 and 4. Naturally, they are narrower because the changes to the right-side are happening simultaneously and independently. That is why for the task type II. (Table 3) we indicate an example of a combination of classical and tolerance analysis. We are only considering changing the two limits of the right side of the linear programming task (the

upper limit of the returns on equity and operating cost indicators), we consider the others to be fixed. This increases the maximum tolerance to $p^* = 1.1\%$ and thus extends the optimal synthetic indicator interval <0.7539; 0.77>. Some denominators in the equation (11) may equal 0, because many original values of b_i equal to zero or the original matrix is very sparse and therefore the inverse matrix B^{-1} has similar properties. Thus, the interval limits of the permissible simultaneous changes to the elements of the right side are infinite.

Accepting some of the Outliers

T a b l e 4

**Classical and Tolerance Approach with Acceptance of the Outliers –
Optimal Intervals of the SI**

| Task type	Approach to sensitivity analysis	
	Classical	Tolerance
I.	<3.0603; 12.6333>	<3.582; 4.6292> $p^* = 12.8\%$
II.	<1.0857; 1.3568>	<1.1298; 1.2252> $p^* = 4.05\%$
III.	<–2.6608; 1.1702>	<–0.9973; –0.8001> $p^* = 10.96\%$
The resulting interval	**<–2.6608; 1.3568> ∨ <3.0603; 12.6333>**	**<–0.9973; –0.8001> ∨ <1.1298; 1.2252> ∨ ∨ <3.582; 4.6292>**

Source: Own work.

Analogously, we can determine the performance intervals based on tolerance analysis.

F i g u r e 2

**Performance Bands According to the Tolerance Approach Depending on the HGN1
Synthetic Indicator Value**
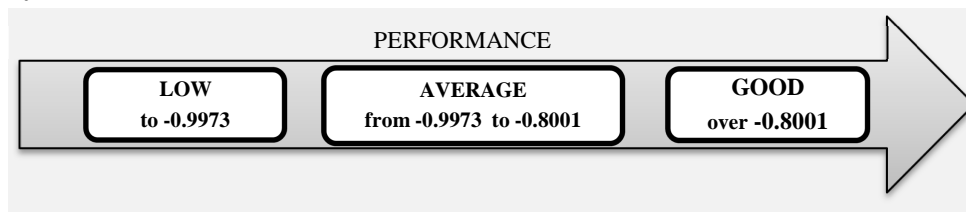
Source: Own work.

4. Discussion

For discussion purposes, the knowledge gained from the application of the HGN model may be summarized into two sets of problems, namely the characterization of the synthetic indicator and the precision of the mathematical apparatus.

As a final indicator, the synthetic indicator expresses the interconnection of the efficiency indicators and the efficiency decreasing indicators. The HGN can be considered to be a comprehensive model that objectively reflects the financial situation and performance, focusing on the problems of many enterprises at the present time, namely indebtedness. From this point of view, we considered important to implement into the model an influence focused on the ability, or rather the inability to repay debt expressed by the indicator *repayment time of foreign resources* (y_2), the measurement unit being the number of years. This indicator appears to be a limiting factor in some enterprises from the database under review. In the HGN2, we eliminate the problem by creating correction coefficients.

To a certain extent, a limiting factor is the content of the indicator *repayment time of foreign resources* expressed in number of years, due to the content of all foreign resources, including the short-term ones. We have eliminated this limiting factor for performance assessment by adjusting the indicator in the HGN2 by narrowing its content to the *repayment time of long-term foreign resources*. In most enterprises from the database, the long-term foreign resources represent predominantly long-term bank loans, so we have achieved a more objective repayment time from profits and depreciation (cash flow).

In order to increase the model's demonstrative ability, we have modified some of the efficiency decreasing indicators contained in the synthetic indicator, but we do not have to consider this to be the final state of the researched problem.

In connection with the specification of the mathematical apparatus, the advantage of the tolerance approach is its greater universality and consideration of the interrelationships between the individual elements of the linear programming task. This approach appears to be useful in case of an interconnection with classical post-optimization analysis. In practice, we encounter cases where we do not need to detect deviations for some elements of the right side (3) and the space for simultaneous and independent changes increases for the tolerance sensitivity analysis of other elements.

Currently, the model can be fully utilized in the HGN1 version and in the HGN2 version it can be used in the practice of profitable enterprises. The nature of these indicators does not anticipate evaluation of loss-making enterprises, which was not even the ambition of the model's authors at this stage of research. In the further development of the model, we shall also try to select the indicators that may link e.g. the effectiveness of business innovations with the performance (Chajdiak, Glatz Ďurechová and Mišota, 2016), or to show the interrelation of the macro and micro views on the performance of a business (Mišota, 2013). The authors also intend to extend the scientific examination with the industry point of view.

Conclusion

Selected ratios for the presented model purposes are in line with the overall strategic goals of an enterprise. Three efficiency indicators and three efficiency decreasing indicators reflect the significant requirements for the company's financial performance. With the three selected efficiency indicators, we have created one aggregate efficiency indicator and the sum of the three efficiency decreasing indicators created one aggregate efficiency decreasing indicator. We have decreased the value of the aggregate efficiency indicator by the value of the aggregate efficiency decreasing indicator and we have obtained a synthetic indicator implying the impact of all six financial ratios.

The fact that the calculation of the model's indicators is not conditioned by the knowledge of inside business information may be considered as strength of the model. The absolute data contained in the ratios are generally available to external evaluators in the registers of accounts.

It logically follows from the method of calculation and the content of the model's indicators that the best enterprise should be the one with the highest value of the synthetic indicator. We deal with the analysis of credibility of such a synthetic indicator result. We apply the classical and tolerance approaches to sensitivity analysis in linear programming tasks and we take into account the possibility of accepting the outliers.

In the following stages of work, we shall focus on further refining and developing the HGN model by solving problems in relation to selecting financial ratios of efficiency and efficiency decreasing indicators, identifying outlying data for these indicators, and choosing model mathematics.

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